CPSC 532L: Models of Strategic Behaviour **Spring 2023**

Assignment #2

Due January 31^{st}, 2023, 1:00pm

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"If we don't know what we are doing, the enemy certainly can't anticipate our future actions!" – United States Army on Twitter

1 2x2 Games Revisited

Problem 1.1. **[4 points]** Recall the following game from assignment #1:

		P_2	
		C	D
P_1	A	$(-21, 0)$	$(10, 10)$
	B	$(6, 6)$	$(0, -21)$

Find all mixed-strategy Nash equilibria.

Problem 1.2. **[3 points]** Recall the following game from assignment #1:

		P_2	
		L	R
P_1	T	(a, e)	(b, f)
	B	(c, g)	(d, h)

What (in)equalities must hold for a Nash equilibrium to exist where player 1 only plays T but player 2 mixes over L and R (playing L with probability p)?

2 Nash Equilibrium in 3-Player Games

Problem 2.1. **[8 points]** Consider the following game:

	D	E
A	$0, 1, 0$	$3, 1, 2$
B	$0, 3, 1$	$2, 3, 1$
C	$2, 3, 0$	$3, 2, 1$

	D	E
A	$3, 1, 1$	$2, 2, 3$
B	$1, 2, 3$	$2, 3, 2$
C	$0, 2, 1$	$3, 2, 2$

Figure 1: A three-player normal form game. Player three's choice of L or R corresponds to the left and right tables.

(a) **[3 pts]** List all of the pure-strategy Nash equilibria of the game in Figure 1.

(b) **[5 pts]** Characterize the set of Nash equilibria in which player two does not play a pure strategy.

3 Linear Programming & Nash Equilibrium

Problem 3.1. **[5 points]** Consider the following game:

	D	E
A	$3,1$	$1,2$
B	$0,3$	$3,1$
C	$2,3$	$2,1$

(a) **[3 pts]** Construct the linear program that the support enumeration method would use to find a full-support Nash equilibrium of this game (i.e., an equilibrium where every action is played with positive probability).

(b) **[2 pts]** Show that this game has no full-support Nash equilibrium, by showing that your linear program is infeasible.

4 Reevaluation Evaluation

Problem 4.1. **[10 points]** We'll be working with Rock-Paper-Scissors. Feel free to write this out by hand or write code.

$$\text{A} = \begin{bmatrix} (0,0) & (-1,1) & (1,-1) \\ (1,-1) & (0,0) & (-1,1) \\ (-1,1) & (1,-1) & (0,0) \end{bmatrix}$$

(a) **[2 pts]** With the following strategies for each player, what is the expected payoff for each player?

$s_1 = [1/2,\ 1/2,\ 0]$
$s_2 = [1/2,\ 0,\ 1/2]$

(b) **[1 pt]** Calculate the entropy of these two strategies.

$s_1 = [1/2,\ 1/2,\ 0]$
$s_2 = [1/2,\ 0,\ 1/2]$

(c) **[2 pts]** If we augment the RPS game to add in copies of two of the agents what is the expected payoff for each player? Use the supplied strategies in your calculation.

$$\text{A'} = \begin{bmatrix} (0,0) & (1,-1) & (-1,1) & (-1,1) & (1,-1) \\ (-1,1) & (0,0) & (1,-1) & (1,-1) & (0,0) \\ (1,-1) & (-1,1) & (0,0) & (0,0) & (-1,1) \\ (1,-1) & (-1,1) & (0,0) & (0,0) & (-1,1) \\ (-1,1) & (0,0) & (1,-1) & (1,-1) & (0,0) \end{bmatrix}$$

$s_1 = [0.5,\ 0.35,\ 0.05,\ 0,\ 0.1]$
$s_2 = [0.45,\ 0.05,\ 0.35,\ 0.1,\ 0.05]$

(d) **[2 pts]** Check that any solution of the following form is an equilibria of the augmented game (A' in the previous question) and what values of alpha/beta makes this strategy have the highest entropy?

Let $\alpha \in [0,1]$, and $\beta \in [0,1]$

nashStrategy = $[\frac{1}{3}, \frac{1-\beta}{3}, \frac{1-\alpha}{3}, \frac{\alpha}{3}, \frac{\beta}{3}]$

(e) **[3 pts]** The lecture about reevaluating evaluation focused on the agent-vs-agent scenario, what about the agent-vs-task scenario complicates the story we've told so far?

Hint: In AvT, raw results are represented as an $(m \times n)$ matrix S: rows are agents, columns are tasks, entries are scores of agent i on task j (e.g. accuracy or total reward). In particular, think about the graph structure of the evaluation game. The graph in the agent-vs-agent scenario is a fully-connected graph. What about the graph is different in the agent-vs-environment scenario; why does this matter and how does it impact the scores matrix?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

1.

2.

3.

List any non-course materials you refered to:

1.

2.

3.

Fill in this page and include it with your assignment submission.