CPSC 532L: Models of Strategic Behaviour	**Spring 2023**

Assignment #1

Due January 24^{th}, 2023, 1:00pm

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*"*All right, *he said.* Let's play poker. No, *I answered.* I hate cards. I always lose*...* Who's talking about cards? *thus Ervinke.* I was thinking of Jewish poker. You think of a number, I also think of a number*, Ervinke explains.* Whoever thinks of a higher number wins. This sounds easy, but it has a hundred pitfalls. All right, *I agreed.* Let's try. *We plunked down five piasters each, and, leaning back in our chairs began to think of numbers. After a while Ervinke signaled that he had one. I said I was ready.* All right, *thus Ervinke.* Let's hear your number. Eleven, *I said.* Twelve, *Ervinke said, and took the money. I could have kicked myself, because originally I had thought of Fourteen, and only at the last moment had I climbed down to Eleven, I really don't know why.* Listen. *I turned to Ervinke.* What would have happened had I said Fourteen? What a question! I'd have lost...*" – Ephraim Kishon in Jewish Poker*

1 Normal-Form Games

Problem 1.1. [6 points] Consider the following game:

		P_2: C	P_2: D
P_1	A	$(-21, 0)$	$(10, 10)$
	B	$(6, 6)$	$(0, -21)$

Where the first number in each square is the payoff of P_1 and the second number is P_2's payoff.

(a) **[2 pts]** Find all Pareto optimal pure strategy profiles.

(b) **[2 pts]** Find the pure strategy Nash equilibria.

(c) **[2 pts]** Which of the above equilibria do you prefer? Suppose P_2 has decided to play according to one of the equilibria from question (b) (but you do not know which.) What would you play as P_1?

Problem 1.2. [5 points] Consider the following game:

		P_2: L	P_2: R
P_1	T	(a, e)	(b, f)
	B	(c, g)	(d, h)

(a) **[2 pts]** Consider the claim "any weakly dominant action a_i for player i must be played in all Nash equilibria." Disprove this claim by example. Specifically, specify values for the game above such that T is a weakly dominant strategy, but BR is a Nash equilibrium.

(b) **[3 pts]** What (in)equalities must hold for the game to have exactly one pure strategy Nash equilibrium (BL), which is Pareto dominated by a pure strategy profile?

2 Bertrand Duopoly

Problem 2.1. [6 points] Two firms produce identical goods, with a production cost of c per unit. Each firm sets a nonnegative price (p_1 and p_2). All consumers buy from the firm with the lower price, if $p_i \neq p_j$. Half of the consumers buy from each firm if $p_i = p_j$. D is the total demand. Firm i's profit is:

- 0 if $p_i > p_j$ (no one buys from firm i);
- $D(p_i - c)/2$ if $p_i = p_j$ (half of customers buy from firm i);
- $D(p_i - c)$ if $p_i < p_j$ (all customers buy from firm i).

(a) **[3 pts]** Identify a *strictly* dominated strategy in this game, and explain why the strategy is strictly dominated.

(b) **[3 pts]** Identify a *weakly* dominated strategy in this game that is not strictly dominated, and explain why the strategy is only weakly dominated.

3 Congestion and Potential Games

Choose one of the following to answer:

Problem 3.1. [10 points] As we all know, choosing where to go out on a Saturday night can be a nightmare. Let's imagine the Facebook group you are a part of is deciding between going out to Fringe or Colony. Denote such a group as a possibly incomplete graph $G = (V, E)$, where each vertex $\{1, \ldots, n\} \in V$ in this game is a member of the group. Each member chooses either Fringe or Colony ($A_i = \{F, G\}$, or equivalently $A_i = \{0, 1\}$) based on where most of their friends will be partying. Represent member i's choice by $b_i \in \{0, 1\}$ and $N(i)$ as the set of neighbors of i in G. The loss $L_i(\mathbf{b})$ for player i is the number of neighbors that she disagrees with:

$$L_i(\mathbf{b}) = \sum_{j \in N(i)} |b_i - b_j|,$$

where $\mathbf{b} = (b_1, \ldots, b_n)$. Define $\Phi(\mathbf{b}) = {}^1\!/_2 \sum_i L_i(\mathbf{b})$.

(a) **[3 pt]** Describe what $\Phi(\mathbf{b})$ means in this setting in words and show that it is a potential function.

(b) **[7 pts]** We know that in a potential game, a series of improving moves, where exactly one player moves in each round, terminates in a pure Nash equilibrium. Now, consider what happens when all players that would improve their payoff by switching their action, do so *simultaneously.* Show that such a process will converge to a cycle of period at most two (i.e., it either stabilizes or alternates between two vectors).

Problem 3.2. [10 points] *Ring Ring!* Roger Baddel, the commissioner of a new hockey league, the XHL, is calling you in a panic. Each of the ℓ teams in his new league has to settle on a city location! As there are few cities in Canada with enough people that can even host an XHL team he gives you a list of cities, denoted $\mathcal{C}$. Being a young commissioner, however, he is worried that the only outcome will be some ugly mixed strategy profile. The horror!

Let u_j be the potential profit (i.e., number of hockey fans) from city $j \in \mathcal{C}$. If k teams select city j, they evenly split the number of fans in the city and obtain a utility of ${}^{u_j}\!/_k$. Let $s = (c_1, \ldots, c_\ell)$ describe the strategy profile where c_i is the city selected by team i. Furthermore, let $n_j(s)$ be the number of teams that select city j for a profile s. Show that this game is a potential game with potential function:

$$\Phi(s) = \sum_{j \in C} \sum_{k=1}^{n_j(s)} \frac{u_j}{k}$$

and therefore has a pure strategy Nash equilibrium.