Lecture Overview

1. Recap
2. Quasilinear Mechanisms
3. Properties
Recap Quasilinear Mechanisms Properties

Revelation Principle

It turns out that truthfulness can always be achieved!

Consider an arbitrary, non-truthful mechanism (e.g., may be indirect)

Recall that a mechanism defines a game, and consider an equilibrium \( s = (s_1, \ldots, s_n) \)
Recap Quasilinear Mechanisms Properties

Revelation Principle

- We can construct a new direct mechanism, as shown above.
- This mechanism is truthful by exactly the same argument that \( s \) was an equilibrium in the original mechanism.
- “The agents don’t have to lie, because the mechanism already lies for them.”
Theorem (Gibbard-Satterthwaite)

Consider any social choice function $C$ of $N$ and $O$. If:

1. $|O| \geq 3$ (there are at least three outcomes);
2. $C$ is onto; that is, for every $o \in O$ there is a preference profile $\succ$ such that $C(\succ) = o$ (this property is sometimes also called citizen sovereignty); and
3. $C$ is dominant-strategy truthful,

then $C$ is dictatorial.
Quasilinear Utility

Definition (Quasilinear preferences)

Agents have quasilinear preferences in an $n$-player Bayesian game when the set of outcomes is

$$O = X \times \mathbb{R}^n$$

for a finite set $X$, and the utility of an agent $i$ given joint type $\theta$ is given by

$$u_i(o, \theta) = u_i(x, \theta) - f_i(p_i),$$

where $o = (x, p)$ is an element of $O$, $u_i : X \times \Theta \mapsto \mathbb{R}$ is an arbitrary function and $f_i : \mathbb{R} \mapsto \mathbb{R}$ is a strictly monotonically increasing function.
Risk Neutrality

(a) Risk neutrality

(b) Risk neutrality: fair lottery

Figure 8.3 Risk attitudes: Risk aversion, risk neutrality, risk seeking, and in each case, utility for the outcomes of a fair lottery.

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Risk Aversion

(c) Risk aversion

(d) Risk aversion: fair lottery
Risk Seeking

(e) Risk seeking

(f) Risk seeking: fair lottery
Lecture Overview

1. Recap
2. Quasilinear Mechanisms
3. Properties
A mechanism in the quasilinear setting (for a Bayesian game setting \((N, O = X \times \mathbb{R}^n, \Theta, p, u)\)) is a triple \((A, \chi, p)\), where

- \(A = A_1 \times \cdots \times A_n\), where \(A_i\) is the set of actions available to agent \(i \in N\),
- \(\chi : A \mapsto \Pi(X)\) maps each action profile to a distribution over choices, and
- \(p : A \mapsto \mathbb{R}^n\) maps each action profile to a payment for each agent.
Direct Quasilinear Mechanism

**Definition (Direct quasilinear mechanism)**

A direct quasilinear mechanism (for a Bayesian game setting \((N, O = X \times \mathbb{R}^n, \Theta, p, u)\)) is a pair \((\chi, p)\). It defines a standard mechanism in the quasilinear setting, where for each \(i\), \(A_i = \Theta_i\).

**Definition (Conditional utility independence)**

A Bayesian game exhibits **conditional utility independence** if for all agents \(i \in N\), for all outcomes \(o \in O\) and for all pairs of joint types \(\theta\) and \(\theta' \in \Theta\) for which \(\theta_i = \theta'_i\), it holds that \(u_i(o, \theta) = u_i(o, \theta')\).
Quasilinear Mechanisms with Conditional Utility Independence

- Given conditional utility independence, we can write $i$’s utility function as $u_i(o, \theta_i)$
  - it does not depend on the other agents’ types
- An agent’s valuation for choice $x \in X$: $v_i(x) = u_i(x, \theta_i)$
  - the maximum amount $i$ would be willing to pay to get $x$
  - in fact, $i$ would be indifferent between keeping the money and getting $x$
- Alternate definition of direct mechanism:
  - ask agents $i$ to declare $v_i(x)$ for each $x \in X$
- Define $\hat{v}_i$ as the valuation that agent $i$ declares to such a direct mechanism
  - may be different from his true valuation $v_i$
- Also define the tuples $\hat{v}$, $\hat{v}_{-i}$
Lecture Overview

1 Recap

2 Quasilinear Mechanisms

3 Properties
Truthfulness

Definition (Truthfulness)
A quasilinear mechanism is **truthful** if it is direct and $\forall i \forall v_i$, agent $i$’s equilibrium strategy is to adopt the strategy $\hat{v}_i = v_i$.

- Our definition before, adapted for the quasilinear setting
Efficiency

Definition (Efficiency)

A quasilinear mechanism is strictly Pareto efficient, or just efficient, if in equilibrium it selects a choice \( x \) such that

\[
\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').
\]

- An efficient mechanism selects the choice that maximizes the sum of agents’ utilities, disregarding monetary payments.
- How is this related to Pareto efficiency from GT?
Efficiency

Definition (Efficiency)
A quasilinear mechanism is strictly Pareto efficient, or just efficient, if in equilibrium it selects a choice $x$ such that

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').$$

• An efficient mechanism selects the choice that maximizes the sum of agents’ utilities, disregarding monetary payments.

• How is this related to Pareto efficiency from GT?
  • if we include the mechanism as an agent, all Pareto-efficient outcomes involve the same choice (and different payments)
  • any outcome involving another choice is Pareto-dominated: some agents could make a side-payment to others such that all would prefer the swap
A quasilinear mechanism is strictly Pareto efficient, or just efficient, if in equilibrium it selects a choice $x$ such that

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').$$

- Called economic efficiency to distinguish from other (e.g., computational) notions
- Also called social-welfare maximization
- Note: defined in terms of true (not declared) valuations.
A quasilinear mechanism is budget balanced when

$$\forall v, \sum_i p_i(s(v)) = 0,$$

where $s$ is the equilibrium strategy profile.

regardless of the agents’ types, the mechanism collects and disburses the same amount of money from and to the agents.
Budget Balance

Definition (Budget balance)

A quasilinear mechanism is **budget balanced** when

\[ \forall v, \sum_i p_i(s(v)) = 0, \]

where \( s \) is the equilibrium strategy profile.

- regardless of the agents’ types, the mechanism collects and disburses the same amount of money from and to the agents.
- relaxed version: **weak budget balance**:

\[ \forall v, \sum_i p_i(s(v)) \geq 0 \]

- the mechanism never takes a loss, but it may make a profit.
Definition (Budget balance)

A quasilinear mechanism is budget balanced when

$$\forall v, \sum_i p_i(s(v)) = 0,$$

where $s$ is the equilibrium strategy profile.

- regardless of the agents’ types, the mechanism collects and disburses the same amount of money from and to the agents
- Budget balance can be required to hold *ex ante*:

$$\mathbb{E}_v \sum_i p_i(s(v)) = 0$$

- the mechanism must break even or make a profit only on expectation
Individual-Rationality

**Definition (Ex interim individual rationality)**

A mechanism is **ex interim individual rational** when
\[ \forall i \forall v_i, \mathbb{E}_{v_{-i}|v_i} v_i(x(s_i(v_i), s_{-i}(v_{-i}))) - p_i(s_i(v_i), s_{-i}(v_{-i})) \geq 0, \]
where \( s \) is the equilibrium strategy profile.

- no agent loses by participating in the mechanism.
- **ex interim** because it holds for *every* possible valuation for agent \( i \), but averages over the possible valuations of the other agents.
Recap Quasilinear Mechanisms

Properties

Individual-Rationality

**Definition (Ex interim individual rationality)**

A mechanism is **ex interim individual rational** when

\[ \forall i \forall v_i, \mathbb{E}_{v_{-i}|v_i} v_i(\chi(s_i(v_i), s_{-i}(v_{-i}))) - p_i(s_i(v_i), s_{-i}(v_{-i})) \geq 0, \]

where \( s \) is the equilibrium strategy profile.

- no agent loses by participating in the mechanism.
- **ex interim** because it holds for every possible valuation for agent \( i \), but averages over the possible valuations of the other agents.

**Definition (Ex post individual rationality)**

A mechanism is **ex post individual rational** when

\[ \forall i \forall v, v_i(\chi(s(v))) - p_i(s(v)) \geq 0, \]

where \( s \) is the equilibrium strategy profile.
Tractability

Definition (Tractability)
A mechanism is **tractable** when $\forall \hat{v}, \chi(\hat{v})$ and $p(\hat{v})$ can be computed in polynomial time.

- The mechanism is computationally feasible.
We can also add an objective function to our mechanism. One example: revenue maximization.

**Definition (Revenue maximization)**

A mechanism is **revenue maximizing** when, among the set of functions \( \chi \) and \( p \) that satisfy the other constraints, the mechanism selects the \( \chi \) and \( p \) that maximize \( \mathbb{E}_\theta \sum_i p_i(s(\theta)) \), where \( s(\theta) \) denotes the agents’ equilibrium strategy profile.

- The mechanism designer can choose among mechanisms that satisfy the desired constraints by adding an objective function such as revenue maximization.
Revenue Minimization

- The mechanism may not be intended to make money.
- Budget balance may be impossible to satisfy.
- Set weak budget balance as a constraint and add the following objective.

**Definition (Revenue minimization)**

A quasilinear mechanism is *revenue minimizing* when, among the set of functions $\chi$ and $p$ that satisfy the other constraints, the mechanism selects the $\chi$ and $p$ that minimize $\max_v \sum_i p_i(s(v))$ in equilibrium, where $s(v)$ denotes the agents’ equilibrium strategy profile.

- Note: this considers the *worst case* over valuations; we could consider average case instead.
**Fairness**

- Fairness is hard to define. What is fairer:
  - an outcome that fines all agents $100 and makes a choice that all agents hate equally?
  - an outcome that charges all agents $0 and makes a choice that some agents hate and some agents like?
Fairness

- Fairness is hard to define. What is fairer:
  - an outcome that fines all agents $100 and makes a choice that all agents hate equally?
  - an outcome that charges all agents $0 and makes a choice that some agents hate and some agents like?
- **Maxmin fairness**: make the least-happy agent the happiest.

**Definition (Maxmin fairness)**

A quasilinear mechanism is **maxmin fair** when, among the set of functions $\chi$ and $p$ that satisfy the other constraints, the mechanism selects the $\chi$ and $p$ that maximize

$$\mathbb{E}_v \left[ \min_{i \in N} v_i(\chi(s(v))) - p_i(s(v)) \right],$$

where $s(v)$ denotes the agents’ equilibrium strategy profile.
When an efficient mechanism is impossible, we may want to get as close as possible.

Minimize the worst-case ratio between optimal social welfare and the social welfare achieved by the given mechanism.

**Definition (Price-of-anarchy minimization)**

A quasilinear mechanism minimizes the price of anarchy when, among the set of functions $\chi$ and $p$ that satisfy the other constraints, the mechanism selects the $\chi$ and $p$ that minimize

$$\max_{v \in V} \frac{\max_{x \in X} \sum_{i \in N} v_i(x)}{\sum_{i \in N} v_i(\chi(s(v)))},$$

where $s(v)$ denotes the agents’ equilibrium strategy profile in the worst equilibrium of the mechanism—i.e., the one in which $\sum_{i \in N} v_i(\chi(s(v)))$ is the smallest.