Online Combinatorial Auctions

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Abstract

This paper explores the feasibility and motivation for online combinatorial auctions. It explains combinatorial auctions and shows an innovative implementation of an iterative combinatorial auction. It then introduces online auctions and highlights some research which bounds the efficiency and revenue comparison to the standard Vickrey auction. Finally, motivations for online combinatorial auctions are included as well as some work that has been done on this topic.

1 Introduction

This paper will explore the idea of online combinatorial auctions. Essentially, this is the idea of auctioning multiple heterogeneous goods to bidders who may arrive and leave at any moment. Therefore the mechanism must be able to make an allocation and charge a payment any time if needed. In the 2nd section, this paper will describe combinatorial auctions. Then in the 3rd section, an example of a innovative new combinatorial auction is provided. In section 4, online auctions are discussed. Section 5 provides the motivation for creating combinatorial online auctions and section 6 shows a framework which provides the bidder support necessary for creating strategies for online combinatorial auctions in a real application. The paper concludes with section 7 explaining some future work needed to successfully implement combinatorial online auctions.

2 Combinatorial Auctions

A combinatorial auction involves bidding on multiple items simultaneously. It is more complicated than just combining multiple direct auctions because goods might not have additive value functions. Instead goods could either be complements or substitutes. If two goods are complements, it means that together they are each worth more than separate. An example of complements is a snowboard and a lift pass. In that case they are only worth having if you have both. If two goods are substitutes, it means they are each worth more when you have just one instead of two. An example of substitutes is a Burton snowboard and a Ride snowboard (of the same style). You only need one board so the value of the second board decreases if you already own the first one.

Therefore in a combinatorial auction, a bidder gives the mechanism the information regarding the relationship between all the items. A bidder tells which items are complements and substitutes by specifying how his value changes for all the different bundles. The most straightforward way for the bidder to specify his valuations would be to tell the mechanism the value for every possible bundle. However, there are an exponential number of bundles, so this is not practical. Bidding languages provide a better way to represent the preferences of different bundles in combinatorial auctions.

Basic bidding languages include OR and XOR or a combination. In the OR bidding language every given atom has an associated value. Bundles can be formed by combining any possible atoms and adding their valuations. This is how it would be done if there were no complements or substitutes. In the XOR bidding language each of the atoms can be seen as bundles which cannot be combined, so only one atom can be put in a bundle. A bidding language that is more efficient is a combination of XOR and OR.

Another bidding language that has been very successful is OR*. OR* is just as expressive as the combination of XOR and OR, and it is also more compact. It simulates XOR by creating new dummy goods. These dummy goods can only be included in a bundle once, so if the same dummy good is added to atoms which are XORed, only one of these atoms can be chosen, thus simulating the XOR.

Combinatorial auctions have many practical applications. They been successfully used in the FCC spectrum auctions[1] and airport gate allocation [2]. In the FCC spectrum auction they were able to generate more revenue and make the bidders more satisfied than earlier methods.

3 ICE: An Iterative Combinatorial Exchange

ICE is a implementation of a combinatorial exchange where bids are encoded as trees. The trees specify a lower and upper bound for a trade. In this way the tree based bidding language expresses a bidder's valuations more efficiently. A combinatorial exchange is a combinatorial auction combined with a double auction. In a double auction bidders and sellers trade units of the same good. For example a simple stock exchange would be a double auction since all the stocks of a certain company are basically identical and buyers and sellers exchange these stocks. In a combinatorial exchange goods are no longer identical and bidders have complicated valuations for them just like in normal combinatorial auctions.

ICE is a iterative combinatorial exchange which means there will be several rounds that the bidders will interact with the mechanism. The bidders specify a range in which their valuation lies through the lower and upper bounds in the trees which they tighten as the rounds progress. The advantage of this approach is that bidders must not reveal their valuation and the mechanism works even if bidders aren't sure of their exact evaluation. The Myerson-Satterthwaite impossibility theorem states that no mechanism can be incentive compatible, efficient, budget-balanced, and individually rational. ICE is trying to walk a middle line by creating an efficient allocation, that is also IC, IR, and BB through the threshold payment rule [3]. Parkes introduced the Threshold payment rule described in [3] which reduces the ex-post incentive to bid untruthful valuations. In this way Parkes [4] introduces a new useful mechanism for combinatorial exchanges that is being tested as a Java applet.

4 Online Auctions

4.1 The classic secretary problem

The classic secretary problem is a scenario where there are a known number of applicants of varying quality vying for the job of a secretary. You have to interview them one by one and have to make an irreversible decision whether or not to hire them immediately after the interview. Since the candidates are ordered randomly, the question is which one do you accept to optimize your chances of hiring the best one.

The optimal strategy is to pick a number x where you reject the first x candidates, and just sample the talent available. Then you accept any candidate after the first x who is more talented than the best candidate of the first x. If you choose x to be n/e, where n is the number of applicants and e is the base of natural logarithms, you will have a $1/e$ chance of selecting the best candidate.

In online auctions, we have the same problem of deciding when to assign the good to a bidder. We can use the same optimal strategy as in the secretary problem, provided we know the total number of bidders.

4.2 Adaptive Limited-Supply Online Auctions

Adaptive limited-supply online auctions are online auctions where there are k identical goods being sold with $k \geq 1$. They are adaptive because they will try to maximize the profit of the auction. Hajiaghayi et. al. [5] use a similar approach to these online auctions as the secretary problem except that they have to account for the fact that more than one bidder could be present at one time.

For the $k = 1$ case, the mechanism works as follows: for a given number j, the mechanism keeps track of the first j bids coming in. After j bids have come in, the mechanism calculates the highest bid, p and sells it to the highest bidder at the second highest price if he is still available. After the jth bidder has arrived, the item is sold to the first bidder whose bid is higher than the highest seen so far at price p. If we let $j = \lfloor \frac{n}{2} \rfloor$ then we get a 4+o(1)-competitive strategyproof online algorithm compared to the offline Vickrey mechanism with respect to revenue and efficiency. If we follow the optimal solution to the secretary problem and let $j = \lfloor \frac{n}{e} \rfloor$, then we trade off efficiency for revenue. We get a more efficient mechanism at $\frac{1}{e} - O(\frac{1}{n})$ efficiency, but $\frac{1}{e}^2 - O(\frac{1}{n})$ revenue.

The same idea can be applied to multi-item auctions ($k > 1$). The bidders are divided into two subsets. $\lceil k/2 \rceil$ identical items are sold to the first subset, and $\lfloor k/2 \rfloor$ items are sold to the second subset. The first subset consists of the first j bidders for some variable j. Sell the $\lceil k/2 \rceil$ items to the first $\lceil k/2 \rceil$ bidders at the price of the highest bid that didn't receive an item, so the $(\lceil k/2 \rceil + 1)$th bid. If $j < \lceil k/2 \rceil$, then let the price be 0 and discard the rest of the items. For the bidders that arrive after the jth bidder (the second set), sell the items at the bid price to any bidders whose bid is greater than $opt^s(b')$ where b' is the first set.

$$F_p^s(b) = p * min(s, n_p(b)),$$
$$opt^s(b) = argmax_p(F_p^s(b))$$

Let b be the set of bids and n_p be the number of bids greater than or equal to p. Therefore $F_p^s(b)$ is the revenue obtained by selling at most s items at price p. $opt^s(b)$ is the price for making the most revenue selling s items to set b.
If we choose j to be drawn from the binomial distribution $B(n, 1/2)$, then this mechanism does not depend on k, the number of items in it's efficiency or revenue. It is C-competitive with respect to the Vickrey auction for efficiency and revenue for C $<$ 48.

5 Motivations for Online Combinatorial Auctions

Combining combinatorial auctions with online auctions would provide solutions to a new set of problems. The mechanism could allocate heterogeneous goods with complicated interrelated valuation functions for which bidders would arrive and leave at any time. Therefore the mechanism would be able to make an allocation decision along with a payment value at any point in time.
This would expand the use of combinatorial auctions beyond what it is today. Combinatorial auctions are already used in spectrum auctions such as the FCC. By making them online, they could become much more widespread. They could be used for business liquidations. Bidders might be interested in a network setup but it would be worth much more to them if they could get all the routers and access points that they needed. The same would be true for office furniture. Bidders might want office furniture, but if there weren't enough desks then the set would be worth a lot less. By creating more feasible online mechanisms, there could be more widespread use. The winner determination problem in combinatorial auctions is NP-complete and thus computationally intractable, but research has been done on winner determination [6].
Combinatorial auctions for land plots could also become more widespread. There could be a general public auction where land could be sold. Whenever a

big plot of land is put up for sale as cities expand, it can be sold on this auction. If a business is looking for a place to build a commercial enterprise, they could go on this auction site and bid given their valuations. There could be a simple version where the items being sold doesn't change, only the number of bidders. Of course the more complicated (and useful) case would include the ability to add new land at any time.

Combinatorial auctions have been used in the FCC spectrum auctions to increase revenue and satisfaction of buyers [1]. The same advantages can be applied to more commonplace events if this could be done online. Collectible goods such as stamps or baseball cards could be traded on a combinatorial exchange. For extremely rare items, a combinatorial exchange might be more effective than just selling the goods in a single item auction. If people can complete a collection, it will be much more valuable to them.

The idea of using combinatorial auctions for airport takeoff and landing rights can be expanded as well. The following example will illustrate my idea. Let's say that the Denver airport was shut down due to a heavy snow storm. With United's hub being in Denver and Chicago, United had a big problem with most of their flights being cancelled. If there were some online auction where airlines could temporarily trade landing and take-off rights, United could temporarily buy new rights at another airport from airlines that didn't need it. Thereby the planes that couldn't fly to Denver could just fly to this new temporary hub and a crisis would be averted.

As the previous examples have shown, the addition of an online mechanism to combinatorial auctions would make them much more applicable. The problem is that the complexity of the mechanism increases dramatically and the mechanism could become intractable. It also becomes much more complicated for the bidder who can now bid at different times and change his bid, therefore increasing his strategy space dramatically.

6 Comprehensive Bidder Support in Online Combinatorial Auctions

In Online Combinatorial Auctions bidders have the chance to have multiple interactions with the mechanism. Therefore the question arises whether the bidder should change his bid, and if so, by how much. In iterative combinatorial auctions it is usually the case that in between rounds, bidders receive very little information which might not even include whether they are winning or not. This leaves them with little to base this decision on. For online combinatorial auctions this information must be available at any time-step, as bidders could arrive or leave at any time. Adomavicius and Gupta [7] address this issue by designing a frame work in which the bidder receives more information to make such a decision. The bidder can query for the set of winning bids and price levels at any time. He can also query whether his bid as a possibility of winning depending on the bids of other agents. For example if he bids 2 for item A,

and another bidder bids 5 for AB, then he does not have the winning bid. But if a third bidder comes and bids 8 for B, then he becomes the highest bidder again without changing his bid. The seller can also query the total revenue of the most efficient allocation at any point.
Experimental results show that it is possible to keep the system updated after every bid with up to twenty-five different items. These results were obtained on an average home computer[1]. These results show that the first requirements have been met to create online combinatorial auctions.

7 Future Work on Online Combinatorial Auctions

We have seen that there is interest in online combinatorial auctions. The groundwork has been done by showing that bidders can get the information they need in real time thus making them tractable (for relatively small auctions with size < 25). The number of possible heterogeneous goods has to be increased to make it even more practical.
The same ideas present in the ICE project can be applied to online combinatorial auctions. By making the bidding language compact and expressive in a tree structure, the communication throughput is reduced. By specifying a range for a valuation, the mechanism could become for useful for the same motivations as ICE. It could also have several rounds of iteration in order to tighten the bounds.
It would be interesting to see if efficient online mechanisms could be built such that they don't need to know the total number of bidders to optimize revenue. Inherently in online auctions, there is much less information available to the mechanism when it has to make its decision. Therefore the efficiency and revenue are much less than in the Vickrey mechanism. The same idea of using the Secretary problem to maximize revenue by splitting the bidders into two groups can be used in the online combinatorial case.
Many of the ideas already used in combinatorial auctions can be expanded to the online case.

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