Propositional Logic: Semantics and Bottom-Up Proofs

CPSC 322 – Logic 2

Textbook §5.2

Lecture Overview

Propositional Definite Clauses: Syntax

Definition (atom)

An atom is a symbol starting with a lower case letter

Propositional Definite Clauses: Syntax

Definition (atom)

An atom is a symbol starting with a lower case letter

Definition (body)

A body is an atom or is of the form $b_1 \land b_2$ where b_1 and b_2 are bodies.

Propositional Definite Clauses: Syntax

Definition (atom)

An atom is a symbol starting with a lower case letter

Definition (body)

A body is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.

Definition (definite clause)

A definite clause is an atom or is a rule of the form $h \leftarrow b$ where h is an atom and b is a body. (Read this as "h if b.")

Propositional Definite Clauses: Syntax

Definition (atom)

An atom is a symbol starting with a lower case letter

Definition (body)

A body is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.

Definition (definite clause)

A definite clause is an atom or is a rule of the form $h \leftarrow b$ where h is an atom and b is a body. (Read this as "h if b.")

Definition (knowledge base)

A knowledge base is a set of definite clauses

Lecture Overview

1. Recap: Syntax
2. Propositional Definite Clause Logic: Semantics

Propositional Definite Clauses: Semantics

Semantics allows you to relate the symbols in the logic to the domain you're trying to model.

Definition (interpretation)

An interpretation I assigns a truth value to each atom.

Propositional Definite Clauses: Semantics

Semantics allows you to relate the symbols in the logic to the domain you're trying to model.

Definition (interpretation)

An interpretation I assigns a truth value to each atom.

We can use the interpretation to determine the truth value of clauses and knowledge bases:

Definition (truth values of statements)

- A body $b_1 \wedge b_2$ is true in I if and only if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if and only if b is true in I and h is false in I.
- A knowledge base KB is true in I if and only if every clause in KB is true in I.

Models and Logical Consequence

Definition (model)

A model of a set of clauses is an interpretation in which all the clauses are *true*.

Models and Logical Consequence

Definition (model)

A model of a set of clauses is an interpretation in which all the clauses are *true*.

Definition (logical consequence)

If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.

- we also say that g logically follows from KB, or that KB entails g.
- In other words, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*false*	*false*	*false*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*true*	*false*
I_5	*true*	*true*	*false*	*true*

Which interpretations are models?

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$
- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$