CPSC 532L: Models of Strategic Behaviour **Spring 2023**

Assignment #10

Due: April 11th, 2023, 1:00 pm

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1 Vickrey-Clarke-Groves

Problem 1.1. [30 points] The VCG mechanism does not violate the Myerson-Satterthwaite Theorem because it is not budget balanced for general quasilinear preferences. But this seems like an easy enough problem to solve—we can just evenly redistribute any money that was collected by the mechanism (or, tax all agents equally if the net payment to the agents was positive). Below is a proposed, budget-balanced version of the VCG Mechanism. It converts what was p_i into a temporary variable t_i. Then, the new payments p_i contains an equal redistribution of the sum of the original payments.

The *Budget-Balanced VCG Mechanism* is a direct mechanism $M(\hat{v}) = (\chi(\hat{v}), p_1(\hat{v}), \ldots, p_n(\hat{v}))$, where

- $\chi(\hat{v}) = \arg\max_{x \in X} \sum_{i \in N} \hat{v}_i(x)$, and
- $t_i(\hat{v}) = \max_{x \in X_{-i}} \sum_{j \neq i} \hat{v}_j(x) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$
- $p_i(\hat{v}) = t_i - \frac{1}{n} \sum_i t_i(\hat{v})$

(a) [**10 points**] Show that this mechanism is not incentive compatible. One option is to prove this directly. Alternately, you can give two valuation functions for an agent i (one that gives his true valuation (v_i), and alternative one (v_i')) and a set of declared valuation functions ($\hat{v}_j, \hat{v}_k, \ldots$) for as many agents other than i that you need, and show that if all other agents declare these valuations, the utility for agent i is higher if he declares v_i' instead of v_i.

(b) [**10 points**] Although our first mechanism failed, we can use a similar idea to make VCG budget-balanced *ex-ante*. Assume that bidders valuations v_i are randomly drawn from some joint commonly known distribution. Consider the following mechanism:

Ex-Ante Budget-Balanced VCG Mechanism is a direct mechanism $M(\hat{v}) = (\chi(\hat{v}), p_1(\hat{v}), \ldots, p_n(\hat{v}))$, where

- $\chi(\hat{v}) = \arg\max_{x \in X} \sum_{i \in N} \hat{v}_i(x)$, and
- $t_i(\hat{v}) = \max_{x \in X_{-i}} \sum_{j \neq i} \hat{v}_j(x) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$
- $p_i(\hat{v}) = t_i(\hat{v}) - \frac{1}{n} \sum_j \mathbb{E}_v[t_j(v)]$

Prove that truth-telling is a dominant strategy in this new mechanism.

(c) [**10 points**] Show that this mechanism is *ex-ante* budged-balanced on expectation (i.e. expected total payment by all agents is zero).

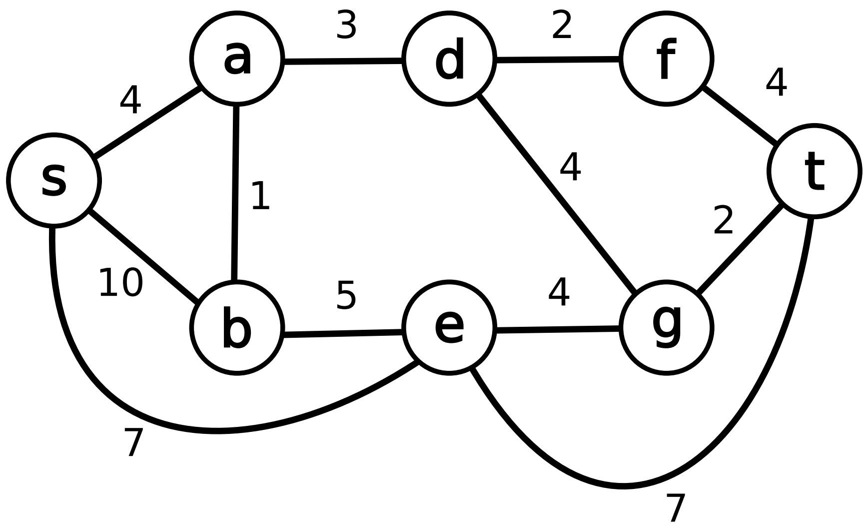

Figure 1: Reported costs for a fun routing network.

Problem 1.2. [5 points] A mechanism designer wishes to find the lowest-cost route from node s to node t in the network of Figure 1. Each edge is controlled by a single agent. Compute the VCG outcome and each agent's payments if every agent declares the costs labelled on the edges.

2 Revenue Maximization in an Unlimited Supply Setting

Problem 2.1. [10 points] Photographer's Dilemma: Suppose you just took a trip to Northern BC and took pictures of some bird species that were previously thought to be extinct on your new iPhone. You're certain that no one else has these pictures, and that various research institutes and nature-focused publications would love to purchase copies of the pictures that you just took.

(a) [**1 point**] These pictures you took can be considered a good with unlimited supply. If you had to explain to a twelve-year old why that is the case, what would you say?

(b) [**4 points**] Before you start selling these pictures, you do some market research and find that there are 20 buyers interested in buying, and they each have a distinct valuations of integers ranging from 1 to 20. What is the price you could charge for the pictures that would give you the highest revenue? What would be the resulting revenue? Assume that a buyer will buy the pictures if they have a valuation that is equal to or higher than the price you set.

(c) [**5 points**] Now suppose that you're in an alternate universe, one where you haven't done the preliminary market research to determine the distribution of buyer valuations (and for the sake of this question, assume that you are unable to do preliminary market research due to buyer-protection laws). You've read about profit extractors, and want to apply them to your situation; to get a target revenue R, you talk with a friend who has been in a similar situation as you and use that friend's advice to set an R. You know that this R you've set is less than the maximum amount of revenue, but you're worried that as you go through the profit extracting algorithm, buyers will misrepresent their valuations and lie to you. Explain to your alternate self why the buyers won't lie. TL;DR - why is the profit extractor algorithm truthful?

3 Quadratic Voting

We saw in Prayus's talk, how a company might want to set its prices according to how much different demographics want to pay. Imagine a company wants to conduct an experiment to find out this information.

(a) [**1 point**] Why might asking people for their willingness to pay not work in this scenario?

(b) [**1 point**] One approach to get people to be truthful would be to randomly sample prices, and see if a focus group is willing to buy at that price. Assuming the focus group only considers the utility for the good, and the price they pay, why might this be better? (i.e. they do not try mislead the company to influence the company's future decisions)

(c) [**1 point**] Let's now consider an agent purchasing a divisible good. Here the agents utility is $xa - c(x)$, where x is the amount of good they purchase, $a > 0$ is how much they value the good, and $c(x)$ is the cost of purchasing x amount where $c(x) = \frac{1}{k}x^k$ for some k. If the agent wants to maximize their utility, what amount x of the good should they buy? Express your answer in terms of k.

$$x = \dots$$

(d) [**1 point**] What is special about the value of $k = 2$? And how does this relate to the earlier questions, i) and ii)?

(e) [**2 points**] We now consider a scenario in which an agent has a budget of 100 tokens that they can spend on two goods, A and B. To get x amount of A it costs x^2 tokens, and to get y amount of B it costs y^2 tokens. The agent values these goods differently valuing A at $a > 0$ and B at $b > 0$, and the agent must spend all their tokens. If the agent wants to maximise their utility: $xa + yb$, subject to $x^2 + y^2 = 100$, what should their choice of x and y be?

(f) [**2 points**] Given two algebraic equations in terms of a and b for x and y

$$x = \dots$$

$$y = \dots$$

What do you notice about the ratio of the amount of goods acquired $x : y$ compared to $a : b$?

(g) [**2 points**] How does this relate to Quadratic Voting?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

1.

2.

3.

List any non-course materials you refered to:

1.

2.

3.

Fill in this page and include it with your assignment submission.