# Game Theory intro

#### CPSC 532A Lecture 3

Game Theory intro

CPSC 532A Lecture 3, Slide 1

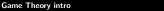
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# Lecture Overview



- 2 Example Matrix Games
- 3 Pareto Optimality
- 4 Best Response and Nash Equilibrium



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- Finite, *n*-person game:  $\langle N, A, u \rangle$ :
  - $\bullet~N$  is a finite set of n players, indexed by i
  - $A = A_1 \times \ldots \times A_n$ , where  $A_i$  is the action set for player i
    - $(a_1, \ldots, a_n) \in A$  is an action profile, and so A is the space of action profiles
  - $u = \langle u_1, \dots, u_n \rangle$ , a utility function for each player, where  $u_i: A \mapsto \mathbb{R}$
- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions  $a \in A_1$ , columns are  $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

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## Games in Matrix Form

Here's the TCP Backoff Game written as a matrix ("normal form").

$$C$$
  $D$ 

$$\begin{array}{c|ccc} C & -1, -1 & -4, 0 \\ \\ D & 0, -4 & -3, -3 \end{array}$$

It's an example of prisoner's dilemma.

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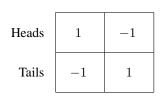
## Games of Pure Competition

Players have exactly opposed interests

- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles  $a \in A$ ,  $u_1(a) + u_2(a) = c$  for some constant c
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it's a one-player game

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#### One player wants to match; the other wants to mismatch.



Heads

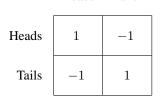
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One player wants to match; the other wants to mismatch.



Heads

Tails

Play this game with someone near you, repeating five times.

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Generalized matching pennies.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

...Believe it or not, there's an annual international competition for this game!

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# Games of Cooperation

Players have exactly the same interests.

• no conflict: all players want the same things

• 
$$\forall a \in A, \forall i, j, u_i(a) = u_j(a)$$

- we often write such games with a single payoff per cell
- why are such games "noncooperative"?

#### Coordination Game

#### Which side of the road should you drive on?

	Lett	Right
Left	1	0
Right	0	1

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Right

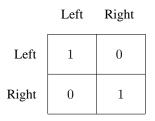
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#### Coordination Game

Which side of the road should you drive on?



Play this game with someone near you. Then find a new partner and play again. Play five times in total.

## General Games: Battle of the Sexes

The most interesting games combine elements of cooperation and competition.

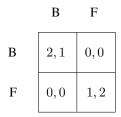
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## General Games: Battle of the Sexes

The most interesting games combine elements of cooperation and competition.



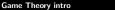
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- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?

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- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?
  - we have no way of saying that one agent's interests are more important than another's
  - intuition: imagine trying to find the revenue-maximizing outcome when you don't know what currency has been used to express each agent's payoff
- Are there situations where we can still prefer one outcome to another?

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- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that *o* Pareto-dominates *o*'.

(3)

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome  $o^\prime$ , and there is some agent who strictly prefers o to  $o^\prime$ 
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• An outcome  $o^*$  is Pareto-optimal if there is no other outcome that Pareto-dominates it.

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- An outcome  $o^*$  is Pareto-optimal if there is no other outcome that Pareto-dominates it.
  - can a game have more than one Pareto-optimal outcome?
  - does every game have at least one Pareto-optimal outcome?

#### Pareto Optimal Outcomes in Example Games

$$C$$
  $D$ 

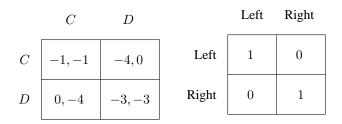
$$\begin{array}{c|c|c} C & -1, -1 & -4, 0 \\ \hline D & 0, -4 & -3, -3 \end{array}$$

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#### Pareto Optimal Outcomes in Example Games



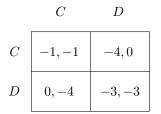
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Right

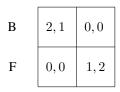
## Pareto Optimal Outcomes in Example Games



Left	1	0
Right	0	1

Left

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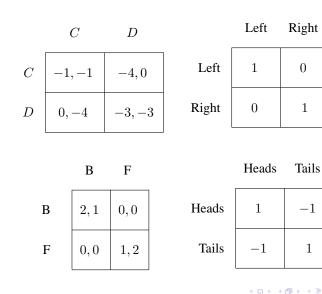


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#### Pareto Optimal Outcomes in Example Games



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#### Best Response

• If you knew what everyone else was going to do, it would be easy to pick your own action

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• If you knew what everyone else was going to do, it would be easy to pick your own action

• Let 
$$a_{-i} = \langle a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n \rangle$$
.

• now 
$$a = (a_{-i}, a_i)$$

• Best response:  $a_i^* \in BR(a_{-i})$  iff  $\forall a_i \in A_i, u_i(a_i^*, a_{-i}) \ge u_i(a_i, a_{-i})$ 

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- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

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- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

- Idea: look for stable action profiles.
- $a = \langle a_1, \ldots, a_n \rangle$  is a ("pure strategy") Nash equilibrium iff  $\forall i, a_i \in BR(a_{-i})$ .

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#### Nash Equilibria of Example Games

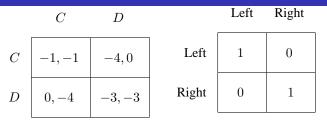
$$\begin{array}{c|c} C & D \\ \hline \\ C & -1, -1 & -4, 0 \end{array}$$

$$D = 0, -4 = -3, -3$$



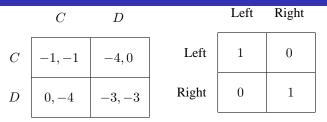
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## Nash Equilibria of Example Games

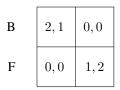




## Nash Equilibria of Example Games

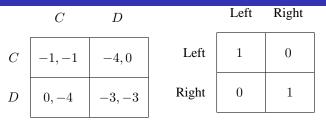


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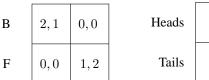
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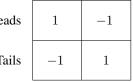
## Nash Equilibria of Example Games



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Heads Tails





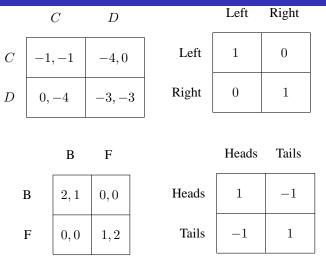
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## Nash Equilibria of Example Games



The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!