Application of Evolutionary Game theory to Social Networks: A Survey

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April 22, 2009

Abstract

Evolutionary game theory has been expanded to many other areas than mere biological concept of evolution. Moreover, social networks determine definite interactions between individuals in social settings. The common nature of these two vast areas, has raised interest in applying evolutionary-game-theoretical approaches to social-network-based problems. This survey paper attempts to explore and roughly categorize them in clear groups, although the vague boundaries between areas make it a difficult task.

1 Introduction

The definition and a brief history of evolutionary game theory as well as a summary of approaches to it come in the following, and then social networks are introduced. The following sections state different categories of research areas in the literature where evolutionary game theory is applied to social networks.

1.1 Evolutionary Game Theory

Generally speaking, evolutionary game theory is the application of game theory to biological concepts. It was first used by Fisher in [1], where the equal sex ratio in mammals is explained implicitly using game theoretical methods. The first use of game theory in the context of biological evolution was made by Lewontin in [2]. In late 20^{th} century, Smith coined the term "evolutionary stable strategy" in [3] as a game theoretical approach, and it sparked global interest in the field not only by biologists but also by social scientists and economists; concepts of evolution and stable state of populations were not confined to pure biological notions any more.

There are two main approaches to evolutionary game theory: the first one employs the concept of evolutionary stable strategies, and the second one analyzes the frequency of different strategies and studies the evolutionary dynamics of populations [4].

An evolutionary stable state of a population takes place when all members adopt an evolutionarily stable strategy (ESS) so that even if mutant strategies (i.e. rare novel strategies) appear in a few members, they cannot prevail and soon will vanish. This means that natural selection is sufficient to keep the population in ESS equilibria, and therefore they are in a refined subset of Nash equilibria.

The main problem being addressed by evolutionary game theory is explanation of emergence of cooperation in populations, where individuals increase others' payoffs at their cost, which is in obvious contradiction to what Darwinian evolution suggests [5]. The second approach addresses cooperation problem using replicator dynamics analysis, which exploits replicator equations (i.e. differential equations based upon population strategy distribution that determine the dynamics of strategies for specific individuals) to analyze the behavior of populations affected by selection.

1.2 Social Networks

A social network is a social graph-based structure, where nodes show entities or agents, and edges show any relation between them; the relations depend on the defined semantics of the network. Therefore, social networks are addressed in a vast range of research areas. Berkowitz is believed ([6]) to have theoretically formalized social networks for the first time in [7], while Wasserman and Faust established a sound platform for social networks analysis in [6]. Afterwards, social networks have stimulated a great interest in various research areas, but in this paper, we tend to consider them from a game theoretical point of view.

Kearns et al. introduce graphical models for game theory, where agents in a multi-agent system are supposed to be nodes of a graph, and their interactions and payoff calculations are limited to their neighbors [8]. However, the results based on this model are mostly restricted to one-stage games or graphs with special characteristics. Kakade et al., in [9], use a graph-based model for a market consisting of producers and costumers to analyze price variations in equilibria, while it tries to simulate social networks that exhibit power law (i.e. the degree of nodes in the graph show a power distribution).

Hauert and Doebeli explore the conditions under which cooperation is evolved in structured populations [10]. Lieberman et al. extend the evolutionary dynamics analysis performed on homogeneous populations to a graph-based model so as to study evolution of structured populations [11]. It can be roughly stated that [10] employs the first approach introduced in the previous subsection, while [11] uses the second one.

1.3 Structure

The following sections of this paper discuss three different categories of research papers where evolutionary game theory or main approaches associated with it are applied to problems modeled by social networks or graphs in general. Since the boundary of research areas in the literature are vaguely defined, clustering them into categories might not be unique. However, this paper has divided them in three groups; approaches associated with evolutionary dynamics come in section 2, the particular problem of evolution of cooperation is discussed in section 3, and section 4 states the widely interesting problem of network creation.

2 Evolutionary Dynamics

Evolutionary dynamics is an attempt to analyze the dynamic behavior of populations as opposed to individuals. In [11], individuals are put on vertices of a graph, and their interactions are limited to their neighbors. The sensitivity of an equilibrium to mutant strategies are studied in this setting. As in Darwinian evolution, the reproduction rate of individuals proportionally depends on their fitness; the fitter the individual, the higher probability of reproduction at each iteration the individual has. Their fitness can be statically determined (i.e. a fitness of 1 for individuals with evolutionarily stable strategy, and fitness of r for mutants), or it could be calculated by averaging over payoffs obtained from playing a number of games with neighbors. The games are traditionally either prisoner's dilemma or snowdrift (aka. chicken) games, payoff tables of which are shown in Table 1. Based on this probabilistic model, a fixation probability is calculated, i.e. the probability that a single mutant strategy takes over the whole population.

Although [12] introduces Folk's theorem of evolutionary game theory, which closely relates the results of dynamical approaches to Nash equilibria (more specifically, ESS equilibria), [13] shows that static equilibrium-based approaches cannot cover all behaviors observed from populations throughout time. It also comprehensively proposes game dynamics by which dynamical analysis can be performed as well.

Table 1: Payoff tables of prisoner's dilemma (left) and snowdrift (right) symmetric games, where ($b < d < a < c$) and ($h < f < e < g$) hold respectively.

	Cooperate	Defect
Cooperate	a	b
Defect	c	d

	Cooperate	Defect
Cooperate	e	f
Defect	g	h

Abramson et al. examine the evolutionary version of prisoner's dilemma on small-world social networks (i.e. a social network with short distance between all individuals) ranging from regular ones to totally random one, and observe different emergent behaviors [14]. Ebel and Bornholdt, in [15], employ a similar setting to show behavior of networks when mutations are incurred to a few individuals. They suggest that networks reestablish ESS equilibria after this so-called avalanche of mutations.

Another interesting problem defined on social networks is spread of influence amongst individuals. In [16], Kleinberg summarizes approaches to this problem where individuals can have two different states (*old* or *new*), and more likeness with neighbors results in more payoff. A contagion threshold is calculated which denotes the minimum number of random individuals in an all-old network to change their state so that all other individuals flip their state. The dynamical behavior is particularly of interest when agents have non-progressive property, which means agents can change their state at any time. Chwe suggests that the structure of network can have a significant impact on spread of influence [17]. [18] calculates the average time of reaching to a consensus when agents can force their opinion to neighbors in a two-candidate voting system.

Another variation of this problem could include the networks where agents can change the state of edges between them. In [19], using evolutionary dynamics, it is shown that a network with *friendly* or *unfriendly* edges, quickly reaches a stable state if agents adjust their edges by just considering local triangular relations.

3 Evolution of Cooperation

While selfish agents act according to their own preferences, sometimes they show actions in favour of the whole society yet opposed to their personal benefits. Therefore, there has been attempts to explain this altruistic behavior observed in social dilemmas, i.e. the choice between private and social interests when they are at odd with each other. One of the first explanations suggests that societies are subject of evolution rather than individuals. Hamilton, in [20], considers kinship relations and explains why family members adopt an altruistic behavior, and explores the conditions under which the cost/benefit analysis requires them to be altruists. [21] models the interaction of organisms using evolutionary stable strategies in prisoner's dilemma game, and shows how mutual cooperation can resist expiration once established enough.

There has been endeavors to characterize evolution of cooperation and the emergence conditions of it in various contexts. Limited scattering of individuals, which is called population viscosity, has shown to make clusters of genetically-determined altruistic individuals in [22]. Ohtsuki et al. show that cooperation is evolved in social networks (ranging from regular lattices to random graphs to small-world networks) when the benefit of the altruistic act divided by cost of it is greater than the average number of neighbors (see [23]).

Hauert and Doebeli, in [10], examine two games of prisoner's dilemma and snowdrift on well-mixed and structured networks while individuals, in different settings, are allowed to adopt pure or mixed strategies. Although defection is the best response to equilibria in prisoner's dilemma, any form of associative interactions or even the fear of future interactions could make individuals cooperate. Nowak and May, in [24], show that spatial populations (as opposed to well-mixed ones) can lead to formation of frozen clusters of cooperative individuals or movement of fronts of, generally speaking, any dominated strategy in a chaotic manner. However, a structured population does not necessarily lead to evolution of cooperation in snowdrift game, as shown in [10].

Dynamic relations between agents, as well as structured populations, can result in evolution of cooperation in social networks. Santos et al. show that when agents can change their social connections in the underlying network, cooperative behavior can resist (see [25]). They also suggest that social viscosity in small-world networks (also known as scale-free networks) might not be enough to evolve cooperation. Fu et al., in [26], define a measure of reputation for individuals in a network, and show that if they can switch their interaction partners to more reputable neighbors, the evolution of cooperation will be more likely to occur.

4 Network Creation

In network creation problems, agents in a network usually have some kinds of incentives to form and maintain edges, which could be costly for them. Therefore, this aspect of network creation problems is similar to the trade off in social dilemmas except for the fact that the contribution made by creating links could be limited to local areas.

Skyrms and Pemantle examine a dynamic social network where the strength of ties changes based on the repeated games played by agents [27]. They show that the dynamical structure of networks allows agents to form clusters with cooperative strategies within each. They also argue that this model gives a better sense of reality yet the computations still remain tractable. [28] also argues emergence of ESS equilibria in the situations where agents are able to change their interaction patterns, but unlike previous settings, it is based on coordination games. In [29], however, the sense of "formation" of networks is more visible. It shows that external incentives (either one-way or two-way) for agents to make and maintain links cause the network to quickly reach an equilibrium.

Another interesting problem in this category, formal definition of which is provided in [30], is the calculation of "price of anarchy" in network creation, which is the amount of reduction in social welfare when agents are not coordinated to form optimal networks anymore. Despite the dynamic nature of this problem, it has been more addressed by pure game-theoretical approaches rather than evolutionary ones. [30] suggests that price of anarchy might be negligible in selfish network creation. [31] addresses the selfish routing problem which is quite analogous to network creation problem, and it concludes that the price of anarchy for various network metrics is modest. Demaine et al. mathematically prove a fairly low upper bound for price of anarchy in the standard network creation problem (see [32]).

5 Conclusion

In this paper, evolutionary game theory is defined as the application of game-theoretical approaches to biological concepts, and a brief definition of social networks follows. Since the application of evolutionary game theory has been expanded to various areas, it touches the platform of social networks in many research areas. A fairly complicated task would be clustering different areas where evolutionary game theory or the approaches associated with it to social networks or graphs in general. This paper puts them in three different categories.

The first category includes social-network-based problems which are addressed by evolutionary dynamics; a common approach in evolutionary game theory. The second category consists of problems where evolution of cooperation is of interest in social settings, and finally the third category covers the particular area of network creation problems.

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