Applications of Auction Theory in Secondary Spectrum Market

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Abstract: Big channel users own large chunks of spectrum. As their spectrum utilization varies dramatically with location and time, they will have idle channel every now and then. On the other hand, spectrum demand is increasing rapidly over the years. More and more networking applications wish to have access to the spectrum that is being exhausted. Consequently, spectrum reallocation becomes not only possible but also necessary. In this paper, several models are discussed, using auction theory to solve problems in secondary spectrum market. Each of them focuses on achieving some of the desirable properties.

1. Introduction

The number of wireless devices has grown very fast in the past decade, which results in unabated growth of wireless bandwidth demand. To allocate resources, we usually resort to auction theory, which could distribute resources fairly and efficiently.

Spectrum usage and availability used to be regulated by government in a static manner. Big spectrum users, like mobile service carrier, own large trunks of spectrum on a long term lease and they take up most of the spectrum available, while emerging users have limited access to the remaining spectrum, which is being exhausted. We refer to the big spectrum users as primary users, who compete in a primary market, and the latecomers as secondary users, who don't have much access to the spectrum.

Static and fixed allocation of spectrum proves to be inefficient in recent studies[1], because the demand of primary users vary dramatically with location and time. As a result, spectrum subleasing is widely acknowledged as a potential way to share the spectrum.

In a secondary market, primary users periodically hold auctions to lease idle portions of their spectrum to unlicensed secondary users. In order to use the spectrum, secondary users pay primary users a certain price in order to access the channel. Primary users wish to achieve as much profit as possible. Such problems can be discussed within the range of a double auction design.

We would like to design mechanism to achieve several properties. First of all, truthfulness is one of the most important properties to implement an auction in order to achieve efficiency. When a mechanism is truthful, each secondary will maximize his profit by telling the truth. This property is extremely important because if a certain user can increase his utility by misreporting his value, the auction will be vulnerable to market manipulation. It will harm both the primary users and the other secondary users' profits.

Efficiency is also one of the desirable properties. We can usually achieve that by maximizing social welfare, which can be defined as the sum of each user's utility. If we maximize social welfare each round, we are guaranteed to have an efficient mechanism, and yet it may lead to unfairness. There are a lot of ways of defining fairness. We may assume that fairness here means that every secondary user should have access to the spectrum if they are as competent as others. This property is important because the auction is held periodically for sufficiently large number of times. If a user does not have access to the spectrum in the first round, he is not likely to do so in the following round if we greedily maximize social welfare in every round: the mechanism is truthful and each user reports his own value and the result of each round will be exactly the same as the first one. If we want to achieve fairness so that every agent is able to have access to the spectrum for at least once, we have to introduce some randomness to the mechanism. Otherwise, it will discourage losing users from continuing participating in the long run. These users, knowing that they don't have a chance to win, may choose to misreport their own values to cause winning users to pay a higher price[2].

Reusability is also one of the important properties an auction should holds in secondary spectrum market because the inherent properties of the channel itself. If several agents are not in conflict with each other, they can be allocated the same channel at the same time. This scheme will make the channel more accessible to buyers and yield more profit at the same time.

Tractability means the mechanism has polynomial time complexity. Sometimes maximizing social welfare is equivalent to the well studied graph coloring problem, which is NP-hard[3]. Approximately optimal solution may have to be used to avoid too much time complexity.

In the next chapter, several different mechanisms will be discussed. Essentially they share most of their basic auction setting and differ in buyer-grouping and winner-selection parts.

2. Auction model

We can model the secondary market as a double auction, where more than one seller (primary users) is selling the same goods (indistinguishable channel access) to more than one buyer (secondary users). The seller has a reserve price, under which the seller would rather not sell the spectrum to a secondary user due to negative profit. There are two different kinds of primary user models, one with a lot of primary users and each has only one idle channel, the other with only one primary user who has more than one idle channel. Although the mechanism itself can be looked as an auctioneer, we can simplify the model by defining the primary user (or users) as the auctioneer, which means they are always truthful about their reserve price and their goals is to distribute the spectrum to secondary users efficiently while trying to achieve more profit, which can be defined as the price they are paid minus their reserve prices. Every secondary user is an agent, or a buyer, who has a value for each channel use, which is private information to them. Agents are selfish and want to maximize their profits, which are their values minus the prices they pay to the primary user. They will misreport their own value if doing so can improve their profits. A truthful mechanism can avoid this so that no agent can increase his utility by misreporting his own value. As a result, all the agents report their true value and rules can be set accordingly.

The auction here is a seal-bid double auction, in which all the sellers and buyers submit their bids at the same time. Sellers' bids are, as they are always truthful, their reserve price, and buyers' bids are how much they are willing to pay for a channel use. Buyers' bids should be their value of one channel use in optimal cases, and yet it is not always the case. The mechanism then decides the winning buyers and sellers and allocates the resource accordingly, charge the buyers and pay the sellers. The mechanism should hold properties like individual rationality and weak budget balance. The former means agents will not lose money if they take part in the auction, and the latter means the mechanism itself won't end up paying something for its own pocket, if not being able to earn something. The latter is very important because here the mechanism is only a set of rules, not an agent who can compensate the discrepancy between income and outcome. If there should exist a gap between the sum of charges from the buyers and the sum of payment to the sellers, it must be the case that payment in total is less than charges. Otherwise there is nowhere to find any payment to compensate dissatisfied sellers.

2.1 McAfee Double Auctions

McAfee double auctions[4] can achieve truthfulness, ex-post budget balance and individual rationality but does not consider reusability because it does use buyer grouping. It will only allocate one channel to one secondary user at the same time. It implement the following procedures:

1. Sort sellers' bids in non-decreasing order and buyers' bids in non-increasing order.

$$B_1^s \leq B_2^s \leq \cdots \leq B_M^s$$

$$B_1^b \geq B_2^b \geq \ldots \geq B_N^b$$

2. Find k = argmax $B_K^s \leq B_K^b$. The first (K-1) sellers and first (K-1) buyers are the auction winners.

3. Charge all the winning buyers equally by the bid of the Kth ranked buyer's bid B_K^b . Pay all the sinning sellers equally with the bid of the kth ranked seller B_K^s .

2.2 VCG Double Auctions

The VCG double auctions[5] differs from McAfee model only in the choice of winners and prices. Instead of choosing the first (K-1) buyers and sellers, it chooses top K pairs so that the last pair will not be sacrificed as in McAfee. It charges each of the k wining buyers $P^b = \max(B_{K+1}^b, B_K^s)$ and pay each of the K wining sellers by $P^s = \min(B_{K+1}^s, B_K^b)$. Note that

$$B_K^s \leq B_{K+1}^s$$

$$B_{K+1}^b \leq B_K^b.$$

$$\therefore \max(B_{K+1}^b, B_K^s) \leq \min(B_{K+1}^s, B_K^b)$$

As a result, $P^b \leq P^s$ so that it is not budget balance and sometime the primary users will have negative profit.

The first two models consider the situation where each primary user has only one idle channel and each secondary user has a demand of only one channel. If we generalize the model by adding a parameter *volume* to each user, where q_i^s indicates the maximum volume of channel seller i intends to trade, and q_j^b indicates the maximum volume buyer j intends to buy, we will have the following new truthful auction model[6]. Note that without losing generality, we can assume that all the users truthfully submit their values of volume (we can actually find in the model that no users except for the one at a certain point can manipulate the auction by misreporting his value of volume but none of the users is able to find out any information about his place on the sorting list).

2.3 Spectrum Double Auction

We sort all the bids from buyers and sellers in strict orders (if there exists duplicate prices, we can combine their volumes to form a new bid with a volume of the sum of the two original volumes):

$$\begin{array}{cccc} B_1^s < B_2^s < \cdots < B_M^s \\ q_1^s \quad q_2^s \quad \cdots \quad q_M^s \\ B_1^b > B_2^b > \cdots < B_N^b \\ q_1^b \quad q_2^b \quad \cdots \quad q_M^b \end{array}$$

The supply volumes are arranged according to the ascending price order of sellers, and the demand volumes are arranged according to the descending price order or buyers.

We try to find a critical point q^* where there are K buyers' bids and L sellers' bids such that

$$B_{L+1}^s \geq B_K^b \geq B_L^s, \text{and} \sum_1^{K-1} q_i^b \leq \sum_1^{L} q_j^s \leq \sum_1^{K} q_i^b$$

or

$$B_K^b \geq B_L^s \geq b_{K+1}^b, \text{and} \sum_1^{K-1} q_i^b \leq \sum_1^{L} q_j^a \leq \sum_1^{K} q_i^b$$

If the first condition holds, $q^* = \sum_1^L q_j^s$. If the second condition holds, $q^* = \sum_1^K q_j^b$.

The total transaction volume is

$$\hat{q} = \min\left(\sum_{1}^{L-1} q_j^s, \sum_{1}^{K-1} q_i^b\right)$$

The winning buyers and sellers are the top (L-1) sellers and the top (K-1) buyers.

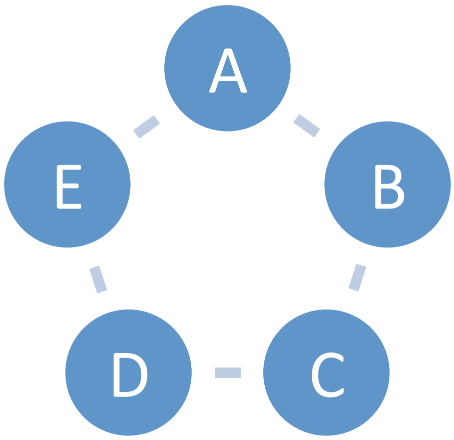
If we set all volumes to 1, meaning each seller has one idle channel and each buyer demands one channel use, the model becomes a McAfee double auction.

2.4 Channel Reuse Models

When considering reusability of the channel, we can divide the buyers into several groups. In each of the groups, we have a set of buyers who can share one channel at the same time. In the light of this, we have two new mechanisms. They have the same buyer grouping method and differ in their models of the primary users as well as how they calculate their groups' bids.

2.4.1 Buyer Grouping

To prevent the buyers from manipulating the auction, we have to group the buyers in a way that is independent of their bids. This property can be achieved building a conflict graph of all the buyers. If two buyers interfere with each other, meaning they can't share a channel, a line will connect these two buyers in the graph. We map all the buyers into the conflict graph and the problem turns into a graph vertex coloring problem in which we are to color the vertices of a graph such that no two adjacent vertices share the same color. Here the number of colors available is the number of channels available in our spectrum allocation problem. Buyer groups are denoted as $G = \{g_1, g_2, \dots, g_l\}$. Each group submit only one group bid, and different groups compete with each other using their group bids.



In a toy network with five buyers above, we can divide them into several patterns. One grouping result has three groups. $g_1 = \{A, C\}$,$g_2 = \{B, D\}$,$g_3 = \{E\}$.

2.4.2 SMALL-Strategy-proof Mechanism for radio spectrum ALLocation

In SMALL auction model[7], we have one primary user who has more than one idle channel for sale and several secondary users, each in need of only one channel. The group bid is calculate as the number of group members minus one, by the smallest bid in that group, i.e.

$$\beta_i = (|g_i - 1|) \cdot \min\,(B_k^b | k \in g_i)$$

Then we sort reserve prices of each channel in non-decreasing order and buyer group bids in non-increasing order

$$B_1^s \le B_2^s \le \cdots \le B_M^s$$

$$\beta_1^b \ge \beta_2^b \ge \cdots \ge \beta_N^b$$

SMALL finds the maximal number of K such that

$$\sum_{i=1}^{K} B_i^s \le \sum_{j=1}^{K} \beta_j^b$$

The winning groups are the first K buyer groups in their non-increasing order and the first K channels in their non-decreasing order. In the case of ties, we randomly choose one of them. While all the winning channels are winners in this auction, not all the members of the winning groups are winners. We exclude the buyer with the smallest bid in each winning group. Each winning buyer is charged the smallest bid of his group, and as there is only one primary user, he collects all the payment$\sum_{i=1}^{K} \beta_i^b$.

2.4.3 TRUST-TRuthful doUble Spectrum aucTions

Unlike SMALL which excludes the buyer with the smallest bid in every group, TRUST[8] counts all the buyers in one group. The group bid is just the smallest bid in that group by the number of buyers in that group

$$\beta_i = |g_i| \min (B_k^b | k \in g_i)$$

Similar to every other mechanism, TRUST sort reserve prices of all sellers in non-decreasing order and buyer group bids in non-increasing order

$$B_1^s \le B_2^s \le \cdots \le B_M^s$$
$$\beta_1^b \ge \beta_2^b \ge \cdots \ge \beta_N^b$$

TRUST also finds the maximal number of K such that

$$B_K^s \le \beta_K^b$$

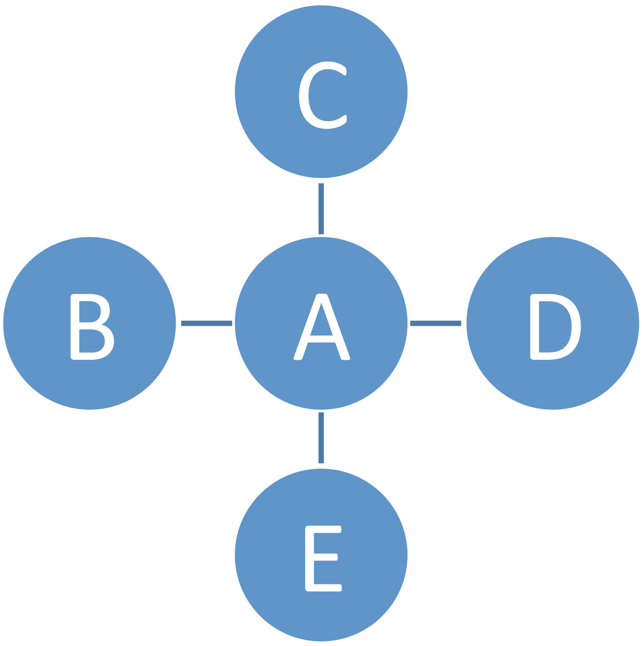
It differs from SMALL in that winners here are the fist (K-1) sellers on the list and top (K-1) buyer group on the list. All members of winning groups are winners who pay the smallest bid of their groups respectively, and each winning seller is paid the Kth seller's bid B_K^s.

When some of the secondary users want to buy more than one channel, we can extend the above two channel reuse model to embrace such situations. In the new models, multiple demands from the same buyer will be denoted as a set of vertices in the graph. Each of them represents a channel demand and any two of them share a common edge, which means they interfere with each other. Although the graph looks similar as before, we can't directly apply previous models. In some cases[7], misreporting will benefit buyers who have more than one channel demand. Modifications are necessary to take special care of critical situations so that the old rules can still apply. For example, it is observed that beneficial misreporting must result in that one radio of a node wins a channel, while the other one does not and holds the smallest bid in that group. When running the algorithm, we can perform a check each round to see if it is the case and take actions accordingly.

3. Conclusion and Analysis

After going through these five models, it seems that the last two models are much better than the previous ones which take no consideration of the channel reusability. If the primary users have the same reserve prices, their payment will be shared among a group of buyers in channel reuse model, while a single user has to cover the expense of a channel access without channel reuse. This will greatly improve buyers' profit. However, channel reuse models are vulnerable to several big problems. On one hand, information about the interference between buyers is not likely to be revealed beforehand. In order to map all the buyers into a conflict graph, channel interference check has to be performed, which takes time. For primary users, there is not much incentive to utilize reuse model if the time of detecting interference takes long because they can use this time to hold auctions without channel reuse and make profit. As the number and locations of secondary users may vary dramatically with time, interference check may have to be performed every time an auction is to be held, which takes even longer time and primary users may refuse to do so. Note that although we can assume the primary users to be truthful, they are not as good as we wish them to be because secondary auctions can not be looked as primary auctions which are held by the government in the hope of distributing the resources efficiently but not much of making more profit, unlike that in the secondary auctions where the goal of the sellers is to make more profit. On the other, graph coloring problem is NP-hard which couldn't be finished within polynomial time. The time complexity of a graph coloring problem is usually an exponential function of the number of vertices[3]. If we hold the auction in a citywide Wi-Fi hotspot channel allocation problem, things may get really tricky.

In terms of the coloring algorithm, we usually start from one vertex, and expand to the whole graph. The starting point can be randomly chosen and different algorithms could be used so that different grouping result may come out. It is very promising when we want to guarantee a minimum service to all the buyers on the condition that we don't exclude the buyer with the smallest bid in every group. If it is not the case, the buyer that is ranked last on the non-increasing list will never have access to the channel because even if his group wins the auction, he will still be excluded. Without channel reuse, the same buyer will never have access to the channel unless the reserve prices of primary users are sufficiently low and the supply of idle channel is no less than the demand of secondary users and VCG instead of McAfee is used because McAfee will still sacrifice the last bidder. In that case, every secondary buyer wins the auction, which is of not much significance. How about the buyer that is ranked first on the list? Is he guaranteed to win the auction? In auctions without channel reuse, he is very like to do so as long as there exists at least one winning buyer. Otherwise no winning buyer exists and thus no winning seller exists, which is no better than every buyer wins the channel. In the channel reuse model, however, it may not be the case. Consider the following conflict graph.



$$V_A = 50, V_B = 45, V_C = 40, V_D = 35, V_E = 30$$

Although A will be ranked first on the list, he will not be able to win a channel as B C

D E are all in a non-conflict group and their group bid will be higher than A's value.

One more problem with auctions of no channel reuse is that there is no way to provide a minimum service guarantee. When the buyer profile remains unchanged, the winning buyers win and losing buyers lose repeatedly. That is obvious considering the sorting method those models utilize. If we want to guarantee a minimum service to every buyer, even for the smallest bidder, we have to add some randomness to the mechanism. For example, we can run a channel lottery every 10 rounds and whoever wins the lottery gets the channel. Such scheme promises an ex-ante minimum service guarantee, but the property of budget balance may not hold ex-post when buyers with bids lower than sellers' reserve prices win the lottery. It is not clear if there exists a mechanism which can achieve fairness by guarantee a minimum service while other desirable properties still hold, and yet it's more like a problem of how we define the term 'fairness' because we can also define fairness in a way such that the smallest bidder should never win, or it will be unfair to buyers with higher bids than him. Well, that can make a fairly good philosophical problem, on which a professor may give an entire semester of lectures without being able to come to a final conclusion in the end, and that is perhaps beyond the range of the discussion here.

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