Logic: Datalog Syntax and Semantics

CPSC 322 Lecture 22

March 9, 2007
Textbook §5

Lecture Overview

1. Recap
2. Datalog Syntax
3. Datalog Semantics

Top-down definite clause interpreter

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

$ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
repeat
  select atom a_i from the body of ac;
  choose clause C from KB with a_i as head;
  replace a_i in the body of ac by the body of C
until ac is an answer.

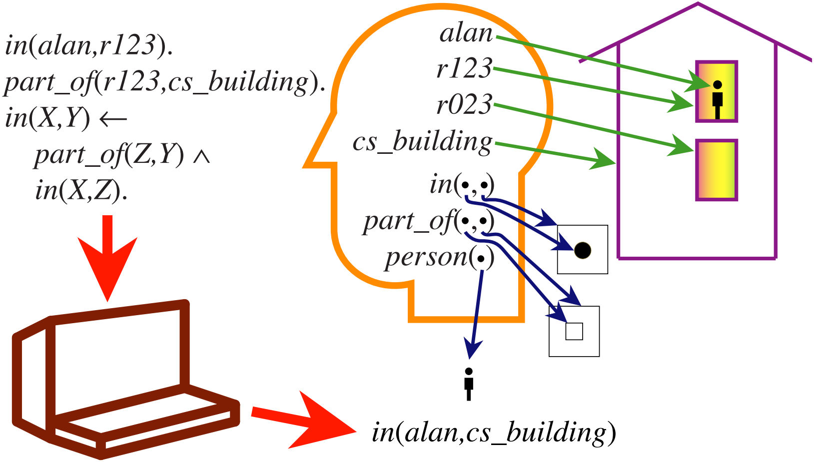
- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives. select
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

Representational Assumptions of Datalog

- An agent's knowledge can be usefully described in terms of *individuals* and *relations* among individuals.
- An agent's knowledge base consists of *definite* and *positive* statements.
- The environment is *static*.
- There are only a finite number of individuals of interest in the domain. Each individual can be given a unique name.

$\Longrightarrow$ Datalog

Example Domain for an RRS



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Syntax of Datalog

Definition (variable)

A variable starts with upper-case letter.

Definition (constant)

A constant starts with lower-case letter or is a sequence of digits.

Definition (term)

A term is either a variable or a constant.

Definition (predicate symbol)

A predicate symbol starts with lower-case letter.

Syntax of Datalog (cont)

Definition (atom)

An atomic symbol (atom) is of the form p or $p(t_1, \ldots, t_n)$ where p is a predicate symbol and t_i are terms.

Definition (definite clause)

A definite clause is either an atomic symbol (a fact) or of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_m}_{\text{body}}$$

where a and b_i are atomic symbols.

Definition (knowledge base)

A knowledge base is a set of definite clauses.

Example Knowledge Base

$$
\begin{aligned}
&in(alan, R) \leftarrow \\
&\qquad teaches(alan, cs322) \wedge \\
&\qquad in(cs322, R). \\
&grandfather(william, X) \leftarrow \\
&\qquad father(william, Y) \wedge \\
&\qquad parent(Y, X). \\
&slithy(toves) \leftarrow \\
&\qquad mimsy \wedge borogroves \wedge \\
&\qquad outgrabe(mome, Raths).
\end{aligned}
$$

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Semantics: General Idea

- Recall: a semantics specifies the meaning of sentences in the language.
  - ultimately, we want to be able to talk about which sentences are true and which are false
- In propositional logic, all we needed to do in order to come up with an interpretation was to assign truth values to atoms
- For Datalog, an interpretation specifies:
  - what objects (individuals) are in the world
  - the correspondence between symbols in the computer and objects & relations in world
    - which constants denote which individuals
    - which predicate symbols denote which relations (and thus, along with the above, which sentences will be true and which will be false)

Formal Semantics

Definition (interpretation)

An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- D, the domain, is a nonempty set. Elements of D are individuals.
- ϕ is a mapping that assigns to each constant an element of D. Constant c denotes individual $\phi(c)$.
- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{$*TRUE*, *FALSE*$\}$.

Example Interpretation

Constants: $phone$, $pencil$, $telephone$.

Predicate Symbol: $noisy$ (unary), $left\_of$ (binary).

- $D = \{$✂, ☎, ✎$\}$.
  - These are actually objects in the world, not symbols
- $\phi(phone) =$ ☎, $\phi(pencil) =$ ✎, $\phi(telephone) =$ ☎.
- $\pi(noisy)$:

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

$\pi(left\_of)$:

⟨✂, ✂⟩	*FALSE*	⟨✂, ☎⟩	*TRUE*	⟨✂, ✎⟩	*TRUE*
⟨☎, ✂⟩	*FALSE*	⟨☎, ☎⟩	*FALSE*	⟨☎, ✎⟩	*TRUE*
⟨✎, ✂⟩	*FALSE*	⟨✎, ☎⟩	*FALSE*	⟨✎, ✎⟩	*FALSE*

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.
- The constants do not have to match up one-to-one with members of the domain. Multiple constants can refer to the same object, and some objects can have no constants that refer to them.
- $\pi(p)$ specifies whether the relation denoted by the n-ary predicate symbol p is true or false for each n-tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.
  - this was the situation in propositional logic

Truth in an interpretation

Definition (truth in an interpretation)

- A constant c denotes in I the individual $\phi(c)$.
- Ground (variable-free) atom $p(t_1, \ldots, t_n)$ is
  - *true* in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) =$ *TRUE*, where t_i denotes t'_i in interpretation I and
  - *false* in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) =$ *FALSE*.
- Ground clause $h \leftarrow b_1 \wedge \ldots \wedge b_m$ is
  - *false* in interpretation I if h is *false* in I and each b_i is *true* in I, and item *true* in interpretation I otherwise.
  - A knowledge base, KB, is *true* in interpretation I if and only if every clause in KB is *true* in I.

- Notice that truth values are only associated with predicates (atomic symbols; clauses), not variables and constants!

Example Truths

In the interpretation given before:

$noisy(phone)$

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	*false*
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	*false*
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	*true*

Variables

How do we determine the truth value of a clause that includes variables?

Definition (variable assignment)

A variable assignment is a function from variables into the domain.

- Given an interpretation and a variable assignment, each term denotes an individual and each clause is either true or false.
- A clause containing variables is true in an interpretation if it is true for all variable assignments.
  - Variables are universally quantified in the scope of a clause.

Models and logical consequences

Definition (model)

A model of a set of clauses is an interpretation in which all the clauses are *true*.

Definition (logical consequence)

If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.

- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.