CPSC 532A, Fall 2006
Final Exam

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1. **[20 points] (Game Theory)**

   Consider the following 3-player game (where player 1 chooses the row, 2 the column and 3 the matrix):

   U:

	L	R
T	3,0,0	1,3,1
B	0,0,0	1,3,2

   D:

	L	R
T	1,2,3	2,1,0
B	0,0,0	1,1,0

   (a) Find all the Pareto optimal pure strategy profiles.

   (b) Find all the dominated strategies and state whether they are strict, weak or very weak.

   (c) Find all the pure strategy Nash equilibria.

   (d) Find all the mixed strategy Nash equilibria.

   (e) What is player 1's minimax value (i.e., the lowest expected utility that the other agents can guarantee that 1 will receive)?

   (f) Find a correlated equilibrium where players 2 and 3 have the same expected utility. Use a fair coin as your only randomizing device.

   (g) Suppose player 1 does not act strategically, but instead chooses action T with probability $p(T)$. Find all the pure strategy equilibria of the resulting game.

   (h) As before, suppose player 1 does not act strategically, but instead chooses action T with probability $p(T)$. Furthermore, now suppose that player 3 is able to observe this move before acting, while player 2 is not able to observe anything. Find all the pure strategy equilibria of the resulting game.

2. **[10 points] (Subgame Perfect Folk Theorem)**

   Consider an infinitely repeated game with average rewards, where the stage game is the following variant of prisoner's dilemma:

	C	D
C	2,2	0,10
D	10,0	1,1

   (a) Construct a Nash equilibrium in which each player gets an average reward of 4, using a trigger strategy as shown in class. Prove that it is an equilibrium.

   One problem with such equilibria is that they are not subgame perfect—an agent might not want to follow through on his punishment threat, as it could hurt him as well as the agent being punished. Intuitively, though, this infinite punishment is unnecessarily strong—after all, the deviating agent only gains a finite amount from his deviation. It turns out that a stronger, subgame perfect folk theorem is possible. The idea is only to punish deviating agents enough to eliminate the rewards they gained by deviating, and then to return to the cooperative equilibrium strategy.

(b) Modify your equilibrium above to make it subgame perfect. Prove both that it is an equilibrium and that it is subgame perfect.

3. **[10 points] (Social Welfare Functions)**

   Consider the "random-dictatorship" social welfare function, whose output is the exact preference ordering of some randomly selected agent.

   (a) Is random-dictatorship Pareto efficient? Prove your answer.

   (b) Is random-dictatorship IIA? Prove your answer.

   (c) Is random-dictatorship dictatorial? Prove your answer.

   (d) Does random-dictatorship contradict Arrow's impossibility theorem? Why or why not?

4. **[20 points] (Mechanism Design and Budget-Balance)**

   Consider the following mechanism: The outcome which maximizes social welfare (assuming agents had truthfully reported their types) is chosen. Each agent i is paid a transfer t_i which is the expected social welfare of all other agents given i's report (again, assuming the other agents report truthfully).

$$t_i(\hat{v}_i) = \mathbb{E}_{v_{-i}}\left[\sum_{j \neq i} v_j(\chi(\hat{v}_i, v_{-i}))\right]$$

   The cost of these payments is divided equally among the agents, so the final payment rule is:

$$p_i(\hat{v}) = \left(\frac{1}{N-1}\sum_{j \neq i} t_j(\hat{v}_j)\right) - t_i(\hat{v}_i)$$

   (a) Is this mechanism truthful in dominant strategies, truthful in *ex-post* Bayes Nash equilibrium, truthful in *ex-interim* Bayes Nash equilibrium, or none of these? Prove the strongest result that holds.

   (b) Is this mechanism budget-balanced? *Ex-ante*, *ex-interim* or *ex-post*? Prove the strongest result that holds.

   (c) Is this mechanism *ex-interim* individual-rational? Prove your answer.

5. **[20 points] (Software Pricing vs. VCG)**

   Consider the following mechanism design problem: A software company can produce an unlimited number of copies of its product at zero marginal (i.e., per-unit) cost. There are a large number of users who each have some valuation for a single unit of the product. Nobody wants more than one unit. The company wants to find a way of selling its software to these users in order to maximize its revenue.

   The company runs the following mechanism: All the users are asked their valuations. The users are then randomly divided into two groups, N_I and N_{II}. The seller identifies price p_I: the amount that would yield maximum profit if offered as a fixed price to the agents in group N_I, presuming that each agent in N_I was honest about his valuation and would buy the good iff its price was less than his valuation. Likewise, the seller identifies price p_{II}. The seller then runs the following mechanism:

- agent i in group N_I is awarded the good and charged p_{II} if $v_i \geq p_{II}$; else, i is not awarded the good and is charged nothing.
- agent j in group N_{II} is awarded the good and charged p_I if $v_j \geq p_I$; else, j is not awarded the good and is charged nothing.

Answer the following questions:

(a) Is the mechanism truthful? Does it have an equilibrium in dominant strategies, in *ex-post* Bayes Nash equilibrium, in *ex-interim* Bayes Nash equilibrium, or none of these? Prove the strongest result that holds.

(b) Is the mechanism individual-rational? *Ex-ante*, *ex-interim* or *ex-post*? Prove the strongest result that holds.

(c) What is the efficient outcome in this setting? Is the mechanism efficient? Prove your answer.

(d) What allocation and payments would VCG choose in this setting? How much revenue would this generate?

6. **[20 points] (Combinatorial Auction Bidding Languages)** Recall that the OR^* bidding language adds "dummy goods" to bids in a combinatorial auction in order to express complex bidding languages within the OR language. Consider a combinatorial auction with n goods. For each valuation function below, explain how to encode it in the OR^* language using as few bids as possible, explain how many dummy goods your encoding requires. Partial marks will be given for encodings that are not minimal.

(a) A set s is valued at $k|s|$ if $|s| \leq j$, otherwise it is valued at 0.

(b) A set s where $|s| \geq n/2$ is valued at 1; all other sets are valued at 0.

(c) Goods come in four colours, cyan, yellow, magenta and black; there are $n/4$ goods of each colour. A set s is valued at $k_c|s|$ if it contains only cyan items, $k_y|s|$ if it contains only yellow items, $k_m|s|$ if it contains only magenta items, $k_b|s|$ if it contains only black items, and 0 if it contains items of more than one colour.

(d) Goods come in pairs. A set s is valued at $|s|$ if it contains at most one item from every pair, and 0 otherwise.

Academic Honesty Form

In my work on this exam, I did not discuss the questions with other students. The only references I consulted were the course notes and the textbook. I did not use any computational tools other than a calculator.

Signature:

Fill in this page and include it with your exam submission. If you submit electronically, typing the text above in your email message can substitute for submitting this page.