Advanced Topics in Behavioral Game Theory II

Modeling Strategic Behavior as a Machine Learning Problem

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Lecture Overview

Explaining anomalies using loss aversion

Risk aversion

Definition

An agent is **risk averse** if they strictly prefer a “sure thing” to a risky prospect with a higher expected value (so long as it isn’t too much higher).

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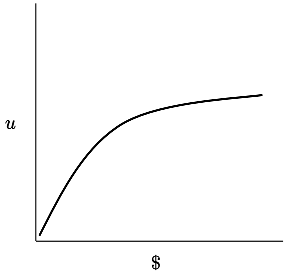
Example

From your current wealth level, would you prefer:

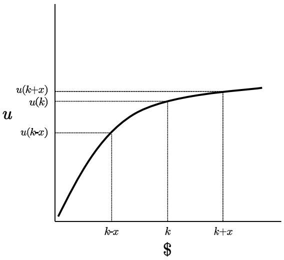
A $0 for sure

B A 50-50 chance of **losing $100** or **gaining $110**

Risk aversion as concave utility



(c) Risk aversion



(d) Risk aversion: fair lottery

- A **concave** utility for money models **decreasing marginal value for money**
- An agent with concave u_i is said to be **risk averse**, because they will **strictly prefer** to receive a fair lottery's expected value than to play the lottery.

Relative Risk Aversion

- Risk aversion of u at x can be measured by **relative risk aversion**:

$$R(x) = \frac{-xu''(x)}{u'(x)}$$

  - Common assumption: Constant relative risk aversion utility (CRRA)
  - i.e., $R(x) = \rho$ for all x, constant ρ
- Can experimentally measure ρ (**how?**)

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- Can experimentally measure ρ (**how?**)
- Estimates are usually around **1.0**

Equity premium puzzle

- Equity premium: Equities (e.g., stocks) typically have a higher return than fixed income securities (i.e., bonds)
- Usual explanation: Risk aversion!
  - Stocks have variable returns
  - Risk averse investor will only buy stocks if their average returns are (enough) greater than bonds
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- Between 1926 and 1995: Average real returns on stocks: 7%, vs. 1% for bonds
- Equity premium puzzle: To explain the historical equity premium, investors would need to have risk aversion greater than **30**

Prospect theory [Kahneman & Tversky, 1979]

- Utility theory: The only thing that matters is the (distribution over the) **final outcome**
- Prospect theory: The only thing that matters is the (distribution over the) **changes in outcome** from current situation
  - i.e., the relevant quantities are **gains** versus **losses**

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An agent who exhibits **loss aversion** is more sensitive to a given loss of $\$x$ from their **current** wealth level than they are to a gain of $\$x$.

E.g., the following prospect-theoretic value function exhibits (linear) loss aversion:

$$v(\delta x) = \begin{cases} \delta x & \text{if } \delta x \geq 0, \\ 2.5\delta x & \text{if } \delta x < 0. \end{cases}$$

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- Loss aversion is a very robust experimental finding
  - People are typically twice as averse to losses as attracted by gains
- A loss averse agent will likely turn down a 50-50 bet between $-\$100$ vs $\$190$, but will accept 100 such bets pooled together (**why?**)

"Myopic Loss Aversion and the Equity Premium Puzzle" [Benartzi & Thaler, 1995]

- Equity premium implies unrealistically-large risk aversion
- Model implication: What degree of loss aversion could explain the equity premium?
  - Simulation study to estimate loss averse values for different portfolios
  - What values of loss aversion yield indifference between bonds and stocks?
- Evaluation of loss averse values depends on aggregation period (**why?**)
- If investors evaluate portfolios annually, implies loss aversion of roughly 2 (previous experimental estimates)

Summary

1. Examples of how to extend BGT models to more complex settings
   - Extensive form games
   - Bayesian games
2. Examples of additional assumptions that can be relaxed
   - Specific decision procedure
   - Utility of outcomes vs. value of changes
3. Examples of different kinds of questions BGT can bear on
   - Normative, reasons that specific models have desirable properties
   - Descriptive, predictions of decisions
   - Explanation of anomalies, implications of models