Designing truthful online mechanisms beyond VCG

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Abstract

Dominance truthful bidding is one of the most desirable features in auction design. In the offline setting, a common technique for ensuring truthful bidding is to start with the Vickrey-Clarke-Groves(VCG) mechanism. However, many difficulties arise when implementing this scheme to an online auction. The main purpose of this paper is to survey recent literature aimed at designing truthful mechanisms in an online auction setting. The paper highlights several scenarios identified in the literature where the application of the VCG scheme is not feasible, and summarizes the recent development in finding alternative solutions. The characterization of truthfulness analogous to results pertaining to the offline setting, the feasibility and potential trade-off in designing alternative solutions are emphasized.

1 Introduction

Auctions are becoming an increasingly prominent part of today's economy. From multi-billion dollar auctions of radio-frequency spectrum to 10 cents auctions of plastic wrist watches on eBay, auctions of various types both provide structured resource allocation in practice and inspire theoretical interests in their properties. In addition to trading of real goods, virtual goods such as online game items are also exchanged in auctions. Auction problems differ from traditional centralized optimization problems because of the individual private interests possessed by its participants, making the prediction of outcome a challenge. To circumvent the complexity of incorporating and anticipating human behavior into the design of auctions with stable and socially optimal outcomes, auction designers frequently seek to design mechanisms that promote truthful bidding[1]. The famous VCG mechanism offers a simple yet powerful starting point in designing truthful mechanisms for auctions where participants are motivated to report their truthful valuations.

However, several issues in computational complexity, privacy and practical implementation are raised since its introduction. Rothkopf in [2], Ausubel and Milgrom in [3] summarize some main concerns in applying VCG scheme[2]. These papers focus on the difficulties in applying VCG in general, without particular emphasis on the unique concerns of implementing VCG scheme in an online auction setting. As internet becomes an increasingly important arena for resource allocation, difficulties in applying VCG mechanism in a dynamic setting motivate efforts in exploring the impact of the online setting on the feasibility of having truthful and speedy auctions. This survey focuses particularly on the design of dominant strategy truthful online mechanisms[3]. Bayes-Nash implementation and other solution concepts are explored in the literature as well, but are beyond the scope of this survey. Friedman and Parkes [6] show a straightforward extension

[1] For a comprehensive survey on auctions and biddings for computer scientists, see the survey by Parson *et.al.*[1].
[2] For a detailed account of applications of Vickrey auctions, see Lucking's book [4].
[3] Parkes give a survey on online mechanisms [5].

of VCG allocation and prices to an online setting, provided that the online choice rule is offline optimal. However, a major difficulty raised in [6] in applying the VCG mechanism in real online setting is the impossibility to delay computation of payments.

The remaining of the paper is organized as follows: section 2 gives a brief description of competitive analysis – the standard technique adopted by online mechanism design literature for comparison against the benchmark–often an optimal algorithm having the full information (i.e., offline) of the same scenario. Section 3 describes some of the main challenges in online mechanism design and defines some notations shared by the subsequent sections before highlighting several scenarios in which the application of VCG mechanism presents difficulty, including unknown total supply, reusable goods, strategic timing of presence declaration and goods with expiration times. Section 4 describes two recent work on prompt mechanism design with motivation in advertisement time allocation.

2 Competitive analysis

The main challenge in designing online algorithms[4] is the need for real time computation without seeing the future events. In the online auction design setting, this means an online auction mechanism has to make allocation and (in many cases) pricing decisions before accessing information of any bidders who have not arrived. If these decisions could be delayed until the end of all possible bidders' arrivals, an offline auction mechanism such as the VCG scheme can then be applied. Competitive analysis is a method of analyzing the relative performance of an online algorithm to a benchmark algorithm(for example, an optimal offline algorithm), and is commonly used in online algorithm design literature. In general, the ratio of some performance measure from an online algorithm to the corresponding measure in the offline algorithm is used as a measure of competence called competitive ratio[5]. In analyzing an online mechanism for an auction setting, efficiency can be one such performance measure. The competitive ratio (also called approximation ratio) in terms of efficiency is calculated by dividing the social welfare achieved by an optimal offline algorithm by that achieved by the online algorithm. For example, an online algorithm is said to be 3-competitive, or to be a 3-approximation in efficiency if the social welfare achieved by an optimal offline algorithm is three times of that achieved by the online algorithm when applied to the same scenario.

Since the performance of an algorithm is not a one-dimensional measure but is instead relative to many factors, upper bound and lower bound of competitive ratios incorporating the standard big-O notation relative to some input parameters (e.g., number of bidders, maximum number of units demanded, etc.) are frequently explored and used in the online auction design literature. Consequently, a upper bound on the competitive ratio suggests the worst-case in the online algorithm's relative performance. Another popular performance measure in the online auction design setting is revenue. Various versions of VCG scheme are frequently used as the benchmark in the competitive analysis of online mechanisms. Depending on the online algorithm analyzed, alternative benchmark may be used[6]. For a detailed introduction to competitive analysis, see [9].

[4]The word 'algorithm' and 'mechanism' are used interchangeably in this section, mainly because the competitive analysis concept applies to a more general setting. See section 3.1 "The general setting" for an description of the distinction between online mechanism and offline mechanism.

[5]The inverse of this measure is sometimes also referred to as competitive ratio. In this survey we convert the measure to the ratio of performance of offline algorithm to that of the online algorithm whenever this is the case.

[6]For examples, see [7] and [8].

3 Scenarios beyond VCG

3.1 The general setting

Compared to the offline setting, mechanisms designed to work online face many additional constraints on the information available to make decisions. From an implementation point of view, delaying the calculation of payment and allocation decision until the end of the auction is often not feasible for various reasons summarized in [10]. The direct consequence of this restriction is that the mechanism cannot delay the computation of payments until the end of the auction. Information about the auction which an offline mechanism naturally has access to (examples include the quantity of available goods, the expiration time of each unit, the timing of each bidder's presence in the auction, the total number of possible bidders, etc.,) cannot be taken for granted by an online mechanism. In the following sections, several papers exploring the impact of the online requirement in different scenarios are summarized as examples in this line of research, starting with the case where the quantity of the items to be sold is unknown to the mechanism. Agents are assumed to be self-interested and have quasi-linear utility, as in most auction theory analysis. Depending on the type of private information each agent hold, the models can be classified as single-parameter models or non single-parameter models, with the former more extensively studied. In the rest of the paper, n denotes the total number of bidders.

3.2 Unknown supply

Babaioff, Blumrosen and Roth in [10] consider the problem of selling identical goods without the mechanism knowing the total supply to a fixed set of n bidders each with unit demand reporting a single valuation. In order to explore the value of information on supply, online mechanisms in a worst case model (what the authors call the "adversarial supply" model) and an alternative model (referred to as the "stochastic supply" model) are examined separately. No assumptions are made on the distribution of total quantity available for sale in the worst case model, whereas the distribution from which supply is drawn is known to the mechanism in the stochastic supply model. The authors show that truthful deterministic mechanisms without assumptions on the total supply of goods and bidder valuations perform poorly in worst case with competitive ratio growing at least linearly with respect to n (number of bidders), and adding randomization to these mechanisms only achieves a lower bound of O(log log n) on the competitive ratio. For the stochastic supply model, the authors prove that constant approximation to social welfare is impossible for a deterministic truthful mechanism without assumptions on the characteristic of the supply distribution. The authors further limit the distributions to those with monotone hazard rate and show in this case a lower bound of approximately 1.63 and 1.17 on the competitive ratios of deterministic and randomized online mechanisms, respectively. These competitive ratios are obtained by comparing social welfare achieved by online mechanisms to the optimal social welfare from offline mechanisms (after seeing all arrivals of items) for every possible quantity of supply in the adversarial supply model, and to the optimal expected social welfare (more precisely, the expectation of that over the assumed distribution of total number of items) by offline mechanisms in the stochastic supply model[7]. The authors suggest that online-envy-free is an important property such online mechanisms should possess. The significance of this property to promoting stable behavior in online auctions would be an interesting question. In addition,

[7]In the absence of prior on the supply distribution in the model, the authors work on a stronger version of dominant-strategy truthfulness where bidding truthfully is a dominant strategy in every possible instantiation of supply and randomization (in the case of randomized mechanism), in addition to being a dominant strategy in every possible set of bids by other bidders.

investigating whether other recently proposed online algorithms satisfy this additional fairness criterium may also provide additional insights.

3.3 Reusable goods and strategic timing

Hajiaghayi, Kleinberg and Mahdian in [11] study the scenario of selling reusable resource to agents having dynamic arrival and departure times and valuation (which are all private information to individual agent) demanding possibly multiple finite units from multiple reusable goods. A special case of this scenario as noted by the authors is equivalent to the scenario of real-time online scheduling of computing resources studied in [12]. The benchmark of competitive analysis used in this work is the offline VCG scheme, with both efficiency and revenue explored as performance measures. The setting here differs from the single parameter setting in the previous section by having multiple types of private information an agent can hold and strategize on. Therefore, the authors start by characterizing truthfully implementable online allocation rules for the non-single parameter setting, and separate subsequent analysis into synchronous and asynchronous (with interruptible jobs) models for different levels of discretization in allocation timing. Starting from the single-unit demand case, the authors proposes a greedy allocation rule for the asynchronous model resulting in a 2-competitive (in terms of efficiency) online mechanism which later is proven by the authors to be the lower bound. A modification of the greedy algorithm is then proposed and shown to a 5-approximation (in terms of efficiency) of the offline benchmark.

Similar approaches can be adapted to the mutiple-unit setting yielding similar competitive ratios, however it is not clear whether these ratios are lower bounds. In analyzing the performance on revenue, the authors introduce a further desired property that considers impatient bidders anonymously, and prove that no constant approximation from a truthful online mechanism is possible. The authors proceed to show that a O(log h) competitive ratio is attainable by a randomized online truthful mechanism, where h denotes the ratio of the upper bound and lower bound of the interval to which all bids belong. Next, variable job lengths (or equivalently, the number of units each bidder demands) as another piece of private information to each bidder is added to the basic setting and a competitive ratio that grow with the range of job length h is attainable by a randomized truthful mechanism. However, the same mechanism is slightly weaker in enforcing truthfulness since the mechanism now needs an additional piece of information (the ratio of the maximum job length to the minimum job length) to ensure truthful revealing of job lengths.

An important assumption behind these results is that only late reports of arrivals and early reports of departures can be allowed. In other words, an agent is allowed to hide but not allowed to pretend to be in the auction. Although payments are calculated based on information received before an agent's departure, the mechanisms analyzed in this work do not reinforce immediate payment at the time of allocation. Cole *et. al.* in [13] propose alternative mechanisms achieving similar competitive ratio while allowing prompt payments[8]. Finally, individual rationality is implied by the rule that no payment will be charged on an gent if no resource is allocated to that agent in the proposed mechanisms.

Porter in [12] studies a special case motivated by processor time sharing where a single reusable good is auctioned. In this setting, agents have private release time (equivalent to the arrival time in the base case of [11]), deadline (equivalent to the departure time in the base case of [11]) and processing time (equivalent to the job lengths/ number of units demanded in the generalized case of [11]). The same benchmark as the one in [11] is used for competitive analysis, but efficiency (in this setting the total value of completed jobs) is used as the sole

[8]See section 4 Prompt mechanism design for online auctions.

performance measure to be maximized with constraints of incentive compatibility and individual rationality. In order to explore the impact of considering self-interested agents on top of some previous work, Porter measures the result of competitive analysis in terms of the ratio between the maximum and minimum value density (defined as the ratio of value to job length). The author proposes an incentive compatible and individual rational deterministic mechanism based on the idea of second-price auction and proves that it is $((1+\sqrt{k})^2+1)$- competitive, and 5-competitive in the special case where agents cannot falsely report job lengths and value densities for all jobs are the same. This competitive ratio is later shown in the same paper to be the lower bound for all incentive compatible and individual rational deterministic mechanisms which never pay any agent. As [11] points out, this competitive ratio is only lower (and thus making it more competitive) than the mechanism proposed in [11] when the variation in job lengths is greater exponentially than the variation in value densities. It is not clear whether the mechanism proposed apply to the equivalent setting as the asynchronous model in [11].

Ng, Parkes and Seltzer in an earlier work [14] also considers this problem with the motivation of data staging, and proposes the Virtual Worlds mechanism which partitions the availability of reusable goods over time into a sequence of auctions. Heydenreich and Muller in [15] extends the setting to multiple machines (equivalent to the synchronous model in the multire-usable goods case in [11] and proposes a 3-competitive deterministic mechanism that is not dominant strategy truthful but only truthful in a weaker solution concept. The question of whether a constant competitive (in terms of efficiency) dominant strategy truthful mechanism in this setting can be obtained remains open. In addition, many questions remain open regarding the revenue competence of online mechanisms in this setting.

3.4 Expiring goods

Perishable goods pose an additional challenge to online auction designers by adding time restrictions to the allocation. Lavi and Nisan in [16] take an initial step by considering a relatively simple case of selling gradually and deterministically expiring items to self-interested bidders with dynamic and private arrival time, departure time and valuation for a single unit of the good which each agent demands. Not restricting the report of arrival and departure times, the authors provide negative results on the competence of online mechanisms (with a lower bound growing with the number of identical items for the competitive ratio in terms of efficiency). In addition to the negative result on the quest for dominant-strategy truthfulness, the authors suggest an alternative solution concept named Set-Nash equilibrium in which agents best respond to a set of recommended strategies by choosing from the same set of recommended strategies. The authors further demonstrates this concept with two examples: an Online Iterated Auction and a Sequential Japanese Auction, both of which provide a constant competitive ratio. This setting can be seen as equivalent to the a generalization of [11], [12] and the "Generalized" model in [13]. More importantly, it provides a starting point from which settings with more complex expiration characteristics of goods can be analyzed. However, a completely new algorithm or even a new approach may be needed to incorporate nondeterministic expiration of goods into the current model.

4 Prompt mechanism design for online auctions

The resource sharing scenario reviewed in section "Reusable goods and strategic timing" is also studied by Cole, Dobzinski and Fleischer in [13], emphasizing prompt calculation of payment where a winning bidder is informed of the payment immediately after winning. The authors argue that promptness is an important property an online mechanism should have in order to make the auction process intuitive to the bidders, reduce uncertainty in the winning bidders cost, alleviate difficulty in payment collection and reduce risk on the bidders. Two particular models of increasing complexity are studied: a Value-Only model where the only private information an bidder has is the valuation of the item, and a Generalized model where both the valuation and departure time are private to each bidder. The Value-Only model studies a scenario equivalent to that in an earlier work by Hajiaghayi, Kleiberg and Parkes [7], whereas the Generalized model is equivalent to a simplified version of the basic model studied in [11] with arrival times being public information once it is available. The authors present a deterministic prompt and truthful mechanism for the Value-Only model that is 2-competitive, and prove that this is in fact a lower bound even for online deterministic mechanisms that are not prompt. In addition, the authors propose a randomized mechanism which essentially conducts second-price auction on uniformly random partitions of bidders, and show that this mechanism provides an approximation to the optimal offline algorithm of at least as good as a 10-approximation, without stating that this is a tight bound. A lower bound of 2 on efficiency is then provided for randomized mechanism in the "Generalized" model.

Koutscoupias and Pierrakos in [8] consider a single-parameter case where only the valuation of agents are private, and agents arrive in a random order, demanding any single unit of M identical items. Aimed at maximizing the profit, the authors take the approach of building truthful online mechanisms from truthful offline mechanisms, and prove that the competitive ratio grows to at most twice its original size. The authors define a family of auctions $BPSF_r$ which is short for "Best Price So Far" auctions, with r denoting the range of highest price among previous bids from which the current offer price will be determined. A conjecture that the competitive ratio of at most 4 in terms of profit is then is put forward, matching that of Random Sampling Optimal Price auction. Since Random Sampling Optimal Price auction partitions all the current bids into two parts and calculates the optimal single price for each, it would be interesting to explore the performance of its online adaptions relative to the size of the bidding group. Exploring the use of online learning techniques in designing approximations to optimal auctions may also provide further insights.

Azar and Khaitsin in [17] also considers the single-parameter case, but with variable job sizes as an additional level of complexity in the model. A video ad placement into time slots is used as motivation, where the expiration date of each ad corresponds to the departure time in previous sections, the size (allowed to be arbitrary in this work) of each ad corresponds to the job length in [12],[15] and number of units demanded in [11]. In the ad placement setting, the authors naturally assume that only the value of an ad is private information possessed by each bidder, the arrival and expiration time along with the size of each ad is known to all the bidders and the mechanism as soon as the owners of the ads arrive into the auction, resulting in a single-parameter setting. Aiming at designing prompt mechanisms, the authors first confirm the inapplicability of the VCG scheme, then propose a truthful and prompt mechanism called the Gravity Algorithm. This algorithm only keep bids with the highest density (defined as the value over size, similar to value density concept in [12]) in two types of tentative schedules: a schedule with a single ad taking the whole time period available, and a schedule with multiple ads sharing the unit time period. At the time of publishing, the current total value of the two schedules are compared to decide which set of ads to reject. This algorithm is shown to be truthful, prompt and have an approximation ratio of 6 in terms of efficiency. The authors go on to show that this competitive ratio can be maintained in an extended setting with time restriction on slot assignments, and even improved in the case where ads are sufficiently small in size.

Looking back, the mechanisms studied in [10] and [15] are prompt, whereas the ones shown in [11], [14] and [12] are not. In the future, a concrete framework generalizing the settings mentioned above would provide benefit to researchers new to the field as well as those seeking to conduct advanced analysis.

5 Conclusion

The scenarios highlighted in this survey have many potential real life applications. The uncertainty in total quantity of supply is a natural characteristic in many environments such as the labor market, scheduling of a dynamic team, market of natural resources, etc. The possibility of strategic announcement of participants is also a prevalent practice in certain markets, for example, customers purchasing airline tickets, futures exchange, and any other markets where customers may find incentive to misreport their presence. The perishable goods scenario is inherent in markets where time-limited resources are exchanged, including fashion goods, transportation capacity, entertainment, agricultural products, surgical hours, hospital beds, computing capacity and many more. Due to the need of online transactions before receiving complete information on all possible participants and bids, the design for online mechanisms not relying on the traditional VCG scheme used for offline setting become an interesting and important area of study. Many results have been produced regarding the possibility of having online mechanisms with limited trade-off in efficiency compared to the offline counterpart having complete information, yet many questions remain open regarding the revenue/profit trade-off, leveraging machine learning techniques and characterization of desirable mechanism properties unique to the online setting. Lastly, empirical analysis on the performance of existing and future algorithms would complement theoretical results and provide new directions of research. Specifically, results from analyzing both controlled experiments (in the form of computer simulation and/or designed experiments with human participants) and real-world auctions would provide insights into the empirical performance of online mechanisms.

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