Game Theory intro

CPSC 532A Lecture 3

September 19, 2006

Lecture Overview

Self-interested agents

- What does it mean to say that an agent is self-interested?
  - not that they want to harm other agents
  - not that they only care about things that benefit them
  - that the agent has its own description of states of the world that it likes, and that its actions are motivated by this description
- Utility theory:
  - quantifies degree of preference across alternatives
  - understand the impact of uncertainty on these preferences
  - utility function: a mapping from states of the world to real numbers, indicating the agent's level of happiness with that state of the world
  - Decision-theoretic rationality: take actions to maximize expected utility.

Preferences Over Outcomes

If o_1 and o_2 are outcomes

- $o_1 \succeq o_2$ means o_1 is at least as desirable as o_2.
  - read this as "the agent weakly prefers o_1 to o_2"
- $o_1 \sim o_2$ means $o_1 \succeq o_2$ and $o_2 \succeq o_1$.
  - read this as "the agent is indifferent between o_1 and o_2."
- $o_1 \succ o_2$ means $o_1 \succeq o_2$ and $o_2 \not\succeq o_1$
  - read this as "the agent strictly prefers o_1 to o_2"

Lotteries

- An agent may not know the outcomes of his actions, but may instead only have a probability distribution over the outcomes.
- A lottery is a probability distribution over outcomes. It is written

$$[p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k]$$

  where the o_i are outcomes and $p_i > 0$ such that

$$\sum_i p_i = 1$$

- The lottery specifies that outcome o_i occurs with probability p_i.
- We will consider lotteries to be outcomes.

Preference Axioms: Completeness

- Completeness: A preference relationship must be defined between every pair of outcomes:

$$\forall o_1 \forall o_2 \; o_1 \succeq o_2 \text{ or } o_2 \succeq o_1$$

Preference Axioms: Transitivity

- Transitivity: Preferences must be transitive:

$$\text{if } o_1 \succeq o_2 \text{ and } o_2 \succeq o_3 \text{ then } o_1 \succeq o_3$$

  - This makes good sense: otherwise $o_1 \succeq o_2$ and $o_2 \succeq o_3$ and $o_3 \succ o_1$.
  - An agent should be prepared to pay some amount to swap between an outcome they prefer less and an outcome they prefer more
  - Intransitive preferences mean we can construct a "money pump"!

Preference Axioms

Monotonicity: An agent prefers a larger chance of getting a better outcome to a smaller chance:

- If $o_1 \succ o_2$ and $p > q$ then

$$[p : o_1, 1 - p : o_2] \succ [q : o_1, 1 - q : o_2]$$

Preference Axioms

Let $P_\ell(o_i)$ denote the probability that outcome o_i is selected by lottery ℓ. For example, if $\ell = [0.3 : o_1; 0.7 : [0.8 : o_2; 0.2 : o_1]]$ then $P_\ell(o_1) = 0.44$ and $P_\ell(o_3) = 0$.

Decomposability: ("no fun in gambling"). If $\forall o_i \in O$, $P_{\ell_1}(o_i) = P_{\ell_2}(o_i)$ then $\ell_1 \sim \ell_2$.

Preference Axioms

- Substitutability: If $o_1 \sim o_2$ then for all sequences of one or more outcomes $o_3, \ldots, o_k$ and sets of probabilities $p, p_3, \ldots, p_k$ for which $p + \sum_{i=3}^{k} p_i = 1$, $[p : o_1, p_3 : o_3, \ldots, p_k : o_k] \sim [p : o_2, p_3 : o_3, \ldots, p_k : o_k]$.

Preference Axioms

Continuity: Suppose $o_1 \succ o_2$ and $o_2 \succ o_3$, then there exists a $p \in [0,1]$ such that $o_2 \sim [p : o_1, 1-p : o_3]$

Preferences and utility functions

Theorem (von Neumann and Morgenstern, 1944)

If an agent's preference relation satisfies the axioms Completeness, Transitivity, Decomposability, Substitutability, Monotonicity and Continuity then there exists a function $u : O \rightarrow [0, 1]$ *with the properties that:*

1. $u(o_1) \geq u(o_2)$ *iff the agent prefers* o_1 *to* o_2*; and*
2. *when faced about uncertainty about which outcomes he will receive, the agent prefers outcomes that maximize the expected value of* u*.*

Proof idea for part 2:

- define the utility of the best outcome $u(\overline{o}) = 1$ and of the worst $u(\underline{o}) = 0$
- now define the utility of each other outcome o as the p for which $o \sim [p : \overline{o}; (1 - p) : \underline{o}]$.

Lecture Overview

Non-Cooperative Game Theory

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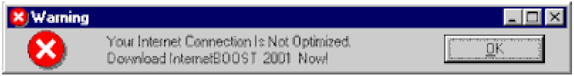
- What is it?
  - mathematical study of interaction between rational, self-interested agents
- Why is it called non-cooperative?
  - while it's most interested in situations where agents' interests conflict, it's not restricted to these settings
  - the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
    - cooperative/coalitional game theory has teams as the central unit, rather than agents

TCP Backoff Game

TCP Backoff Game



Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?

- Consider this situation as a two-player game:
  - both use a correct implementation: both get 1 ms delay
  - one correct, one defective: 4 ms delay for correct, 0 ms for defective
  - both defective: both get a 3 ms delay.

TCP Backoff Game

- Consider this situation as a two-player game:
  - both use a correct implementation: both get 1 ms delay
  - one correct, one defective: 4 ms delay for correct, 0 ms for defective
  - both defective: both get a 3 ms delay.
- Questions:
  - What action should a player of the game take?
  - Would all users behave the same in this scenario?
  - What global patterns of behaviour should the system designer expect?
  - Under what changes to the delay numbers would behavior be the same?
  - What effect would communication have?
  - Repetitions? (finite? infinite?)
  - Does it matter if I believe that my opponent is rational?

Defining Games

- Finite, n-person game: $\langle N, A, u \rangle$:
  - N is a finite set of n players, indexed by i
  - $A = A_1, \ldots, A_n$ is a set of actions for each player i
    - $a \in A$ is an action profile
  - $u = \{u_1, \ldots, u_n\}$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

Lecture Overview

Recap: Utility Theory

Game Theory

Example Matrix Games

Games in Matrix Form

Here's the TCP Backoff Game written as a matrix ("normal form").

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

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Play this game with someone near you, repeating five times.

More General Form

Prisoner's dilemma is any game

	C	D
C	a, a	b, c
D	c, b	d, d

with $c > a > d > b$.

Games of Pure Competition

Players have exactly opposed interests

- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant c
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it's a one-player game

Matching Pennies

One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
Tails	-1	1

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Rock-Paper-Scissors

Generalized matching pennies.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

...Believe it or not, there's an annual international competition for this game!

Games of Cooperation

Players have exactly the same interests.

- no conflict: all players want the same things
- $\forall a \in A, \forall i, j,\ u_i(a) = u_j(a)$
- we often write such games with a single payoff per cell
- why are such games "noncooperative"?

Coordination Game

Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

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