REALLOCATING SPECTRUM: THE INCENTIVE AUCTION

Presented by Alexandre Fréchette
Outline

1. Spectrum Auctions
   - Motivation
   - Experimenting
   - Risks & Dangers

2. Incentive Auction
   - Ambition
   - Proposed Procedure
   - Reverse Auction

3. Deferred Acceptance Auction
   - Mechanism
   - Three Interesting Properties

4. Conclusion
   - Extensions & Real Life
Spectrum Auctions

Governments *auction off* public electromagnetic spectrum.

http://wireless.fcc.gov/auctions/default.htm?job=auction_summary&id=73

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**Successful applications:**

- Since July 1994, the Federal Communications Commission (FCC) has conducted 87 spectrum auctions, which raised over $60 billion for the U.S. Treasury.
- A UK auction in 2000 generated €38.3 billion in revenue.
- The latest Canadian auction held in February 2014 raised $5.3 billion.

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A Game - *Reenacting the Turkish Auction of 2000* [1]

**Spectrum Blocks:**

1. You have your **private value/budget**.
2. Auction off **one block at a time**, using **first price auction**.
3. **Reserve/starting price** of block \(i\) is final price of block \(i - 1\).
4. Your utility is the **fraction of sold blocks you own**.
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Spectrum Blocks:

![Spectrum Blocks](image)

Value distribution - normal with mean 10 and standard deviation 3.
Spectrum auction design is a complex problem, and has serious consequences when done inadequately [2].
High-Stakes Does Not Guarantee High Quality

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The FCC wants to provide more spectrum for mobile companies to fuel the next generation of products by buying some off broadcast television companies, and selling it to telecoms.

http://www.hlspectrumreview.com/2012/10/articles/auctions/
united-states-rulemaking-for-incentive-auction-of-broadcaster-spectrum/
Freening Up Spectrum

Assume some TV stations agree to go off air.

Repack the remaining (on-air) stations on a smaller range of channels without causing interferences.

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Nick Arnosti, Auctionomics
Three components acting in unison:

1. **Reverse auction** to buy spectrum off TV stations, and meet a clearing target.

2. **Forward (ascending prices) auction** to sell cleared spectrum to mobile companies; A slight adaptation of the successful clock auction previously used.

3. Coordination mechanism to direct reallocation goals; Adjusts clearing target based on global efficiency.
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Reverse Auction

**Which stations** should we compensate, and **how much** should we give them? “That’s easy, just use VCG!”

Let $N$ be the set of stations, let $\mathcal{F} \subseteq 2^N$ be the collection of subsets of stations that can feasibly be repacked. Then given bids $\hat{b} \in \mathbb{R}^N$,

$$\chi^{\text{VCG}}(\hat{b}) = \arg \min_{S \in \mathcal{F}} \sum_{i \in S} \hat{b}_i.$$  

This is the Minimum Weight Graph Colouring Problem on a very large graph, an NP-complete optimization problem that is very hard to solve in practice.
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Deferred Acceptance Auction

Alternate mechanism for the reverse auction [4, 3].

“Deferred-acceptance (DA) auctions choose allocations by an iterative process of rejecting the least attractive bid.”

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Allocation Rule

At each step $t$ of the DA auction, we have a set $A_t \subseteq N$ of **active bidders**.

The DA auction is specified by **scoring functions** for any active set $A \subseteq N$ and station $i \in A$

$$s_i^A : B_i \times B_{N \setminus A} \rightarrow \mathbb{R}^+,$$

where $B_i$ is the “bid space” of station $i$. Scoring functions must be **non-decreasing in their first argument**.

Then at each step the DA auction **removes from its active set the bidders with highest non-zero bid scores**, and otherwise returns $\chi^{DA}(\hat{b}_t) = A_t$ if all scores are zero.
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Simplest DA Algorithm for the Reverse Auction

For the reverse auction, $A$ can be thought of as the “packable” stations. Then a simple scoring function give non-zero score only to repackable bidders:

$$s_i^A(\hat{b}) = \begin{cases} 0 & \text{if } N \setminus A \cup \{i\} \notin \mathcal{F} \\ \hat{b}_i & \text{otherwise} \end{cases}$$

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Payment Rules

Pay-as-bid / first-price payments:

\[ \phi_i^{FP}(\hat{b}) = \begin{cases} \hat{b}_i & \text{if } i \in A \\ 0 & \text{otherwise} \end{cases} \]

Threshold price - highest bid without changing outcome:

\[ \phi_i^{TP}(\hat{b}) = \max \{ b \in B_i : i \in \chi^{DA}(b, \hat{b}_{-i}) \} . \]
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Proposition

A threshold price, deferred acceptance auction \((N, B, \chi^{DA}, \phi^{TP})\) is dominant strategy incentive compatible.
Flexibility with Scoring Rules

Added resilience through various scoring rules:

- use imperfect feasibility checking,
- impose auction budget constraints, or
- bound efficiency with interference graph specific scoring rules.
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*DA threshold auctions are equivalent to (finite) clock auctions in which bidders are restricted to cutoff strategies.*

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Conclusion

The nature of the reverse auction and intractability of standard techniques required the design of novel deferred acceptance auction.

This is an ongoing FCC auction design project, with still much to address:

- Generalize to more than two outcomes; e.g. possibility of going to lower quality spectrum instead of just off-air.
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References

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