Logic: Resolution Proofs

CPSC 322 – Logic 5

Textbook §5.2
Lecture Overview

1 Recap

2 Resolution Proofs
Soundness and completeness of bottom-up proofs

- Proved soundness of bottom-up proof procedure
  - assuming that there is a $g$ such that $KB \vdash g$ and $KB \not\models g$ leads to a contradiction

- Proved completeness of bottom-up
  - construct a minimal model
    - every atom in $C$ is true, all others are false
  - proved this is indeed a model
  - used the existence of the minimal model to show that $KB \models g$ implies that $KB \vdash g$. 
Lecture Overview

1. Recap

2. Resolution Proofs
Top-down Ground Proof Procedure

Idea: search backward from a query to determine if it is a logical consequence of $KB$.

An answer clause is of the form:

$$yes \leftarrow a_1 \land a_2 \land \ldots \land a_m$$

The SLD Resolution of this answer clause on atom $a_i$ with the clause:

$$a_i \leftarrow b_1 \land \ldots \land b_p$$

is the answer clause

$$yes \leftarrow a_1 \land \cdots \land a_{i-1} \land b_1 \land \cdots \land b_p \land a_{i+1} \land \cdots \land a_m.$$
An answer is an answer clause with $m = 0$. That is, it is the answer clause $yes \leftarrow \cdot$.

A derivation of query "$q_1 \land \ldots \land q_k$" from $KB$ is a sequence of answer clauses $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that:

- $\gamma_0$ is the answer clause $yes \leftarrow q_1 \land \ldots \land q_k$,
- $\gamma_i$ is obtained by resolving $\gamma_{i-1}$ with a clause in $KB$, and
- $\gamma_n$ is an answer.
Top-down definite clause interpreter

To solve the query \(?q_1 \land \ldots \land q_k\):

\[ ac := \text{"yes} \leftarrow q_1 \land \ldots \land q_k" \]

repeat

select atom \(a_i\) from the body of \(ac\);
choose clause \(C\) from \(KB\) with \(a_i\) as head;
replace \(a_i\) in the body of \(ac\) by the body of \(C\)

until \(ac\) is an answer.

Recall:

- Don’t-care nondeterminism If one selection doesn’t lead to a solution, there is no point trying other alternatives. select
- Don’t-know nondeterminism If one choice doesn’t lead to a solution, other choices may. choose
Example: successful derivation

\[ a \leftarrow b \land c. \quad a \leftarrow e \land f. \quad b \leftarrow f \land k. \]
\[ c \leftarrow e. \quad d \leftarrow k. \quad e. \]
\[ f \leftarrow j \land e. \quad f \leftarrow c. \quad j \leftarrow c. \]

Query: \( ?a \)

\[ \gamma_0 : \ yes \leftarrow a \]
\[ \gamma_1 : \ yes \leftarrow e \land f \]
\[ \gamma_2 : \ yes \leftarrow f \]
\[ \gamma_3 : \ yes \leftarrow c \]
\[ \gamma_4 : \ yes \leftarrow e \]
\[ \gamma_5 : \ yes \leftarrow \]
Example: failing derivation

\[ a \leftarrow b \land c. \quad a \leftarrow e \land f. \quad b \leftarrow f \land k. \]
\[ c \leftarrow e. \quad d \leftarrow k. \quad e. \]
\[ f \leftarrow j \land e. \quad f \leftarrow c. \quad j \leftarrow c. \]

Query: \(?a\)

\[ \gamma_0 : \quad yes \leftarrow a \]
\[ \gamma_1 : \quad yes \leftarrow b \land c \]
\[ \gamma_2 : \quad yes \leftarrow f \land k \land c \]
\[ \gamma_3 : \quad yes \leftarrow c \land k \land c \]
\[ \gamma_4 : \quad yes \leftarrow e \land k \land c \]
\[ \gamma_5 : \quad yes \leftarrow k \land c \]
Search Graph

Recap Resolution Proofs

a ← b ∧ c. a ← g.
a ← h. b ← j.
b ← k. d ← m.
d ← p. f ← m.
f ← p. g ← m.
g ← f. k ← m.
h ← m. p.
?a ∧ d

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