Lecture Overview

1. Self-interested agents
2. What is Game Theory?
3. Example Matrix Games
Self-interested agents

- What does it mean to say that an agent is **self-interested**?
  - not that they want to harm other agents
  - not that they only care about things that benefit them
  - that the agent has its own description of states of the world that it likes, and that its actions are motivated by this description
Self-interested agents

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We capture this by saying that each agent has a utility function: a mapping from states of the world to real numbers, indicating level of happiness with that state of the world

- quantifies degree of preference across alternatives
- allows us to understand the impact of uncertainty on these preferences
- Decision-theoretic rationality: take actions to maximize expected utility.
Why Utility?

Why would anyone argue with the idea that an agent's preferences could be described using a utility function?
Why Utility?

- Why would anyone argue with the idea that an agent’s preferences could be described using a utility function?
  - why should a single-dimensional function be enough to explain preferences over an arbitrarily complicated set of alternatives?
  - Why should an agent’s response to uncertainty be captured purely by the expected value of his utility function?

- It turns out that the claim that an agent has a utility function is substantive.

- There’s a famous theorem (von Neumann & Morgenstern, 1944) that derives the existence of a utility function from a more basic preference ordering and axioms on such orderings.
  - see Theorem 3.1.18 in the book, which includes a proof.
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Non-Cooperative Game Theory

- What is it?
Non-Cooperative Game Theory

What is it?
- mathematical study of interaction between rational, self-interested agents
Non-Cooperative Game Theory

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- Why is it called non-cooperative?
Non-Cooperative Game Theory

- **What is it?**
  - mathematical study of interaction between rational, self-interested agents

- **Why is it called non-cooperative?**
  - while it’s most interested in situations where agents’ interests conflict, it’s not restricted to these settings
  - the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
    - cooperative/coalitional game theory has teams as the central unit, rather than agents
TCP Backoff Game

Consider this situation as a two-player game:

- Both use a correct implementation: both get 1 ms delay.
- One correct, one defective: 4 ms delay for correct, 0 ms for defective.
- Both defective: both get a 3 ms delay.

Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?
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- Play this game with someone near you. Then find a new partner and play again. Play five times in total.
TCP Backoff Game

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- Questions:
  - What action should a player of the game take?
  - Would all users behave the same in this scenario?
  - What global patterns of behaviour should the system designer expect?
  - Under what changes to the delay numbers would behavior be the same?
  - What effect would communication have?
  - Repetitions? (finite? infinite?)
  - Does it matter if I believe that my opponent is rational?
Defining Games

- Finite, $n$-person game: $\langle N, A, u \rangle$:
  - $N$ is a finite set of $n$ players, indexed by $i$
  - $A = A_1 \times \ldots \times A_n$, where $A_i$ is the action set for player $i$
    - $a \in A$ is an action profile, and so $A$ is the space of action profiles
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player
Games in Matrix Form

Here’s the TCP Backoff Game written as a matrix ("normal form").

\[
\begin{array}{cc|c}
C & D \\
\hline
C & -1, -1 & -4, 0 \\
D & 0, -4 & -3, -3 \\
\end{array}
\]

The term 'Prisoners' Dilemma' for this famous game theoretic situation derives from the original story accompanying the numbers. Imagine the players of the game are two prisoners suspected of a crime rather than network users, that you each can either Confess to the crime or Deny it, and that the absolute values of the numbers represent the length of jail term each of you will get in each scenario.

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Prisoner’s dilemma is any game

\[
\begin{array}{c|cc}
  & C & D \\
\hline
C & a, a & b, c \\
D & c, b & d, d \\
\end{array}
\]

with \(c > a > d > b\).
Games of Pure Competition

Players have **exactly opposed** interests

- There must be precisely two players (otherwise they can’t have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant $c$
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it’s a one-player game
One player wants to **match**; the other wants to **mismatch**.

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<thead>
<tr>
<th></th>
<th>Heads</th>
<th>Tails</th>
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<tbody>
<tr>
<td>Heads</td>
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<td>-1</td>
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Matching Pennies

One player wants to **match**; the other wants to **mismatch**.

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Play this game with someone near you, repeating five times.
Rock-Paper-Scissors

Generalized matching pennies.

<table>
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<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
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</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0</td>
<td>−1</td>
<td>1</td>
</tr>
<tr>
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<td>0</td>
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</tr>
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<td>−1</td>
<td>1</td>
<td>0</td>
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...Believe it or not, there’s an annual international competition for this game!
Games of Cooperation

Players have exactly the same interests.

- no conflict: all players want the same things
- $\forall a \in A, \forall i, j, u_i(a) = u_j(a)$
- we often write such games with a single payoff per cell
- why are such games “noncooperative”? 
Coordination Game

Which side of the road should you drive on?

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Play this game with someone near you. Then find a new partner and play again. Play five times in total.
General Games: Battle of the Sexes

The most interesting games combine elements of cooperation \textit{and} competition.

\[
\begin{array}{c|cc}
   & B & F \\
\hline
B & 2, 1 & 0, 0 \\
F & 0, 0 & 1, 2 \\
\end{array}
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