Analyzing Games: Mixed Strategies

CPSC 532A Lecture 5

September 26, 2006

Analyzing Games: Mixed Strategies

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Lecture Overview

Recap

Mixed Strategies

Fun Game

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Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there some agent who strictly prefers o to o'
 - in this case, it seems reasonable to say that o is better than o'
 - ▶ we say that *o* Pareto-dominates *o*′.

An outcome o* is Pareto-optimal if there is no other outcome that Pareto-dominates it.

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Best Response

 If you knew what everyone else was going to do, it would be easy to pick your own action

• Let
$$a_{-i} = \langle a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n \rangle$$
.

• now
$$a = (a_{-i}, a_i)$$

▶ Best response:
$$a_i^* \in BR(a_{-i})$$
 iff
 $\forall a_i \in A_i, u_i(a_i^*, a_{-i}) \ge u_i(a_i, a_{-i})$

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Nash Equilibrium

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

- Idea: look for stable action profiles.
- $a = \langle a_1, \ldots, a_n \rangle$ is a Nash equilibrium iff $\forall i, a_i \in BR(a_{-i})$.

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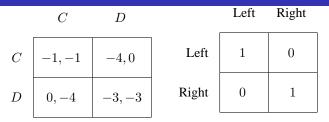
$$\begin{array}{c|c} C & D \\ \hline C & -1, -1 & -4, 0 \\ D & 0, -4 & -3, -3 \end{array}$$

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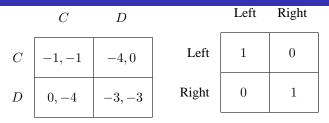
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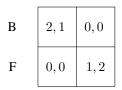


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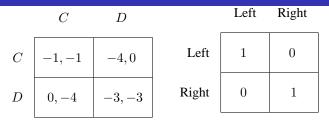


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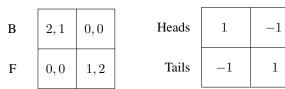
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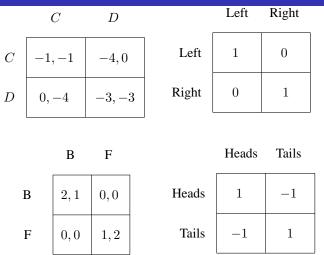


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The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!

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Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy s_i for agent i as any probability distribution over the actions A_i.
 - pure strategy: only one action is played with positive probability
 - mixed strategy: more than one action is played with positive probability
 - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be S_i
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$.

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile s ∈ S?
 - We can't just read this number from the game matrix anymore: we won't always end up in the same cell

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Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile s ∈ S?
 - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$
$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

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Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

- Best response:
 - ► $s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i, u_i(s_i^*, s_{-i}) \ge u_i(s_i, s_{-i})$

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- Nash equilibrium:
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Every finite game has a Nash equilibrium! [Nash, 1950]
 e.g., matching pennies: both players play heads/tails 50%/50%

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	В	F
В	2, 1	0,0
F	0,0	1, 2

- It's hard in general to compute Nash equilibria, but it's easy when you can guess the support
- For BoS, let's look for an equilibrium where all actions are part of the support

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- Let player 2 play B with p, F with 1 p.
- ▶ If player 1 best-responds with a mixed strategy, player 2 must make him indifferent between *F* and *B* (why?)



- Let player 2 play B with p, F with 1 p.
- If player 1 best-responds with a mixed strategy, player 2 must make him indifferent between F and B (why?)

$$u_1(B) = u_1(F)$$

 $2p + 0(1-p) = 0p + 1(1-p)$
 $p = \frac{1}{3}$



- Likewise, player 1 must randomize to make player 2 indifferent.
 - Why is player 1 willing to randomize?



- Likewise, player 1 must randomize to make player 2 indifferent.
 - Why is player 1 willing to randomize?
- Let player 1 play B with q, F with 1-q.

$$u_2(B) = u_2(F)$$

$$q + 0(1 - q) = 0q + 2(1 - q)$$

$$q = \frac{2}{3}$$
Thus the mixed strategies (²/₃, ¹/₃), (¹/₃, ²/₃) are a Nash equilibrium.

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Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy? Different interpretations:

- Randomize to confuse your opponent
 - consider the matching pennies example
- Players randomize when they are uncertain about the other's action
 - consider battle of the sexes
- Mixed strategies are a concise description of what might happen in repeated play: count of pure strategies in the limit
- Mixed strategies describe population dynamics: 2 agents chosen from a population, all having deterministic strategies. MS is the probability of getting an agent who will play one PS or another.

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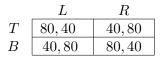
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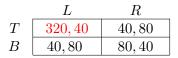


Play once as each player, recording the strategy you follow.

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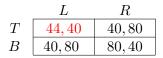
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Play once as each player, recording the strategy you follow.

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- Play once as each player, recording the strategy you follow.
- What does row player do in equilibrium of this game?
 - row player randomizes 50-50 all the time
 - that's what it takes to make column player indifferent
- What happens when people play this game?
 - with payoff of 320, row player goes up essentially all the time
 - with payoff of 44, row player goes down essentially all the time

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