Algorithm Portfolios through Empirical Hardness Models
Case Studies on Combinatorial Auction Winner Determination and Satisfiability

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The Algorithm Selection Problem

• What is the **best algorithm** for a given problem?
  – worst-/average-case measure doesn’t tell the whole story
  – ideally, select algorithm on a per-instance basis [Rice]

• Our **approach**:
  – Identify:
    • a target **distribution of problem instances**, $D$
    • a **set of algorithms**, where each algorithm has a significant probability of outperforming the others on instances drawn from $D$
    • polytime-computable **features** of problem instances
  – Learn per-algorithm **empirical hardness models**
  – Use the models to construct an **algorithm portfolio** by choosing the algorithm with the best predicted runtime
Combinatorial Auction Winner Determination

- Equivalent to weighted set packing
- Input: $n$ goods, $m$ bids $< S_i, p_i >$, $S_i \subseteq \{1, \ldots, n\}$
- Objective: find revenue-maximizing non-conflicting allocation

\[
\begin{align*}
\text{maximize:} & \quad \sum_{i=1}^{m} x_i p_i \\
\text{subject to:} & \quad \sum_{i \mid g \in S_i} x_i \leq 1 \quad \forall g \\
& \quad x_i \in \{0, 1\} \quad \forall i
\end{align*}
\]
WDP: Runtime Variation

- Complete algorithms:
  - CPLEX [ILOG Inc.]
  - CASS [Leyton-Brown et.al],
  - GL [Gonen and Lehman]

- Gathered runtime data using various distributions
  - randomly sampled generator's parameters for each instance

- Even holding problem size constant, runtimes vary by many orders of magnitude across and within distributions
WDP: Features

1. **Linear Programming**
   - $L_1$, $L_2$, $L_\infty$ norms of integer slack vector

2. **Price**
   - $\text{stdev}(\text{prices})$
   - $\text{stdev}(\text{avg price per good})$
   - $\text{stdev}(\text{average price per sqrt(good)})$

3. **Bid-Good graph**
   - node degree stats (max, min, avg, stdev)

4. **Bid graph**
   - node degree stats
   - edge density
   - clustering coefficient (CC), stdev
   - avg min path length (AMPL)
   - ratio of CC to AMPL
   - eccentricity stats (max, min, avg, stdev)
WDP: Empirical Hardness Models

- Quadratic regression can be used to learn very accurate models
  - predicting $\log_{10}$ of CPLEX runtime
  
  ![Graph showing predicted log(runtime) vs. log(actual runtime)]

  - Root mean squared error: 0.216 (test data)
WDP: From Models to a Portfolio

Optimal Algorithm Selection

Portfolio Algorithm Selection

- CASS
- GL
- CPLEX

Time (s)

CPLEX Portfolio Optimal

0 100 200 300 400 500 600 700 800
SATZilla: A Portfolio for SAT

- **Algorithms** in the portfolio:
  - 2clseq [Bacchus]  Limmat [Biere]
  - OKsolver [Kullmann]  relsat [Bayardo]
  - Satz-Rand [Kautz, Li]  SATO [Zhang]
  - zChaff [Zhang]  Jerusat [Nadel]

- **Satzilla2** (Hors-Concours) added:
  - eqsatz [Li]  HeerHugo [Groote]
  - AutoWalkSat [Patterson, Kautz] (preprocessing)

- Developed in just over two weeks!
SATzilla: Features

1. **Problem Size**: \#vars, \#clauses, \#vars/\#clauses
   - rest of features are normalized by these

2. **Graphs**:
   - **Variable-Clause** (VCG, bipartite)
   - **Variable** (VG, edge whenever two variables occur in the same clause)
   - **Clause** (CG, edge whenever two clauses share a variable with opposite sign)
     - compute stats=(max, min, stdev, mean, entropy) over node degrees
     - for VCG, both for vars and clauses
     - # of unary, binary, ternary clauses
     - stats of the CG clustering coefficients
SATzilla: Features

3. Stats of \#positive/\#negative literals in each clause
4. Stats of \#positive/\#negative occurrences for each var
5. Horn clauses
   – total \#horn clauses
   – stats of \#horn occurrences for each var
6. LP relaxation features
   – objective value
   – stats of integer slacks
   – \#vars set to an integer
7. Probing features
   • DPLL probing features (to depth 256)
     – \#unit props after reaching depths 1, 4, 16, 64, 256
   • Local search probing (100 probes, each probe runs to plateau/max)
     – stats of climb height (in \#clauses) – stats of \#steps taken
     – stats of fraction of satisfied clauses – stats of break counts/\#vars
   • Search space size probing (5000 random search paths with unit-prop)
     – average depth to contradiction, estimate log-num-nodes in search tree
k1  # pos/# neg: should be abs(0.5 - #pos / (#pos + #neg)) so that flipping all pos and neg doesn't change the stat

kevinlb, 1/1/2004
SATzilla: Models and Portfolio

• Learned **linear regression models** for each algorithm
  
  – trained on more than 20000 instances
  
  • included 2002 competition instances
  
  • highly skewed towards random instances
  
  – training set preprocessed to **exclude instances** that were solved by all solvers, or by none of them
  
  – **terrible RMSE** on test set
  
  – enough predictive power to **discriminate** well

• On the training set, SATzilla’s choice takes on average **92 seconds** longer to run than the optimal choice
  
  – gives SATzilla an edge over its subsolvers, especially on harder instances
SATzilla: SAT-2003 Competition

• 2nd in Random instances track
• 3rd in Handmade track; 2nd in Handmade track, SAT only

• Only solver with good performance in more than one track
• Success measured in #series solved, not #benchmarks solved
  – Satzilla 2 solved more random instances than kcnfs
SATzilla: Areas for Improvement

• Add **new algorithms** to the portfolio
  – SATzilla outperformed all its constituent algorithms

• Construct **better models**
  – as we continue to study and analyze SAT data, our model accuracy is increasing

• Spend more development time to **eliminate bugs**
  – LP features timed out on many industrial benchmarks
    • instead of using a fallback solver (zChaff), SATzilla picked one essentially at random, but most don’t do well on industrial
    – some “random” instances were solved but didn’t count!
      • Relsat was chosen, and actually solved them, but it had an output bug 😞
Conclusions

• WDP
  – models: very mature, high accuracy
  – algorithms: one is dominant, limiting the size of possible gains from a portfolio approach

• SAT
  – models: more of a proof of concept, much room for improvement. However, discrimination accuracy is much better than prediction accuracy.
  – algorithms: many are strong and correlation is fairly low, making this an excellent domain for future study
Overall, our techniques provide a quick and relatively automatic blueprint for building algorithm portfolios, suitable when there are:

- two or more algorithms with relatively uncorrelated runtimes
- a set of good features
- lots of data