Algorithm Portfolios through Empirical Hardness Models

Case Studies on Combinatorial Auction Winner Determination and Satisfiability

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The Algorithm Selection Problem

- What is the best algorithm for a given problem?
  - worst-/average-case measure doesn't tell the whole story
  - ideally, select algorithm on a per-instance basis [Rice]

- Our approach:
  - Identify:
    - a target distribution of problem instances, D
    - a set of algorithms, where each algorithm has a significant probability of outperforming the others on instances drawn from D
    - polytime-computable features of problem instances
  - Learn per-algorithm empirical hardness models
  - Use the models to construct an algorithm portfolio by choosing the algorithm with the best predicted runtime

Combinatorial Auction Winner Determination

- Equivalent to weighted set packing
- Input: n goods, m bids $< S_i, p_i >$, $S_i \subseteq \{1, \ldots, n\}$
- Objective: find revenue-maximizing non-conflicting allocation

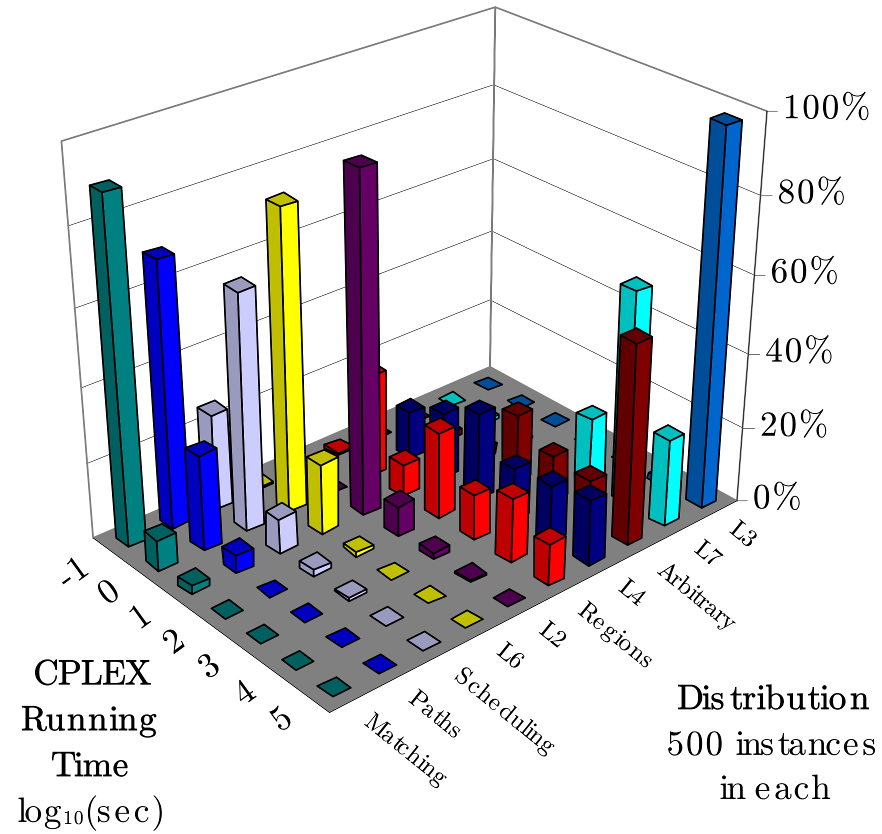
$$
\begin{array}{lll}
\text{maximize:} & \displaystyle\sum_{i=1}^{m} x_i p_i & \\
\text{subject to:} & \displaystyle\sum_{i \mid g \in S_i} x_i \leq 1 & \forall g \\
 & x_i \in \{0, 1\} & \forall i
\end{array}
$$

WDP: Runtime Variation

- Complete algorithms:
  - CPLEX [ILOG Inc.]
  - CASS [Leyton-Brown et.al],
  - GL [Gonen and Lehman]

- Gathered runtime data using various distributions
  - randomly sampled generator's parameters for each instance

- Even holding problem size constant, runtimes vary by many orders of magnitude across and within distributions



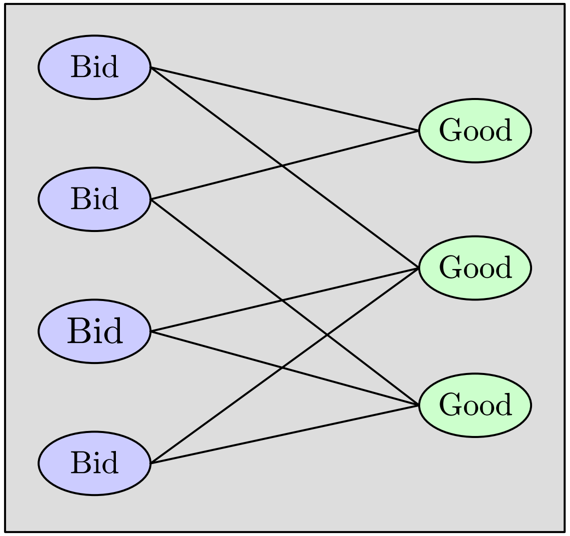
WDP: Features

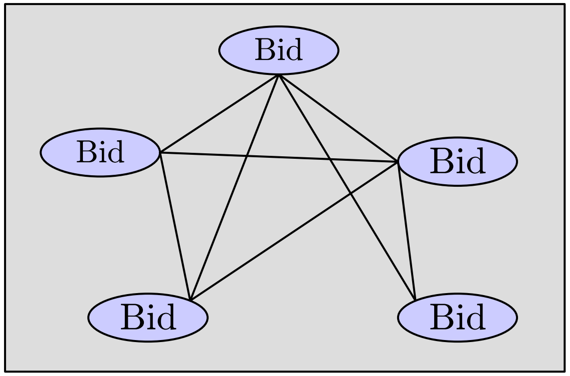
$$\text{maximize: } \sum_{i=1}^{m} x_i p_i$$

$$\text{subject to: } \sum_{i|g \in S_i} x_i \leq 1 \quad \forall g$$

$$0 \leq x_i \leq 1 \quad \forall i$$

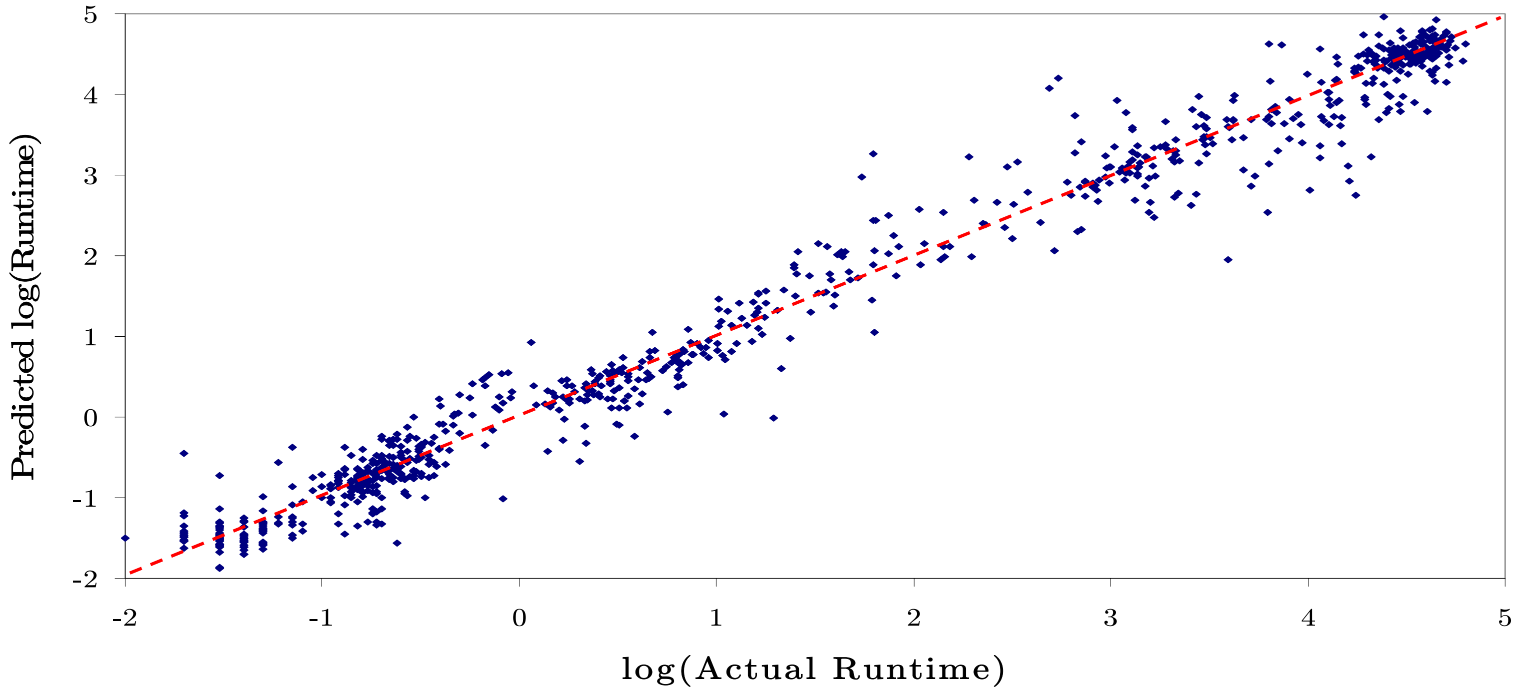
1. Linear Programming
   – L_1, L_2, L_∞ norms of integer slack vector
2. Price
   – stdev(prices)
   – stdev(avg price per good)
   – stdev(average price per sqrt(good))
3. Bid-Good graph
   – node degree stats (max, min, avg, stdev)
4. Bid graph
   – node degree stats
   – edge density
   – clustering coefficient (CC), stdev
   – avg min path length (AMPL)
   – ratio of CC to AMPL
   – eccentricity stats (max, min, avg, stdev)





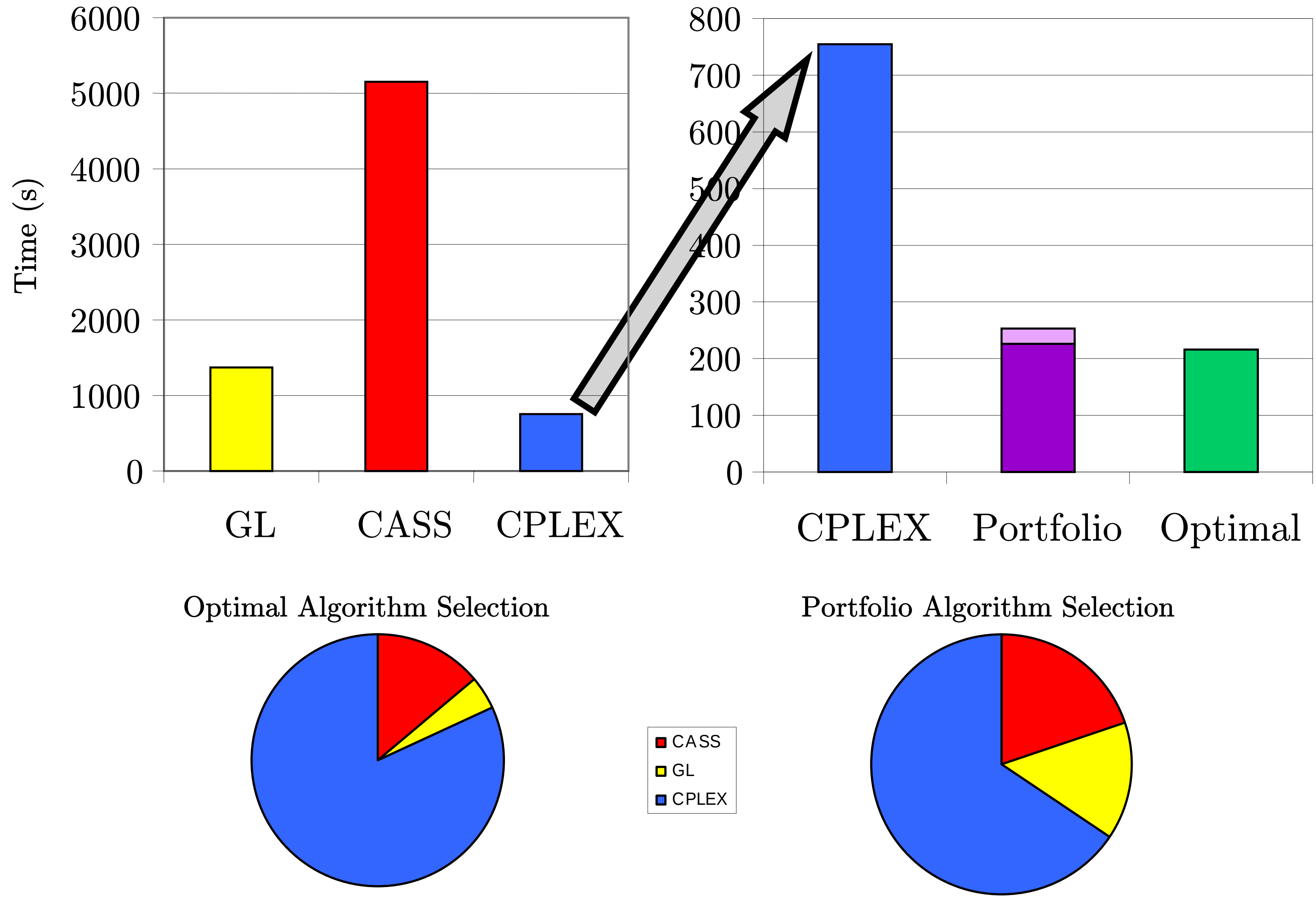
WDP: Empirical Hardness Models

- Quadratic regression can be used to learn very accurate models
  - predicting $\log_{10}$ of CPLEX runtime



  - Root mean squared error: 0.216 (test data)

WDP: From Models to a Portfolio



SATZilla: A Portfolio for SAT

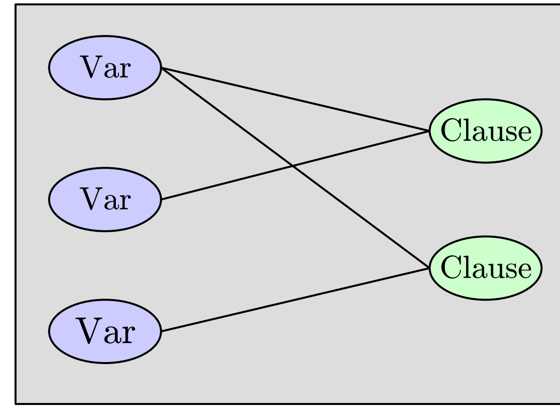
- Algorithms in the portfolio:
  - 2clseq [Bacchus]
  - OKsolver [Kullmann]
  - Satz-Rand [Kautz, Li]
  - zChaff [Zhang]
  - Limmat [Biere]
  - relsat [Bayardo]
  - SATO [Zhang]
  - Jerusat [Nadel]

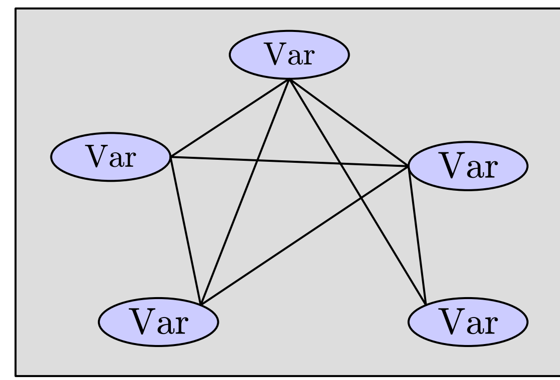
- Satzilla2 (Hors-Concours) added:
  - eqsatz [Li]
  - HeerHugo [Groote]
  - AutoWalkSat [Patterson, Kautz] (preprocessing)

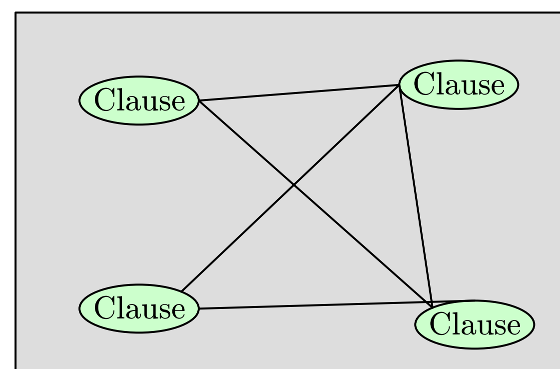
- Developed in just over two weeks!

SATzilla: Features

1. Problem Size: #vars, #clauses, #vars/#clauses
   - rest of features are normalized by these
2. Graphs:
   - Variable-Clause (*VCG*, bipartite)
   - Variable (*VG*, edge whenever two variables occur in the same clause)
   - Clause (*CG*, edge whenever two clauses share a variable with opposite sign)
     - compute stats=(max, min, stdev, mean, entropy) over node degrees
       - for VCG, both for vars and clauses
     - # of unary, binary, ternary clauses
     - stats of the CG clustering coefficients







SATzilla: Features

k1

3. Stats of #positive/#negative literals in each clause
4. Stats of #positive/#negative occurrences for each var
5. Horn clauses
   - total #horn clauses
   - stats of #horn occurrences for each var
6. LP relaxation features
   - objective value
   - stats of integer slacks
   - #vars set to an integer
7. Probing features
   - DPLL probing features (to depth 256)
     - #unit props after reaching depths 1, 4, 16, 64, 256
   - Local search probing (100 probes, each probe runs to plateau/max)
     - stats of climb height (in #clauses)
     - stats of fraction of satisfied clauses
     - stats of #steps taken
     - stats of break counts/#vars
   - Search space size probing (5000 random search paths with unit-prop)
     - average depth to contradiction, estimate log-num-nodes in search tree

k1 # pos/# neg: should be abs(0.5 - #pos / (#pos + #neg)) so that flipping all pos and neg doesn't change the stat
kevinlb, 1/1/2004

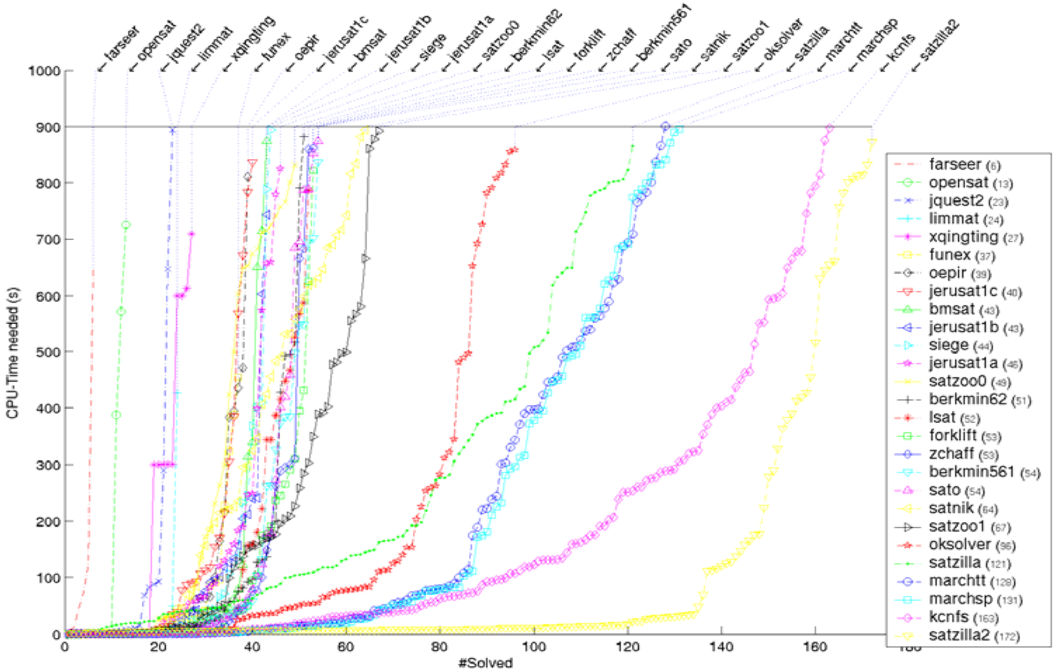
SATzilla: Models and Portfolio

- Learned linear regression models for each algorithm
  - trained on more than 20000 instances
    - included 2002 competition instances
    - highly skewed towards random instances
  - training set preprocessed to exclude instances that were solved by all solvers, or by none of them
  - terrible RMSE on test set
  - enough predictive power to discriminate well

- On the training set, SATzilla's choice takes on average 92 seconds longer to run than the optimal choice
  - gives SATzilla an edge over its subsolvers, especially on harder instances

SATzilla: SAT-2003 Competition

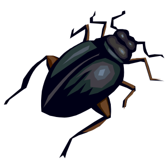
- 2nd in Random instances track
- 3rd in Handmade track; 2nd in Handmade track, SAT only



- Only solver with good performance in more than one track
- Success measured in #series solved, not #benchmarks solved
  - Satzilla 2 solved more random instances than kcnfs

SATzilla: Areas for Improvement

- Add new algorithms to the portfolio
  - SATzilla outperformed all its constituent algorithms
- Construct better models
  - as we continue to study and analyze SAT data, our model accuracy is increasing
- Spend more development time to eliminate bugs
  - LP features timed out on many industrial benchmarks



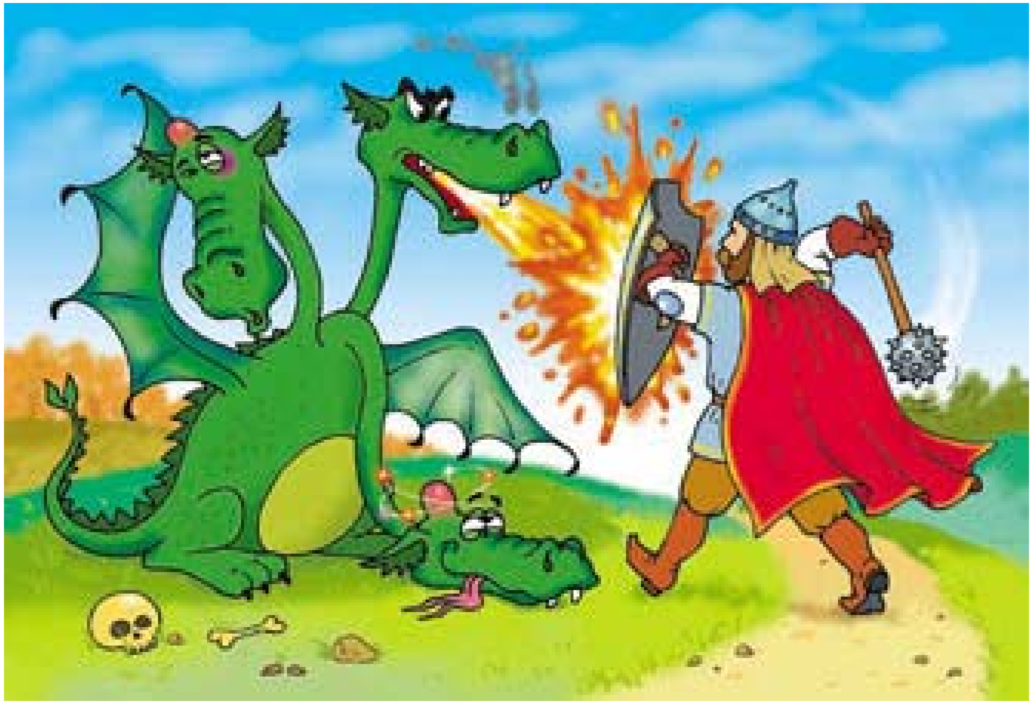
    - instead of using a fallback solver (zChaff), SATzilla picked one essentially at random, but most don't do well on industrial
  - some "random" instances were solved but didn't count!
    - Relsat was chosen, and actually solved them, but it had an output bug ☹

Conclusions

- WDP
  - models: very mature, high accuracy
  - algorithms: one is dominant, limiting the size of possible gains from a portfolio approach

- SAT
  - models: more of a proof of concept, much room for improvement. However, discrimination accuracy is much better than prediction accuracy.
  - algorithms: many are strong and correlation is fairly low, making this an excellent domain for future study

Conclusions



Overall, our techniques provide a quick and relatively automatic blueprint for building algorithm portfolios, suitable when there are:

- two or more algorithms with relatively uncorrelated runtimes
- a set of good features
- lots of data