Behavioral Game Theory

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1. Introduction

“Game theory is a study of mathematical models of conflict and cooperation between intelligent and rational decision makers” -wikipedia

Behavioral Game theory is about what players actually do, driven by empirical observation (mostly experiment).

- Start with a game experiment
- Think of plausible explanations for the differences
- Extend formal game theory
This is a very simple game.
- Two players, a Proposer and a Responder
- Bargain over some amount, e.g. $10
- The Proposer offers $x to the Responder
- If the Responder take the offer, the Responder gets $x and the Proposer gets $10-x
- If the Responder can reject it, they both get nothing.

Fun game!

The formal game theory’s analysis is like this:
- Players are self-interested
- The Proposer has all the bargaining power
➢ Half of offers of $2 or less are rejected

➢ When bargaining over $100, similar pattern of results occur
Why do Responders reject substantial sums?

- Players have a preference for being treated fairly!
- “Negative reciprocity”

Where does this fairness preference come from?
- Methodology, demography, culture, structure...

- Rejections in ultimatum games do not necessarily reject the strategic principles.
Can we model the social preference?

- Maintain the assumptions that players maximize their utilities
- Allow the utility to reflect a social preference

Find new utility functions!

- Inequality-Aversion Theories
- Fairness Equilibrium
Fairness Equilibrium-two player games

- Strategies: \( a_i \)
- \( i \)'s beliefs about the strategy of the other player: \( b_{3-i} \)
- \( i \)'s beliefs about what player \( j \) believes about player \( i \)'s strategy: \( c_i \)

How to describe kindness and utilities?

- Suppose player 1 has the belief \( b_2 \) about what player 2 will do.
- Call the highest, lowest and fair payoffs for player 2, \( \pi_2^\text{max}(b_2) \pi_2^\text{min}(b_2) \pi_2^\text{fair}(b_2) \)
- Player 1’s kindness toward 2, which depends on her actual choice \( a_1 \), is

\[
f_1(a_1, b_2) = \frac{\pi_2(b_2, a_1) - \pi_2^\text{fair}(b_2)}{\pi_2^\text{max}(b_2) - \pi_2^\text{min}(b_2)}
\]

Similarly, player 1 also forms a perceived kindness number:

\[
\tilde{f}_2(b_2, c_1) = \frac{(\pi_1(c_1, b_2) - \pi_1^\text{fair}(c_1))}{(\pi_1^\text{max}(c_1) - \pi_1^\text{min}(c_1))}
\]

- Player 1’s utility function including social preferences:

\[
U_1(a_1, b_2, c_1) = \pi_1(a_1, b_2) + \alpha \tilde{f}_2(b_2, c_1) + \alpha \tilde{f}_2(b_2, c_1) \cdot f_1(a_1, b_2)
\]
Fairness Equilibrium-two player games

- Fairness Equilibrium
  - Choose strategy with highest utility
  - Beliefs about strategies are correct— $a_i = b_i = c_i$

- Example: Prisoners’ dilemma with social preferences

<table>
<thead>
<tr>
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<th>Cooperate</th>
<th>Defect</th>
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<tbody>
<tr>
<td>Cooperate</td>
<td>$4 + 0.75\alpha$, $4 + 0.75\alpha$</td>
<td>$0 - 0.5\alpha$, $6$</td>
</tr>
<tr>
<td>Defect</td>
<td>$6$, $0 - 0.5\alpha$</td>
<td>$0$, $0$</td>
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- When $\alpha$ is large enough, both cooperation is a fairness equilibrium!
3. Beauty Contest

Review:
- N players simultaneously choose a number in the interval [0,100]
- Take an average of the numbers and multiplied by p<1 (e.g. 0.7)
- The player whose number is closest of this target wins a fixed prize.

The formal game theory’s analysis
- All players choose a same number 0!

This game is a “dominance solvable game”
First round choices are around 21-40!
How can we explain this phenomenon?

- An interesting story...

- Why it’s difficult to achieve Nash Equilibrium?
  - Iterated reasoning
  - Consider other players’ reasoning and beliefs

- Limited iterated reasoning to understand the initial choices!

- A theory of learning to explain players’ movement
How can we explain this phenomenon?

- Iterated reasoning is limited to a couple of steps

- Limited iterated reasoning could be modeled formally
  - Quantal Response Equilibrium
  - Cognitive Hierarchy
  - Level-k
  - ...

Introduction
Ultimatum Game
Beauty Contest
Conclusion
4. Conclusion

- Behavioral Game theory studies **what players actually do** in a game.
- It’s based on **experimental facts**.
- It extends formal game theory by including **feeling, thinking and learning**
  - Feeling: players express **social preferences**
  - Thinking: limited strategic thinking- **bounded rationality**
  - Learning: players **change behaviors** in the process of a game


Thank You!