Lecture Overview

1. What is Game Theory?

2. Example Matrix Games
Non-Cooperative Game Theory

What is it?
Non-Cooperative Game Theory

• What is it?
  • mathematical study of interaction between rational, self-interested agents
What is Game Theory?

Non-Cooperative Game Theory

- What is it?
  - mathematical study of interaction between rational, self-interested agents

- Why is it called non-cooperative?
Non-Cooperative Game Theory

What is it?
- mathematical study of interaction between rational, self-interested agents

Why is it called non-cooperative?
- while it’s most interested in situations where agents’ interests conflict, it’s not restricted to these settings
- the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
  - cooperative/coalitional game theory has teams as the central unit, rather than agents
TCP Backoff Game

Game Theory

Consider this situation as a two-player game:

- Both use a correct implementation: both get 1 ms delay.
- One correct, one defective: 4 ms delay for correct, 0 ms for defective.
- Both defective: both get a 3 ms delay.

Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?
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Questions:
- What action should a player of the game take?
- Would all users behave the same in this scenario?
- What global patterns of behaviour should the system designer expect?
- Under what changes to the delay numbers would behavior be the same?
- What effect would communication have?
- Repetitions? (finite? infinite?)
- Does it matter if I believe that my opponent is rational?
Defining Games

- Finite, $n$-person game: $\langle N, A, u \rangle$:
  - $N$ is a finite set of $n$ players, indexed by $i$
  - $A = A_1 \times \ldots \times A_n$, where $A_i$ is the action set for player $i$
    - $a \in A$ is an action profile, and so $A$ is the space of action profiles
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \rightarrow \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player
Games in Matrix Form

Here’s the TCP Backoff Game written as a matrix (“normal form”).

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>−1, −1</td>
<td>−4, 0</td>
</tr>
<tr>
<td>D</td>
<td>0, −4</td>
<td>−3, −3</td>
</tr>
</tbody>
</table>
Lecture Overview

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Prisoner’s dilemma is any game

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<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>a, a</td>
<td>b, c</td>
</tr>
<tr>
<td>D</td>
<td>c, b</td>
<td>d, d</td>
</tr>
</tbody>
</table>

with $c > a > d > b$. 
Games of Pure Competition

Players have **exactly opposed** interests

- There must be precisely two players (otherwise they can’t have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant $c$
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it’s a one-player game
One player wants to **match**; the other wants to **mismatch**.

<table>
<thead>
<tr>
<th></th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>1</td>
<td>-1</td>
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What is Game Theory?

Example Matrix Games

Matching Pennies

One player wants to match; the other wants to mismatch.

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Play this game with someone near you, repeating five times.
Rock-Paper-Scissors

Generalized matching pennies.

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0</td>
<td>−1</td>
<td>1</td>
</tr>
<tr>
<td>Paper</td>
<td>1</td>
<td>0</td>
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...Believe it or not, there’s an annual international competition for this game!
Games of Cooperation

Players have exactly the same interests.
- no conflict: all players want the same things
- \( \forall a \in A, \forall i, j, u_i(a) = u_j(a) \)
- we often write such games with a single payoff per cell
- why are such games “noncooperative”? 
What is Game Theory?

Example Matrix Games

Coordination Game

Which side of the road should you drive on?

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Play this game with someone near you. Then find a new partner and play again. Play five times in total.
General Games: Battle of the Sexes

The most interesting games combine elements of cooperation and competition.

\[
\begin{array}{cc}
\text{B} & \text{F} \\
\text{B} & 2, 1 & 0, 0 \\
\text{F} & 0, 0 & 1, 2 \\
\end{array}
\]
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