Analyzing Games

CPSC 532A Lecture 4

September 21, 2006

Lecture Overview

Defining Games

- Finite, n-person game: $\langle N, A, u \rangle$:
  - N is a finite set of n players, indexed by i
  - $A = \langle A_1, \ldots, A_n \rangle$ is a tuple of action sets for each player i
    - $a \in A$ is an action profile
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

Prisoner's dilemma

Prisoner's dilemma is any game

	C	D
C	a, a	b, c
D	c, b	d, d

with $c > a > d > b$.

Matching Pennies

A zero-sum game: players have exactly opposed interests.
One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
Tails	-1	1

Coordination Game

A cooperative game: players have exactly the same interests.

Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

Lecture Overview

General Games: Battle of the Sexes

The most interesting games combine elements of cooperation *and* competition.

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

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Play this game with someone near you, repeating five times.

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Analyzing Games

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- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?

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- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?
  - we have no way of saying that one agent's interests are more important than another's
  - intuition: imagine trying to find the revenue-maximizing outcome when you don't know what currency has been used to express each agent's payoff
- Are there situations where we can still prefer one outcome to another?

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.

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- An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it.
  - can a game have more than one Pareto-optimal outcome?
  - does every game have at least one Pareto-optimal outcome?

Pareto Optimal Outcomes in Example Games

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C	$-1, -1$	$-4, 0$
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- Let $a_{-i} = \langle a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n \rangle$.
  - now $a = (a_{-i}, a_i)$

- Best response: $a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i,\ u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$

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- Idea: look for stable action profiles.
- $a = \langle a_1, \ldots, a_n \rangle$ is a Nash equilibrium iff $\forall i,\ a_i \in BR(a_{-i})$.

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The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!