One more example

Analyzing Games

CPSC 532A Lecture 4

September 21, 2006

Lecture Overview

Recap

One more example

Pareto Optimality

Best Response and Nash Equilibrium

Defining Games

- ▶ Finite, n-person game: $\langle N, A, u \rangle$:
 - N is a finite set of n players, indexed by i
 - $lack A = \langle A_1, \dots, A_n \rangle$ is a tuple of action sets for each player i
 - \bullet $a \in A$ is an action profile
 - \bullet $u = \langle u_1, \dots, u_n \rangle$, a utility function for each player, where $u_i:A\mapsto\mathbb{R}$
- Writing a 2-player game as a matrix:
 - row player is player 1, column player is player 2
 - rows are actions $a \in A_1$, columns are $a' \in A_2$
 - cells are outcomes, written as a tuple of utility values for each player

Prisoner's dilemma

Prisoner's dilemma is any game

$$egin{array}{c|c} C & D \\ \hline C & a,a & b,c \\ \hline D & c,b & d,d \\ \hline \end{array}$$

with c > a > d > b.

Matching Pennies

A zero-sum game: players have exactly opposed interests. One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
Tails	-1	1

Coordination Game

One more example

A cooperative game: players have exactly the same interests. Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

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General Games: Battle of the Sexes

The most interesting games combine elements of cooperation and competition.

	В	F
В	2,1	0,0
F	0,0	1, 2

General Games: Battle of the Sexes

The most interesting games combine elements of cooperation and competition.

	В	F
В	2,1	0,0
F	0,0	1, 2

Play this game with someone near you, repeating five times.

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- ▶ We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?

Analyzing Games

One more example

- ▶ We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?
 - we have no way of saying that one agent's interests are more important than another's
 - intuition: imagine trying to find the revenue-maximizing outcome when you don't know what currency has been used to express each agent's payoff
- ▶ Are there situations where we can still prefer one outcome to another?

- \triangleright Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
 - \blacktriangleright in this case, it seems reasonable to say that o is better than o'
 - we say that o Pareto-dominates o'.

One more example

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- \triangleright An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it
 - can a game have more than one Pareto-optimal outcome?
 - does every game have at least one Pareto-optimal outcome?

	C	D
C	-1, -1	-4,0
D	0, -4	-3, -3

	C	D
C	-1, -1	-4,0
D	0, -4	-3, -3

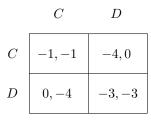
One more example

	Left	Right
Left	1	0
Right	0	1

	C	D
C	-1, -1	-4,0
D	0, -4	-3, -3

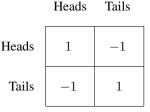
	Left	Right
Left	1	0
Right	0	1

 $\begin{array}{c|cccc} & B & F \\ \\ B & 2,1 & 0,0 \\ \\ F & 0,0 & 1,2 \end{array}$



$$\begin{array}{c|cccc} & Left & Right \\ \hline Left & 1 & 0 \\ \hline Right & 0 & 1 \\ \hline \end{array}$$

 $\begin{array}{c|cccc} & & B & & F \\ & & & \\ B & & & \\ F & & & \\ \hline \\ 0,0 & & \\ 1,2 & \\ \end{array}$



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Best Response and Nash Equilibrium

Best Response

▶ If you knew what everyone else was going to do, it would be easy to pick your own action

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Pareto Optimality

- ▶ Let $a_{-i} = \langle a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n \rangle$.
 - now $a = (a_{-i}, a_i)$

▶ Best response: $a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i, u_i(a_i^*, a_{-i}) \ge u_i(a_i, a_{-i})$

Nash Equilibrium

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

Nash Equilibrium

One more example

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

- Idea: look for stable action profiles.
- $ightharpoonup a = \langle a_1, \ldots, a_n \rangle$ is a Nash equilibrium iff $\forall i, a_i \in BR(a_{-i})$.

$$C = \begin{bmatrix} -1, -1 & -4, 0 \\ 0, -4 & -3, -3 \end{bmatrix}$$

Nash Equilibria of Example Games

	<i>C</i>	D
C	-1, -1	-4,0
D	0, -4	-3, -3

	Left	Right
Left	1	0
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Nash Equilibria of Example Games

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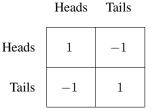
Nash Equilibria of Example Games

	C	D
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	Left	Right
Left	1	0
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В

F



D: 1.4

. C

Nash Equilibria of Example Games

	C	D		Left	Right
C	-1, -1	-4,0	Left	1	0
D	0, -4	-3, -3	Right	0	1
	В	F		Heads	Tails

	Б	Г		Heads	14118
В	2,1	0,0	Heads	1	-1
F	0,0	1,2	Tails	-1	1

The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!