Reasoning Under Uncertainty: Belief Network Inference

CPSC 322 – Uncertainty 5

Textbook §10.4
Lecture Overview

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2 Belief Network Examples
Components of a belief network

Definition (belief network)

A belief network consists of:

- a directed acyclic graph with nodes labeled with random variables
- a domain for each random variable
- a set of conditional probability tables for each variable given its parents (including prior probabilities for nodes with no parents).
Belief network summary

- A belief network is a directed acyclic graph (DAG) where nodes are random variables.
  - A belief network is automatically acyclic by construction.
- The parents of a node \( n \) are those variables on which \( n \) directly depends.
- A belief network is a graphical representation of dependence and independence:
  - A variable is conditionally independent of its non-descendants given its parents.
Relating BNs to the joint

Belief networks are compact representations of the joint.

To encode the joint as a BN:

1. **Totally order** the variables of interest: $X_1, \ldots, X_n$

2. Write down the **chain rule decomposition** of the joint, using this ordering:
   \[
   P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|X_{i-1}, \ldots, X_1)
   \]

3. For every variable $X_i$, **find the smallest set** $pX_i \subseteq \{X_1, \ldots, X_{i-1}\}$ such that
   \[
   P(X_i|X_{i-1}, \ldots, X_1) = P(X_i|pX_i).
   \]
   - If $pX_i \neq \{X_1, \ldots, X_{i-1}\}$, $X_i$ is conditionally independent of some of its ancestors given $pX_i$.

4. **Now we can write**
   \[
   P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|pX_i)
   \]

5. **Construct the BN:**
   - **Nodes** are variables
   - **Incoming edges** to each variable $X_i$ from each variable in $pX_i$
   - **Conditional probability table** for variable $X_i$: $P(X_i|pX_i)$
Lecture Overview

1. Recap

2. Belief Network Examples
Example: Fire Diagnosis

Suppose you want to diagnose whether there is a fire in a building
- you receive a noisy report about whether everyone is leaving the building.
- if everyone is leaving, this may have been caused by a fire alarm.
- if there is a fire alarm, it may have been caused by a fire or by tampering
- if there is a fire, there may be smoke
Example: Fire Diagnosis

First you choose the variables. In this case, all are boolean:

- **Tampering** is true when the alarm has been tampered with
- **Fire** is true when there is a fire
- **Alarm** is true when there is an alarm
- **Smoke** is true when there is smoke
- **Leaving** is true if there are lots of people leaving the building
- **Report** is true if the sensor reports that people are leaving the building
Example: Fire Diagnosis

- Next, you order the variables: Fire; Tampering; Alarm; Smoke; Leaving; Report.
- Now evaluate which variables are conditionally independent given their parents:
  - Fire is independent of Tampering (learning that one is true would not change your beliefs about the probability of the other)
  - Alarm depends on both Fire and Tampering: it could be caused by either or both.
  - Smoke is caused by Fire, and so is independent of Tampering and Alarm given whether there is a Fire
  - Leaving is caused by Alarm, and thus is independent of the other variables given Alarm.
  - Report is caused by Leaving, and thus is independent of the other variables given Leaving.
Example: Fire Diagnosis

This corresponds to the following belief network:

Of course, we’re not done until we also come up with conditional probability tables for each node in the graph.
The belief network also specifies:

- The domain of the variables:
  \( W_0, \ldots, W_6 \in \{\text{live, dead}\} \)
  \( S_{1\_pos}, S_{2\_pos}, \text{ and } S_{3\_pos} \) have domain \( \{\text{up, down}\} \)
  \( S_{1\_st} \) has \( \{\text{ok, upside\_down, short, intermittent, broken}\} \).

- Conditional probabilities, including:
  \[
P(W_1 = \text{live}|s_{1\_pos} = \text{up} \land S_{1\_st} = \text{ok} \land W_3 = \text{live})
  
P(W_1 = \text{live}|s_{1\_pos} = \text{up} \land S_{1\_st} = \text{ok} \land W_3 = \text{dead})
  
P(S_{1\_pos} = \text{up})
  
P(S_{1\_st} = \text{upside\_down})
  \]
Example: Circuit Diagnosis

The power network can be used in a number of ways:

- Conditioning on the status of the switches and circuit breakers, whether there is outside power and the position of the switches, you can simulate the lighting.
- Given values for the switches, the outside power, and whether the lights are lit, you can determine the posterior probability that each switch or circuit breaker is \textit{ok} or not.
- Given some switch positions and some outputs and some intermediate values, you can determine the probability of any other variable in the network.
Example: Liver Diagnosis

Source: Onisko et al., 1999