Auctions Introduction

CPSC 532A Lecture 20

November 21, 2006

Lecture Overview

1. Recap
2. VCG caveats
3. Auctions
4. Standard auctions
5. More exotic auctions

Groves Uniqueness

Theorem

An efficient social choice function $C : \mathbb{R}^{Xn} \rightarrow X \times \mathbb{R}^n$ *can be implemented in dominant strategies for agents with unrestricted quasilinear utilities only if* $p_i(v) = h(v_{-i}) - \sum_{j \neq i} v_j(\chi(v))$.

- it turns out that the same result also holds for the broader class of Bayes-Nash incentive-compatible efficient mechanisms.

VCG

Definition (Vickrey-Clarke-Groves (VCG) mechanism)

The *Vickrey-Clarke-Groves mechanism* is a direct quasilinear mechanism $(\mathbb{R}^{|X|n}, \chi, p)$, where

$$\chi(\hat{v}) = \arg\max_x \sum_i \hat{v}_i(x)$$

$$p_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j\left(\chi(\hat{v}_{-i})\right) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$$

Two definitions

Definition (Choice-set monotonicity)

An environment exhibits choice-set monotonicity if $\forall i, |X_{-i}| \leq |X|$.

- removing any agent weakly decreases—that is, never increases—the mechanism's set of possible choices X

Definition (No negative externalities)

An environment exhibits no negative externalities if $\forall i \forall x \in X_{-i}, v_i(x) \geq 0$.

- every agent has zero or positive utility for any choice that can be made without his participation

VCG Individual Rationality

Theorem

The VCG mechanism is ex-post individual rational when the choice set monotonicity and no negative externalities properties hold.

Another property

Definition (No single-agent effect)

An environment exhibits no single-agent effect if $\forall x, \forall i$ such that $\exists v_{-i}$ where $x \in \arg\max \sum_j v_j(x)$ there exists a choice x' that is feasible without i and that has $\sum_{j \neq i} v_j(x') \geq \sum_{j \neq i} v_j(x)$.

Theorem

The VCG mechanism is weakly budget-balanced when the no single-agent effect property holds.

Bad news

Theorem

No dominant strategy incentive-compatible mechanism is always both efficient and weakly budget balanced, even if agents are restricted to the simple exchange setting.

Theorem

No Bayes-Nash incentive-compatible mechanism is always simultaneously efficient, weakly budget balanced and ex-interim individual rational, even if agents are restricted to quasilinear utility functions.

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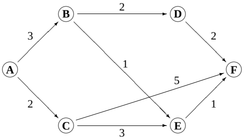
Frugality



- VCG can end up paying arbitrarily more than an agent is willing to accept (or equivalently charging arbitrarily less than an agent is willing to pay)
- Consider AC, which is not part of the shortest path.
  - If the cost of this edge increased to 8, our payment to AB would increase to $p_{AB} = (-12) - (-2) = -10$.
  - If the cost were any $x \geq 2$, we would select the path $ABEF$ and would have to make a payment to AB of $p_{AB} = (-4 - x) - (-2) = -(x + 2)$.
  - The gap between agents' true costs and the payments that they could receive under VCG is unbounded.

Privacy

- VCG requires agents to fully reveal their private information
- this private information may have value to agents that extends beyond the current interaction
  - for example, the agents may know that they will compete with each other again in the future
- it is often preferable to elicit only as much information from agents as is required to determine the social welfare maximizing outcome and compute the VCG payments.

Collusion

Example

Agent	U(build road)	U(do not build road)	Payment
1	200	0	150
2	100	0	50
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by $50?

Collusion

Example

Agent	U(build road)	U(do not build road)	Payment
1	250	0	
2	150	0	
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by $50?

Collusion

Example

Agent	U(build road)	U(do not build road)	Payment
1	250	0	100
2	150	0	0
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by $50?
- The outcome is unchanged, but both of their payments are reduced.
- Thus, while no agent can gain by changing his declaration, groups *can*.

Returning profits to the agents

- we may want to use VCG to induce agents to report their valuations honestly, but may not want to make a profit by collecting money from the agents.
- Thus, we might want to find some way of returning the mechanism's profits back the agents.
- However, the possibility of receiving a rebate after the mechanism has been run changes the agents' incentives.
- In fact, even if profits are given to a charity that the agents care about, or spent in a way that benefits the local economy and hence benefits the agents, the VCG mechanism is undermined.
- Thus, burning the money collected by the mechanism is the only way ensuring that the agents' incentives are not altered!

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Motivation

- Auctions are any mechanisms for allocating resources among self-interested agents
- Very widely used
  - government sale of resources
  - privatization
  - stock market
  - request for quote
  - FCC spectrum
  - real estate sales
  - eBay

CS Motivation

- resource allocation is a fundamental problem in CS
- increasing importance of studying distributed systems with heterogeneous agents
- markets for:
  - computational resources (JINI, etc.)
  - SETI, etc.
  - autonomous agents
  - P2P systems
  - network bandwidth
- currency needn't be real money, just something scarce
  - that said, real money trading agents are also an important motivation

Formal Model

- while we think of auctions in terms of a guy with a gavel, "going-going-gone!", they're actually a much broader theoretical framework for resource allocation
- another way of thinking of an auction: any negotiation mechanism which is
  - mediated (auctioneer)
  - well-specified (follows rules)
  - market-based (determines an exchange in terms of currency)

Modeling Auctions

Every resource allocation mechanism in a setting with quasilinear utilities can be understood as an auction

- ascending auction: an extensive-form game with imperfect information
- sealed-bid auction: direct mechanism; a variety of payment functions are possible here
  - give the good to the person who says they need it the most
  - a non-incentive compatible mechanism with a payment function $p_i = 0$
  - charge a fixed price for the good, sell a unit of it to anyone who wants one
  - trivial allocation rule, constant payment function
- stock market
  - both buyers and sellers make bids
  - market-maker clears the market and keeps the spread between ask and buy

Auction Dimensions

- rules for bidding
  - who can bid, when
  - what is the form of a bid
  - restrictions on offers, as a function of:
    - bidder's own previous bid
    - auction state (others' bids)
    - eligibility (e.g., budget constraints)
    - expiration, withdrawal, replacement
- rules for what information is revealed
  - when to reveal what information to whom
- rules for clearing
  - when to clear
  - at intervals
  - on each bid
  - after a period of inactivity
  - allocation (who gets what)
  - payment (who pays what)

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Some popular auctions

- English
- Dutch
- First-Price
- Second-Price

English Auction

- auctioneer starts the bidding at some “reservation price”
- bidders then shout out ascending prices
- once bidders stop shouting, the high bidder gets the good at that price

Dutch Auction

- the auctioneer starts a clock at some high value; it descends
- at some point, a bidder shouts "mine!" and gets the good at the price shown on the clock

First-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount of his bid

Second-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount bid by the second-highest bidder

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Some more exotic auction types

- Japanese auction
- All-pay auction
- Continuous double auction
- Call market ("periodic clear")

Japanese Auction

- Same as an English auction except that the auctioneer calls out the prices
- all bidders start out standing
- when the price reaches a level that a bidder is not willing to pay, that bidder sits down
  - once a bidder sits down, they can't get back up
- the last person standing gets the good
- analytically more tractable than English because jump bidding can't occur
  - consider the branching factor of the extensive form game...

All-pay auction

- sealed bid auction
- everyone pays the amount of their bid regardless of whether or not they win