Wheelchair Collision Avoidance: An Application for Differential Games

Pooja Viswanathan

December 27, 2006

Abstract

This paper discusses the design of intelligent wheelchairs that avoid obstacles using differential games. Models for similar applications are investigated. Techniques are provided to satisfy multiple performance requirements of the intelligent wheelchair system. Finally, areas of future work are identified.

1 Introduction

Mobility has been identified as a key component of physical well-being and happiness, enabling people to interact with their surroundings. Unfortunately, the mobility and independence of many older adults is often reduced due to physical disabilities. Physically disabled older adults with cognitive impairments, such as dementia, lack the skills required to safely manoeuvre powered wheelchairs. Exclusion from the use of powered wheelchairs and the lack of strength needed for manual wheelchair use result in greatly impaired or non-existent mobility for a large number of LTC residents. Reduced mobility often leads to decreased opportunities to socialize, thus leading to social isolation and depression [11]. Loss of mobility also results in increased dependence on care-givers in order to fulfill daily tasks. It is thus imperative to design wheelchairs that enable safe and independent mobility, thereby improving the quality of life of wheelchair users while simultaneously reducing the burden on caregivers.

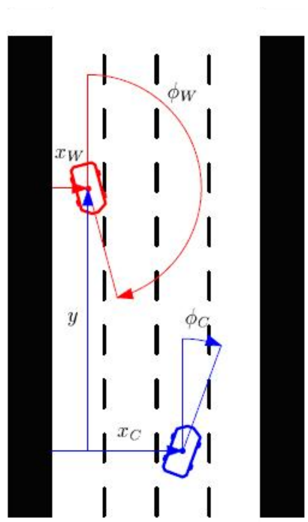
To address this need, several intelligent wheelchairs that prevent collisions have been developed in the past decade [11, 13, 9]. In most cases, the wheelchair is stopped upon physical contact of sensors with an obstacle. It has been reported that 73-80% of older adults experience a trip or fall after even a minor collision with a wheelchair [2], and 5-10% of these falls result in a fracture, particularly a hip fracture [10]. Hip fractures have serious consequences for this population, usually leading to a severe reduction in mobility and up to a 40% mortality rate within six months as a result of complications [4]. Thus, a non-contact method of collision avoidance is needed to ensure the safety of residents in LTC facilities. In [9], a stereovision camera is used to detect obstacles. Maps are constructed of the wheelchair's surroundings and the wheelchair is stopped if an object is detected within a predefined distance threshold. Although this method is successful at detecting stationary objects, it does not work as well for moving objects. The authors suggest that the time to collision with an obstacle be used instead of distance. However, this method assumes that the moving obstacles continue along the same path. In the context of long-term care facilities, where almost 63% of residents have dementia or Alzheimer's disease [5], movements of residents can be quite unpredictable. Thus, we need to compute optimal steering and velocity control strategies for each wheelchair user against the worst possible maneuvers of other wheelchair users. This worst-case scenario can be modeled as a pursuit evasion game described in the next section. Subsequent sections explain differential games and their application to collision avoidance. In addition to preventing collisions, we might also want to optimize over other criteria such as minimal deviation from route and wheelchair user comfort. Techniques for implementing multiple performance requirements, as well as future areas of research are discussed.

2 Pursuit Evasion Games

A pursuit-evasion game involves two main players: the pursuer, denoted by P, and the evader, denoted by E. In more complex games, these players can represent two groups of vehicles opposing each other, however, in this paper, P and E govern a single vehicle. Thus, the symbols represent the vehicles themselves, which can be steered by either a human pilot or an automatic mechanism. Specifically, P represents some fixed point on the pursuing vehicle such that P's coordinates specify the geometric location of the vehicle [3]. The pursuit game usually ends as soon as a capture occurs, when the distance between P and E is less than some pre-specified threshold. A natural method to value the outcome of this conflict is to use the capture time as the payoff [1]. If P can enforce capture, he wants to minimize capture time, while E wants to maximize it. This payoff can be incorporated into differential games described in the following section.

3 Overview of Differential Games

Differential game theory has been used in varying fields, such as collision avoidance, economy and ecology. This paper investigates the use of differential games for the task of collision avoidance. Several authors have used the methods described in this paper to avoid collisions on highways, control air traffic, and prevent collisions at sea [6, 7, 12, 14]. Models used for collision prevention on highways are explored in this paper due to their high applicability to the wheelchair collision avoidance problem. Specifically, the paper deals with two-person zero-sum differential games with variable terminal time and complete



State Variable Equalities:

$$\dot{x}_W = v_W \sin\phi_W \quad (1)$$

$$\dot{x}_C = v_C \sin\phi_C \quad (2)$$

$$\dot{y} = v_W \cos\phi_W - v_C \cos\phi_C \quad (3)$$

$$\dot{v}_C = b_C\, \eta_C \quad (4)$$

$$\dot{\phi}_W = w_W\, u_W \quad (5)$$

$$\dot{\phi}_C = w_C(v_C)\, u_C \quad (6)$$

Control Variable Inequalities:

$$-1 \le u_W \le +1 \quad (7)$$

$$-1 \le u_C \le +1 \quad (8)$$

$$-0.1 \le \eta_C \le +1 \quad (9)$$

State Variable Inequalities:

$$0\,\mathrm{m} \le x_W \le 15\,\mathrm{m} \quad (10)$$

$$0\,\mathrm{m} \le x_C \le 15\,\mathrm{m} \quad (11)$$

Figure 1: Kinematic Equations for the Wrong (W) and Correct (C) Drivers

information.

In order to model moving vehicles, we need control and state variables. Players make their decisions by choosing the values of specific control variables. These values, in turn, govern the values of certain other quantities known as state variables [3]. State variables satisfy the property that if their values are known at any instant, the state of the game at that instant is fully determined. For the purposes of this paper, only games with complete information are treated, thus we assume that the current values of the state variables are always known to both players.

4 Differential Game Formulation

A formulation for collision avoidance on highways is provided in [6]. On a highway, in the worst case scenario, a correct driver is faced with a wrong driver ahead. This scenario is modeled as a pursuit-evasion differential game, with the correct driver represented by an evader, and the wrong driver by a pursuer. For this collision avoidance problem, the kinematic equations are modeled with point mass models as shown in Figure 1. The equations have independant variable t, state variables x_W, x_C (distance of W and C, respectively, from the left freeway side) and y (distance between W and C along the freeway) that encode the positions, and state variables ϕ_W, ϕ_C (driving direction of W and C respectively), v_W and v_C (velocity of W and C respectively) for the velocity vectors. The control variables are u_W and u_C (turn rate of W and C respectively) and η_C (velocity change rate). In this model, v_W is assumed to be constant, and the maximum angular velocities $w_W(v_W)$ and $w_C(v_C)$ are given car dependant. The control variable inequality constraints (7) and (8) define kinematic constraints of bounded radius of curvature, and the constraint (9) defines kinematic constraints of acceleration and deceleration. In order to stay on the freeway, W and C must obey constraints (10) and (11) respectively. These constraints can be included in the wheelchair application, since most long-term care facilities consist mainly of hallways of fixed width. The objective function of the game is defined as

$$J(\gamma_W, \gamma_C; t = 0, z = z_0) := t_f$$

where vector $z := (x_W, x_C, y, v_c, \phi_W, \phi_C)^\top$ of state variables, γ_W and γ_C are the (feedback) strategies of W and C respectively, and t_f is the capture time (which is ∞ if capture is not possible). W tries to drive the state z from the initial state z_0 to the terminal state in minimum time. C tries to avoid capture, or maximize capture time if escape is impossible.

$$\max_{\gamma_C} \min_{\gamma_W} J \equiv \max_{\gamma_C} \min_{\gamma_W} t_f \texttt{ for all } \gamma_C, \gamma_W \in C_p^0[0, \infty[$$

where $C_p^0[0, \infty[$ denotes the set of all piecewise continuous functions defined on the interval $[0, \infty[$ [1]. It is assumed that the operators min and max commute, therefore

$$\max_{\gamma_C} \min_{\gamma_W} J \equiv \min_{\gamma_W} \max_{\gamma_C} J$$

The parameters required for these equations can be obtained by using the available devices/sensors on each wheelchair . For example, the speed of a wheelchair can be obtained from an odometer installed on the wheelchair. The wheelchair's position is estimated using a probabilistic framework as described in [9]. A wheelchair can also exchange this information with other wheelchairs through a central server.

5 Hybrid Systems and Zero-Sum Differential Games

Lygeros et al. [7] model the collision avoidance problem in an Automated Highway System (AHS) using a hierarchical hybrid controller. Hybrid systems are those that involve the interaction of discrete and continuous dynamics. In the AHS problem, the continuous dynamics reflect the movement of vehicles, and discrete dynamics reflect possible collisions between vehicles. The system is assumed to be fully decentralized, which means that agent only has access to local information that is available through its sensors. The controller design can be described as a game between the controller and a disturbance created by the environment, which includes the actions of other agents. Safety is the main priority of this application and is achieved by modeling the system as a zero-sum differential game. A similar model can be used to model collision avoidance for wheelchairs.

5.1 Hybrid System Modeling Formalism

In [12], a hybrid system H is defined to be the tuple $H = (Q \times M, U \times D, \sum, I, \text{Inv}, E, f)$, in which

- $Q \times M$ is the state space, with $Q = \{q_1, q_2, ..., q_m\}$ a finite set of discrete states and M an n-manifold. A state of the system is a pair $(q_i, x) \in Q \times M$;
- $U \times D \in \mathbb{R}^u \times \mathbb{R}^d$ is the product of the input set and disturbance set. The space of acceptable control and disturbance trajectories are denoted by $\mathcal{U}$ and $\mathcal{D}$;
- $\sum$ is a finite set of transition labels;
- $I \in Q \times M$ is the set of initial conditions;
- Inv: $Q \rightarrow 2^M$ is the invariant associated with each discrete state, i.e. state (q, x) may flow within q only if $x \in \text{Inv}(q)$;
- $E \subset Q \times M \times \sum \times Q \times M$ is the set of discrete jumps with $(q, x, \sigma, q', x') \in E$, i.e. if the current state is (q, x), the system may instantaneously take a discrete transition σ to state (q', x');
- $f : Q \times M \times U \times D \rightarrow TM$ is a map that associates a control system $f(q, x, u, d)$ (denoted $f_q(x, u, d)$) with each discrete state q.

5.2 Two-Player Zero-Sum Differential Games

Using the definition of the hybrid system above, we can now define a two-player, zero-sum differential game as follows [7]. A two-player, zero-sum differential game $(T, \mathcal{X}, \mathcal{U}, \mathcal{D}, \mathcal{Y}, H, J)$ where

- $T = [t_i, t_f]$ is a time interval
- $\mathcal{X}$ is a trajectory space;
- $\mathcal{U}$ is an input space;
- $\mathcal{D}$ is a disturbance;
- $\mathcal{Y}$ is an output space;
- H is a hybrid system;
- $J : \mathcal{Y} \times \mathcal{U}; \times \mathcal{D} \rightarrow \mathbb{R}$ is a cost function (as defined in Section 4).

Player 1 controls the input $u \in \mathcal{U}$ and receives reward $-J(y, u, d)$ for a given play, while player 2 controls the disturbance $d \in \mathcal{D}$ and receives reward $J(y, u, d)$. As in section 4, player 1 tries to minimize J while player 2 tries to maximize it.

For this class of games, we are interested in finding a global saddle solution [7], which is a pair of input and disturbance trajectories $(u^*, d^*) \in \mathcal{U} \times \mathcal{D}$ such that for all $(q^0, x^0) \in X$, all $u \in \mathcal{U}$ and all $d \in \mathcal{D}$:

$$J((q^0, x^0), u^*, d) \leq J((q^0, x^0), u^*, d^*) \leq J((q^0, x^0), u, d^*)$$

A saddle solution has the property that any unilateral deviation from it leaves the player who deviates worse off. The saddle solution thus enables us to compute the set of safe states and classify all the safe controls. Although existence and uniqueness of the saddle solution cannot be guaranteed in general, the games considered in this paper have unique saddle solutions [7].

5.3 Multi-Objective Hybrid Controllers

In addition to ensuring safety, we might want to also optimize efficiency, for example provide a comfortable ride for the wheelchair users. In the AHS system, comfort implies small control inputs and is encoded using another cost function:

$$J'(q^0, x^0, u, d) = \sup_{t \geq 0} |u(t)|$$

A maneuver is said to be comfortable if $J'(q^0, x^0, u, d) \leq C'$ where C' is defined as $2.5ms^{-3}$ in the AHS application. The authors then deal with the performance requirements of both safety and efficiency by solving a sequence of nested games.

Since higher priority is given to safety, the game for J is solved first. Assuming that the game has a saddle solution, we know there exist input and disturbance trajectories, u^* and d^* such that:

$$J^*(q^0, x^0) = \max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} J(q^0, x^0, u, d) = \min_{u \in \mathcal{U}} \max_{d \in \mathcal{D}} J(q^0, x^0, u, d) = J(q^0, x^0, u^*, d^*)$$

Let the set $V = \{(q, x) \in X | J^*(q, x) \leq C\}$ contain all states for which there exists a control such that the objective on J is satisfied for any allowable disturbance. Note that u^*, if used as a control law, guarantees that J is minimized for the worst possible disturbance, and guarantees satisfaction of the safety requirement if the initial state is in V.

Next, to include efficiency in the controller design, let $\mathcal{U}(q^0, x^0) = \{u \in \mathcal{U} | J(q^0, x^0, u, d^*) \leq C\}$. $\mathcal{U}$ thus maps to each state the least restrictive class of safe controls. In order to select the control that minimizes cost function J' within this class, the authors again pose the problem as a zero-sum differential game. They assume that a saddle solution (u'^*, d'^*) exists and let $J'(q^0, x^0)$ be the corresponding cost. Then the set $V' = \{(q, x) \in X | J'^*(q, x) \leq C'\}$ contains the initial conditions for which there exists a control such that the requirements on both J and J' are met for any allowable disturbance. Note that since the minimax problem can only be posed when $\mathcal{U}(q^0, x^0) \neq \emptyset$, i.e. $(q^0, x^0) \in V$ since $u^* \in \mathcal{U}(q^0, x^0)$, we assume that $V' \subset V$. Thus, there might be states for which the requirement for safety can be satisfied, however that for efficiency can not. Additional performance requirements can be included by creating new zero-sum differential games and solving them sequentially as shown above.

6 Conclusion

This paper summarizes existing methods to model the collision avoidance problem. Differential equations are required to capture the dynamics in the system. Cost functions are derived for each performance requirement, and nested two-person zero-sum differential games are solved to satisfy the performance requirements, in their order of priority. When complete information is available, the model described in this paper can be used to ensure collision avoidance in wheelchairs. However, sensors used in real-life applications are usually noisy. For example, a wheelchair's sensing device might only give it a probability distribution of the location of the obstacle. In this case, further research can be conducted to model differential games with incomplete information as described in Chapter 12 of [3]. Implementing and testing the model described in this paper for the intelligent wheelchair application can help identify its strengths and weaknesses, based on which the model can be modified to improve performance. Techniques for real-time computation of the optimal strategies in the games discussed here have been described in [6]. It is hoped that the methods described in this paper will ensure safe navigation by cognitively-impaired wheelchair users, thus helping to restore their mobility and improve their quality of life.

References

[1] M. H. Breitner, *Complex differential games of pursuit-evasion type with state constraints, part 1: Necessary conditions for optimal open-loop strategies*, Journal of optimization theory and applications **78** (1993), no. 3, 419.

[2] T. A. Corfman, R. A. Cooper, M. J. Dvorznack, M. L. Boninger, W. A. Ammer, M. Sheesley, R. Cooper, and T. Thorman, *A video-based analysis of trips and falls during electric powered wheelchair driving*, Presented at RESNA Annual Conference, Reno, NV, 2001.

[3] R. Isaacs, *Differential games: A mathematical theory with applications to warfare and pursuit, control and optimization.*

[4] S. Jaglal, P. G. Sherry, and J. Schatzker, *The impact and consequences of hip fracture in ontario*, Canadian Journal of Surgery **39** (1996), 105–111.

[5] R. A. Kane and A. L. Caplan (eds.), *Everyday ethics: Resolving dilemmas in nursing home life*, Springer Publishing Company, New York, 1990.

[6] R. Lachner, *Collision avoidance as a differential game: real-time approximation of optimal strategies using higher derivatives of the value function*, vol. 3, 1997, pp. 2308–2313 vol.3.

[7] J. Lygeros, D. N. Godbole, and S. Sastry, *Verified hybrid controllers for automated vehicles*, Automatic Control, IEEE Transactions on **43** (1998), no. 4, 522–539.

[8] J. Lygeros, C. Tomlin, and S. Sastry, *Multi-objective hybrid controller synthesis: least restrictive controls*, Proceedings of the 36th Conference on Decision & Control, San Diego, California USA, December 1997, vol. 1, pp. 127–132.

[9] A. Mihailidis, P. Elinas, J. Boger, and Jesse Hoey, *An intelligent powered wheelchair to enable mobility of cognitively impaired older adults: An anti-collision system*, IEEE Transactions on Neural Systems & Rehabilitation Engineering (in press) (2006).

[10] M. C. Nevitt, S. R. Cummings, S. Kidd, and D. Black, *Risk factors for recurrent nonsyncopal falls*, Journal of the American Medical Association **261** (1989), 2663–2668.

[11] Richard C. Simpson, *Smart wheelchairs: A literature review*, Journal of rehabilitation research and development **42** (2005), no. 4, 423.

[12] C. Tomlin, G. J. Pappas, and S. Sastry, *Conflict resolution for air traffic management: a study in multiagent hybrid systems*, Automatic Control, IEEE Transactions on **43** (1998), no. 4, 509–521.

[13] J. K. Tsotsos, *Playbot: A visually-guided robot for physically disabled children*, Image and vision computing **16** (1998), no. 4, 275.

[14] T. L. Vincent, *Collision avoidance at sea*, Proceedings of a Workshop on Differential Games and Applications, Enschede, Netherlands, 16-25 March 1977, pp. 205–221.