CPSC 532L: Models of Strategic Behaviour	**Spring 2023**

Assignment #3

Due January 24^{th}, 2023, 1:00pm

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1 Maxmin and Minmax

Problem 1.1. [**17 points**] Consider a game with n players. Denote the maxmin strategy for player i as $\bar{s}_i$ and the maxmin value of i as $\bar{v}(i)$. Denote the minmax strategy of some agent $j \neq i$ against i as $\underline{s}_{j,i}$ and the minmax value of i as $\underline{v}(i)$. Denote by $\underline{s}_{-i,i}$ the minmax strategy profile of all players other than i, denoted by $-i$, against i.

(a) [**6 pts**] Prove that for all games, the maxmin value of player i is no greater than the minmax value of player i, i.e. $\bar{v}(i) \leq \underline{v}(i)$.

(b) [**5 pts**] Prove that in all two-player games the maxmin value of player i is equal to the minmax value of player i, in other words $\bar{v}(i) = \underline{v}(i)$. Hint: you can use the minmax theorem, but note that it only applies to two-player *zero-sum* games.

(c) [**6 pts**] Now we demonstrate that the result in (b) does not apply to n-player games with $n > 2$, by the following counterexample. Consider the following three-player game; player 1 chooses the row, player 2 chooses the column, and player 3 chooses the matrix.

If P_3 chooses U:

		P_2: L	P_2: R
P_1	T	$(1, 4, 0)$	$(1, 2, -2)$
	B	$(5, 6, 0)$	$(5, 5, 0)$

If P_3 chooses D:

		P_2: L	P_2: R
P_1	T	$(3, 3, 0)$	$(2, 6, 0)$
	B	$(3, 4, -2)$	$(5, 3, 0)$

Compute $\bar{v}(3)$, and show that $\bar{v}(3) < \underline{v}(3)$. (Hint: the minmax value is hard to calculate for this game, but you don't need to compute it exactly in order to show that $\bar{v}(3) < \underline{v}(3)$.)

2 Rationalizability, Correlated Equilibria

Problem 2.1. [**13 points**] Consider the following two-player game:

		P_2: D	P_2: E	P_2: F
P_1	A	$(9, 10)$	$(3, 5)$	$(5, 4)$
	B	$(1, 6)$	$(17, 9)$	$(8, 5)$
	C	$(0, 5)$	$(2, 6)$	$(6, 13)$

(a) [**3 pts**] Find a pure strategy a_2 for player 2 and prove that it is not rationalizable.

(b) [**2 pts**] Find a different pure strategy a_1' of player 1 and prove that it *is* rationalizable.

(c) [**8 pts**] Find a correlated equilibrium of the game where player 1 achieves an expected payoff of 14. As randomizing devices, you have three publicly observable, fair coins: a nickel, a dime and a quarter. You may not use any other randomizing devices.

3 Dominance and Mixing

Problem 3.1. [6 points] Prove or disprove the claim "If finite game G has no strictly dominated pure strategies, then G has no strictly dominated mixed strategies."

4 Altruism and Evolutionary Stability

We'll be studying the work of Bester and Guth's paper, "Is Altruism Evolutionarily Stable?"

Problem 4.1. [10 points] Suppose P_1 and P_2 are playing a game, where they each pick a number greater than or equal to 0. P_1 chooses the number x and P_2 chooses the number y. Bester and Guth studied these *evolutionary success functions*:

$$U_1(x, y) = x(ky + m - x)$$
$$U_2(x, y) = y(kx + m - y)$$

subject to $-1 < k < 1$ and $m > 0$. Recall that these are not the same as *subjective utility functions* which they define as follows:

$$V_1 = \alpha U_1 + (1 - \alpha)U_2$$
$$V_2 = \beta U_2 + (1 - \beta)U_1$$

In this game, both players know the value of α and β.

(a) **[5 pts]** For egoists, the altruism parameters are $\alpha = 1$ and $\beta = 1$. Consider two egoists playing against one another. As a function of m and k, what are the equilibrium values x and y that P_1 and P_2 should play? Show your work.

(b) **[5 pts]** For altruists, the altruism parameters can be anywhere in the range $0.5 \leq \alpha < 1$ and $0.5 \leq \beta < 1$. Let $x^*(\alpha, \beta)$ and $y^*(\alpha, \beta)$ denote the equilibrium values of x and y that P_1 and P_2 should play knowing that their preference parameters are α and β respectively, and let $R(\alpha, \beta) = U_1(x^*(\alpha, \beta), y^*(\alpha, \beta))$. We will not ask you to calculate these equilibrium values yourself, but they are derived from V_1 and V_2, and are a function of α, β, m and k.

We will consider the preference parameter α^* to be *evolutionarily stable* if

$$R(\alpha^*, \alpha^*) \geq R(\alpha, \alpha^*) \text{ for all } \alpha, \text{ and}$$
$$R(\alpha^*, \alpha^*) > R(\alpha^*, \alpha) \text{ for all } \alpha \neq \alpha^*$$

Given that the partial derivative

$$\frac{\partial R(\alpha, \beta)}{\partial \alpha} = 0$$

is equivalent to

$$\alpha = [4\beta + k(2 - k)]/[4(\beta + k)],$$

find the two potential evolutionarily stable preference parameters α^* as a function of k. Show your work. (In fact, only one of these preference parameters is actually evolutionarily stable, and only in the case that $k > 0$. But it's still pretty cool to see that choosing a value of $\alpha < 1$ can result in an evolutionarily stable outcome - so in this case, altruism is an evolutionarily stable strategy.)

5 Bounded Iterative Reasoning

Problem 5.1. [**5 points**] Recall the Beauty Contest Game from the lecture. Given n players, each player chooses a number $s_i \in \{1, 2, 3, ..., 99\}$. The player who names the integer closest to $\frac{2}{3}$ of the average wins. Ties are broken at random.

(a) [**1 pt**] What is the Nash Equilibrium of this game?

(b) [**2 pts**] Under level-k model, calculate level 1 and level 2 responses.

(c) [**2 pts**] Under cognitive hierarchy model, say that each player assumes a uniform distribution over lower-level players. Calculate level 1 and level 2 responses.

6 Quantal Response Equilibrium

Problem 6.1. [**6 points**] Consider the following two-player **symmetric** game and its equilibria:

		P_2	
		L	R
P_1	U	$(3, 3)$	$(0, 0)$
	D	$(0, 0)$	$(1, 1)$

(a) [**1 pt**] Identify all (pure and mixed strategy) Nash Equilibria of this game.

(b) [**3 pts**] Say that player 1 believes that player 2 plays L with probability p, find the QRE using Logit specification. Show your work, leave it as an equation(s) in terms of λ and p.

(c) [**2 pts**] Solve for p (analytically or numerically) for $\lambda \in \{0, 10, 100\}$. Discuss the results.

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

1.
2.
3.

List any non-course materials you refered to:

1.
2.
3.

Fill in this page and include it with your assignment submission.