Theorem 1  Truth telling is a dominant strategy under the Groves mechanism

Consider a situation where every agent \( j \) other than \( i \) follows some arbitrary strategy \( \hat{v}_j \). Consider agent \( i \)'s problem of choosing the best strategy \( \hat{v}_i \). As a shorthand, we will write \( \hat{v} = (\hat{v}_{-i}, \hat{v}_i) \). The best strategy for \( i \) is one that solves

\[
\max_{\hat{v}_i} (v_i(x(\hat{v})) - p(\hat{v})) \tag{1}
\]

Substituting in the payment function from the Groves mechanism, we have

\[
\max_{\hat{v}_i} \left( v_i(x(\hat{v})) - h_i(\hat{v}_{-i}) + \sum_{j \neq i} \hat{v}_j(x(\hat{v})) \right) \tag{2}
\]

Since \( h_i(\hat{v}_{-i}) \) does not depend on \( \hat{v}_i \), it is sufficient to solve

\[
\max_{\hat{v}_i} \left( v_i(x(\hat{v})) + \sum_{j \neq i} \hat{v}_j(x(\hat{v})) \right) . \tag{3}
\]

The only way in which the declaration \( \hat{v}_i \) influences the maximization above is through the choice of \( x \). Thus, \( i \) wants to pick the declaration \( \hat{v}_i \) that will lead the mechanism to pick an \( x \in X \) which solves

\[
\max_x \left( v_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right) . \tag{4}
\]

Under the Groves mechanism,

\[
x(\hat{v}) = \arg \max_x \left( \sum_i \hat{v}_i(x) \right) = \arg \max_x \left( \hat{v}_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right) . \tag{5}
\]

The Groves mechanism will choose \( x \) in a way that solves the maximization problem in Equation (4) when \( i \) declares \( \hat{v}_i = v_i \). Because this argument does not depend in any way on the declarations of the other agents, truth-telling is a dominant strategy for agent \( i \).