Game Representations

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Lecture Overview

Normal-Form

Repeated

Extensive Form

Bayesian Games

Repeated Games

Play the same game **over and over again**.

Discuss in your groups what happens:

- …when you play Prisoner's Dilemma 3 times in a row?
- …if you play PD infinitely?
- …if you play PD repeatedly, stopping each time if two coin flips both come up heads?
- …if you play Battle of the Sexes repeatedly (in any of these conditions)?
- …if you play Matching Pennies repeatedly (in any of these conditions)?

If you like, **actually play the games** to see what happens.

Then let's discuss together as a class.
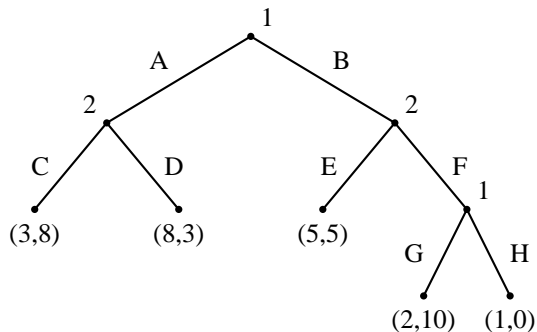
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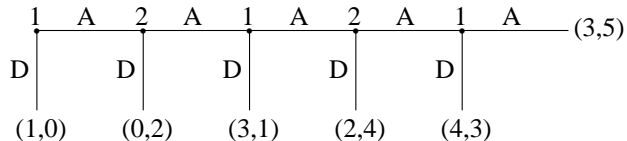
Extensive Form

Bayesian Games

Perfect-Information Extensive-Form Games

- Game **unfolds over time**
- Players can **see each other's moves**
- Can be written as a tree; leaves labelled with payoffs

Fun Games

Centipede

Ultimatum

- Player 1 proposes how to split $50 between the two players
- If player 2 accepts, both keep their portion of the split
- If player 2 rejects, both get $0

Play in a breakout room

Stochastic Games

- Combines perfect-information extensive form with repeated games
- Multiplayer generalization of a Markov Decision Process (MDP):
  - state: which game is being played
  - actions: set of alternatives for each player in that game
  - reward: payoffs in that game
  - transition function: mapping from all players' actions to the next state

Imperfect-Information Extensive-Form Games

- Generalizes perfect-information extensive-form games by allowing for **imperfect observation** of the previous player's moves
  - Some actions might be observed perfectly
  - Sometimes the second player might not be able to tell anything except that the first player moved
  - Most generally, the second player can observe which of a set of **equivalence classes** contains the first player's move
- Examples:
  - Battleship
  - Starcraft (without random starting locations or random races)

Stackelberg Games

A special case of imperfect-information extensive-form games

- One player **commits to a strategy**, which is observed by the second player
  - If the strategy is randomized, the second player can't see random draws
- Motivation: the same game is played repeatedly; the second player can see the first player's actions and hence figure out her strategy
- Example: security games; wildlife poaching games

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Fun Game

- Choose a phone number none of your breakout room members knows; consider its last four digits to be DEFG

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- Choose a phone number none of your breakout room members knows; consider its last four digits to be DEFG
  1. take "DE" as your valuation. Play a first-price auction with three neighbours, where your **utility is your valuation minus the amount you pay**
  2. play the auction again, same neighbours, same valuation
  3. play again, with "FG" as your valuation

- Can we model this interaction as a normal-form game?

Bayesian Games

- **Uncertainty about payoffs** (either one's own or others')
- Interesting when there is **asymmetric information** about this uncertainty
  - otherwise, just play the game where payoffs are expected values of payoffs for each action
- Different from Imperfect Information Extensive Form, which is uncertainty about another agent's *moves*