CSPs: Representation and Search

CPSC 322 – CSPs 2

Textbook §4.3 – 4.5
Lecture Overview

1. Recap
2. CSPs
3. Search
We define the state of the world as an assignment of values to a set of variables:
- variable: a synonym for feature
- we denote variables using capital letters
- each variable $V$ has a domain $\text{dom}(V)$ of possible values

Variables can be of several main kinds:
- **Boolean**: $|\text{dom}(V)| = 2$
- **Finite**: the domain contains a finite number of values
- **Infinite but Discrete**: the domain is countably infinite
- **Continuous**: e.g., real numbers between 0 and 1

We'll call the set of states that are induced by a set of variables the set of possible worlds.
Constraints

Constraints are restrictions on the values that one or more variables can take

- **Unary constraint**: restriction involving a single variable
  - of course, we could also achieve the same thing by using a smaller domain in the first place

- **$k$-ary constraint**: restriction involving the domains of $k$ different variables
  - it turns out that $k$-ary constraints can always be represented as binary constraints, so we’ll often talk about this case

- Constraints can be specified by
  - giving a list of valid domain values for each variable participating in the constraint
  - giving a function that returns true when given values for each variable which satisfy the constraint

- A possible world **satisfies** a set of constraints if the set of variables involved in each constraint take values that are consistent with that constraint
Lecture Overview

1 Recap

2 CSPs

3 Search
Definition

A constraint satisfaction problem consists of:

- a set of variables
- a domain for each variable
- a set of constraints

Definition

A model of a CSP is an assignment of values to variables that satisfies all of the constraints.
Constraint Satisfaction Problems: Variants

We may want to solve the following problems with a CSP:

- determine whether or not a model exists
- find a model
- find all of the models
- count the number of models
- find the best model, given some measure of model quality
  - this is now an optimization problem
- determine whether some property of the variables holds in all models
It turns out that even the simplest problem of determining whether or not a model exists in a general CSP with finite domains is $\mathcal{NP}$-hard.

- we can’t hope to find an efficient algorithm.

However, we can try to:

- find algorithms that are fast on “typical” cases
- identify special cases for which algorithms are efficient (polynomial)
- find approximation algorithms that can find good solutions quickly, even they may offer no theoretical guarantees
- develop parallel or distributed algorithms so that additional hardware can be used
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CSPs as Search Problems

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- **nodes**: assignments of values to a subset of the variables
- **neighbours** of a node: nodes in which values are assigned to one additional variable
- **start node**: the empty assignment (no variables assigned values)
- **leaf node**: a node which assigns a value to each variable
- **goal node**: leaf node which satisfies all of the constraints

Note: the path to a goal node is not important.
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CSPs as Search Problems

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  - the tree is always finite and has no cycles, so DFS is better than BFS
- How can we prune the DFS search tree?
  - once we reach a node that violates one or more constraints, we know that a solution cannot exist below that point
  - thus we should backtrack rather than continuing to search
  - this can yield us exponential savings over unpruned DFS, though it’s still exponential
Example

Problem:

- Variables: $A, B, C$
- Domains: $\{1, 2, 3, 4\}$
- Constraints: $A < B, B < C$
Example

Note: the algorithm’s efficiency depends on the order in which variables are expanded