Propositional Logic: Semantics and an Example

CPSC 322 – Logic 2

Textbook §5.2
Lecture Overview

1. Recap: Syntax
2. Propositional Definite Clause Logic: Semantics
3. Using Logic to Model the World
Propositional Definite Clauses: Syntax

**Definition (atom)**
An **atom** is a symbol starting with a lower case letter.

**Definition (body)**
A body is an atom or is of the form $b_1 \land b_2$ where $b_1$ and $b_2$ are bodies.

**Definition (definite clause)**
A definite clause is an atom or is a rule of the form $h \leftarrow b$ where $h$ is an atom and $b$ is a body. (Read this as "$h$ if $b$".)
Propositional Definite Clauses: Syntax

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Semantics allows you to relate the symbols in the logic to the domain you’re trying to model.

Definition (interpretation)

An interpretation $I$ assigns a truth value to each atom.
Propositional Definite Clauses: Semantics

Semantics allows you to relate the symbols in the logic to the domain you’re trying to model.

**Definition (interpretation)**

An **interpretation** $I$ assigns a truth value to each atom.

We can use the interpretation to determine the truth value of clauses and knowledge bases:

**Definition (truth values of statements)**

- A body $b_1 \land b_2$ is true in $I$ if and only if $b_1$ is true in $I$ and $b_2$ is true in $I$.
- A rule $h \leftarrow b$ is false in $I$ if and only if $b$ is true in $I$ and $h$ is false in $I$.
- A knowledge base $KB$ is true in $I$ if and only if every clause in $KB$ is true in $I$. 

Models and Logical Consequence

Definition (model)

A model of a set of clauses is an interpretation in which all the clauses are true.
Models and Logical Consequence

**Definition (model)**

A *model* of a set of clauses is an interpretation in which all the clauses are *true*.

**Definition (logical consequence)**

If $KB$ is a set of clauses and $g$ is a conjunction of atoms, $g$ is a *logical consequence* of $KB$, written $KB \models g$, if $g$ is *true* in every model of $KB$.

- we also say that $g$ logically follows from $KB$, or that $KB$ entails $g$.
- In other words, $KB \models g$ if there is no interpretation in which $KB$ is *true* and $g$ is *false*. 
Example: Models

\[ KB = \begin{cases} 
  p \leftarrow q. \\
  q. \\
  r \leftarrow s. 
\end{cases} \]

<table>
<thead>
<tr>
<th>(I_1)</th>
<th>(p)</th>
<th>(q)</th>
<th>(r)</th>
<th>(s)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>true</td>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>(I_2)</td>
<td>false</td>
<td>false</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>(I_3)</td>
<td>true</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>(I_4)</td>
<td>true</td>
<td>true</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>(I_5)</td>
<td>true</td>
<td>true</td>
<td>false</td>
<td>true</td>
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Which interpretations are models?
Example: Models

\[ KB = \left\{ \begin{array}{c}
p \leftarrow q. \\
q. \\
r \leftarrow s.
\end{array} \right. \]

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<th></th>
<th>( p )</th>
<th>( q )</th>
<th>( r )</th>
<th>( s )</th>
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<tbody>
<tr>
<td>( I_1 )</td>
<td>true</td>
<td>true</td>
<td>true</td>
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</tr>
<tr>
<td>( I_2 )</td>
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Example: Models

\[ KB = \begin{cases} 
  p \leftarrow q, \\
  q, \\
  r \leftarrow s. 
\end{cases} \]

<table>
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<tr>
<th>( I )</th>
<th>( p )</th>
<th>( q )</th>
<th>( r )</th>
<th>( s )</th>
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</table>
| \( I_1 \) | true | true | true | true | is a model of \( KB \)  
| \( I_2 \) | false | false | false | false | not a model of \( KB \)  
| \( I_3 \) | true | true | false | false | is a model of \( KB \)  
| \( I_4 \) | true | true | true | false | is a model of \( KB \)  
| \( I_5 \) | true | true | false | true | not a model of \( KB \)  

Which of the following is true?

- \( KB \models q \), \( KB \models p \), \( KB \models s \), \( KB \models r \)
Example: Models

\[ KB = \begin{cases} 
  p \leftarrow q, \\
  q, \\
  r \leftarrow s.
\end{cases} \]

\[ \begin{array}{c|cccc}
I & p & q & r & s \\
--- & --- & --- & --- & --- \\
I_1 & true & true & true & true & \text{is a model of } KB \\
I_2 & false & false & false & false & \text{not a model of } KB \\
I_3 & true & true & false & false & \text{is a model of } KB \\
I_4 & true & true & true & false & \text{is a model of } KB \\
I_5 & true & true & false & true & \text{not a model of } KB \\
\end{array} \]

Which of the following is true?

- \( KB \models q, \ KB \models p, \ KB \models s, \ KB \models r \)
- \( KB \models q, \ KB \models p, \ KB \not\models s, \ KB \not\models r \)
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User’s view of Semantics

1. Choose a task domain: **intended interpretation.**
2. Associate an atom with each proposition you want to represent.
3. Tell the system clauses that are true in the intended interpretation: **axiomatizing the domain.**
4. Ask questions about the intended interpretation.
5. If $KB \models g$, then $g$ must be true in the intended interpretation.
6. The user can interpret the answer using their intended interpretation of the symbols.
Computer’s view of semantics

- The computer doesn’t have access to the intended interpretation.
  - All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
  - If $KB \models g$ then $g$ must be true in the intended interpretation.
  - If $KB \not\models g$ then there is a model of $KB$ in which $g$ is false.
    This could be the intended interpretation.
Electrical Environment

Propositional Logic: Semantics and an Example
Representing the Electrical Environment

\begin{align*}
l_{1} & \leftarrow \text{live}_{1} \quad \text{live}_{1} & \leftarrow \text{live}_{0} \\
l_{2} & \leftarrow \text{live}_{0} \quad \text{live}_{0} & \leftarrow \text{live}_{1} \wedge \text{up}_{2} \\
d_{1} & \leftarrow \text{live}_{1} \quad \text{live}_{1} & \leftarrow \text{live}_{3} \wedge \text{up}_{1} \\
d_{2} & \leftarrow \text{live}_{2} \quad \text{live}_{2} & \leftarrow \text{live}_{3} \wedge \text{down}_{2} \\
d_{3} & \leftarrow \text{live}_{2} \quad \text{live}_{2} & \leftarrow \text{live}_{4} \\
o_{1} & \leftarrow \text{live}_{3} \quad \text{live}_{3} & \leftarrow \text{live}_{5} \wedge \text{ok}_{1} \\
o_{2} & \leftarrow \text{live}_{5} \quad \text{live}_{5} & \leftarrow \text{live}_{6} \wedge \text{ok}_{2} \\
ok_{c} & \leftarrow \text{live}_{6} \\
ok_{c2} & \leftarrow \text{live}_{5} \wedge \text{ok}_{c2} \\
\text{live}_{outside} & \leftarrow \text{live}_{5} \wedge \text{live}_{outside}.
\end{align*}
Role of semantics

In user’s mind:
- \( l2\text{\_}\text{broken} \): light \( l2 \) is broken
- \( \text{sw3\_up} \): switch is up
- \( \text{power} \): there is power in the building
- \( \text{unlit\_l2} \): light \( l2 \) isn’t lit
- \( \text{lit\_l1} \): light \( l1 \) is lit

In Computer:
- \( l2\text{\_}\text{broken} \leftarrow \text{sw3\_up} \land \text{power} \land \text{unlit\_l2} \)
- \( \text{sw3\_up} \)
- \( \text{power} \leftarrow \text{lit\_l1} \)
- \( \text{unlit\_l2} \)
- \( \text{lit\_l1} \)

Conclusion: \( l2\text{\_}\text{broken} \)
- The computer doesn’t know the meaning of the symbols
- The user can interpret the symbols using their meaning