# Understanding Game-Theoretic Algorithms: The Game Matters

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http://gamut.stanford.edu

# Game-Theoretic Algorithms?

- Game Theory is useful in many areas of CS
  - Can model multiagent interactions arising in:
    - AI
    - Distributed Systems
    - Networking
- GT also gives rise to some very interesting computational problems
  - compute a sample Nash equilibrium
  - multiagent adaptation (learning)
- So far:
  - very few theoretical results are available
  - even fewer empirical studies

# Outline

- What is GAMUT?
  - Introduction
  - Definitions
  - Classes of Games
  - Implementation
- Experimental Results
  - Computing a sample Nash Equilibrium
  - Multiagent Adaptation

# What is GAMUT?

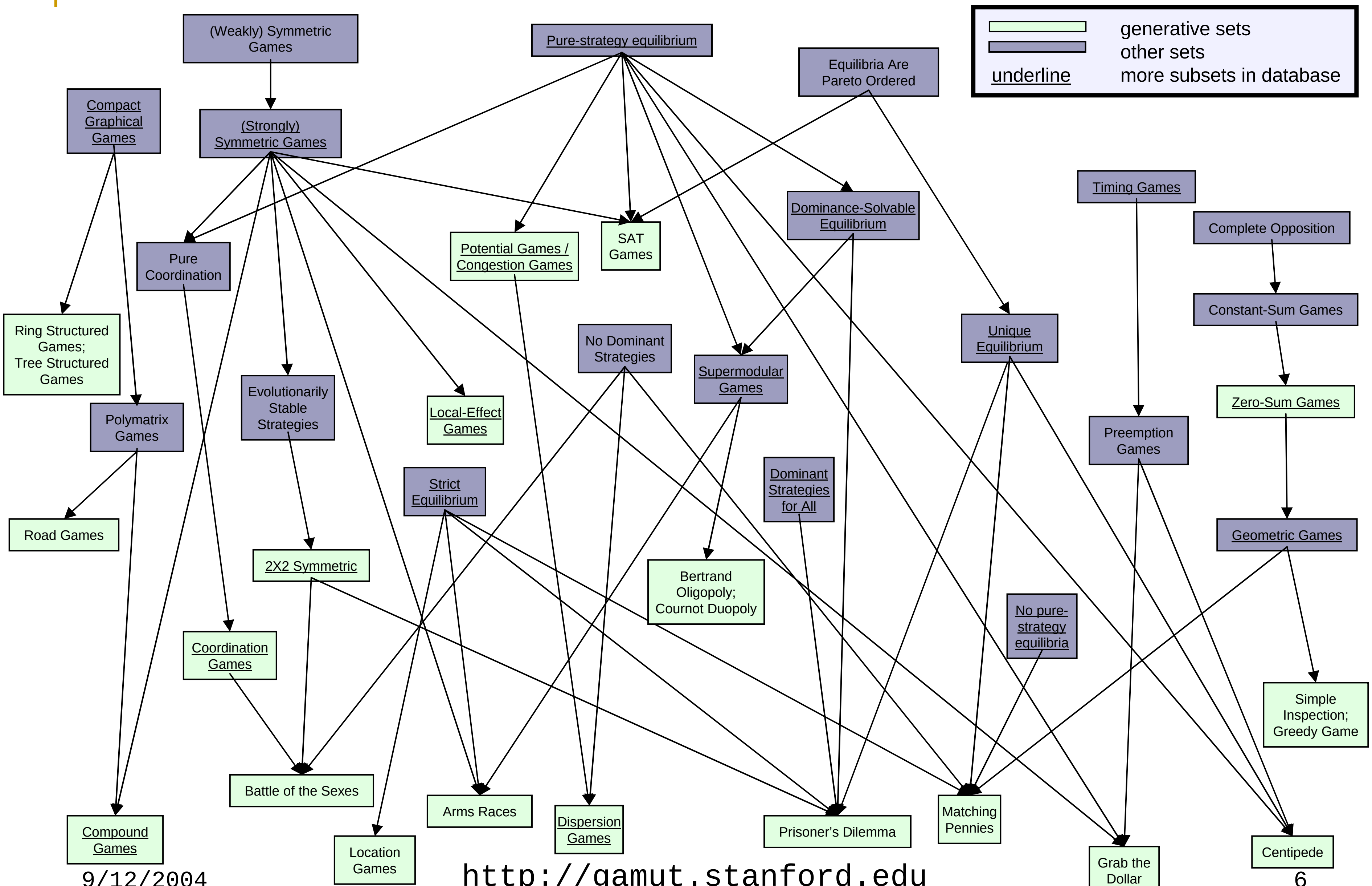
1. A database of classes of games discussed in the literature
   - containment relation between classes
   - distinguishes between *generative* and non-generative sets
2. Implementation of generators for these classes

- Not to be confused with *Gambit*
  - a library of GT software

# Games in GAMUT

- Searched through hundreds of books and papers
  - Game Theory, CS, Political Science, Economics etc.
- We identified 122 interesting sets of games
- 71 of these admit finite-time generative procedures
  - the rest are either too broad or defined implicitly
    - e.g. games with a pure strategy NE
- Sets vary from tiny to huge
  - Prisoners' Dilemma
  - games compactly representable as graphical games
- GAMUT 1.0 contains games that can reasonably be stored in normal form

# How are the games related?



# But isn't everything a game?

- Why not generate payoffs at random?
  - all classes of interest that we discovered are *non-generic* w.r.t. uniformly random sampling
- General lessons of empirical algorithmics:
  - algorithms' behavior varies substantially across "reasonable" input distributions
  - in practice, structure is at least as important as problem size
  - uniformly-random inputs often have very different computational properties

# How was GAMUT built?

- Implemented in Java
- Focus on extensibility and ease of use
  - big software engineering effort
- Most important entity is a `Generator`
  - 35 Java classes suffice to generate games from our 71 sets
  - can pick generators from subset/intersection of classes according to our taxonomy
  - can create subdistributions by partially settings parameters
- Other basic entities include `Outputs, Graphs, Functions`
- Incorporates many utilities:
  - powerful parameter handling mechanism
  - fixed-point conversion and normalization
  - ability to sample parameters at random

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# Running the GAMUT

- Goal: demonstrate empirical variance w.r.t different instance distributions
- Two computational problems:
  - computing a sample Nash equilibrium
  - multiagent adaptation
- Cluster of 12 dual-CPU 2.4GHz Xeons; Linux 2.4.2
- All runtimes reported in seconds
  - runs capped at 1800 seconds (30 minutes)
- Total of over 120 CPU-days of data

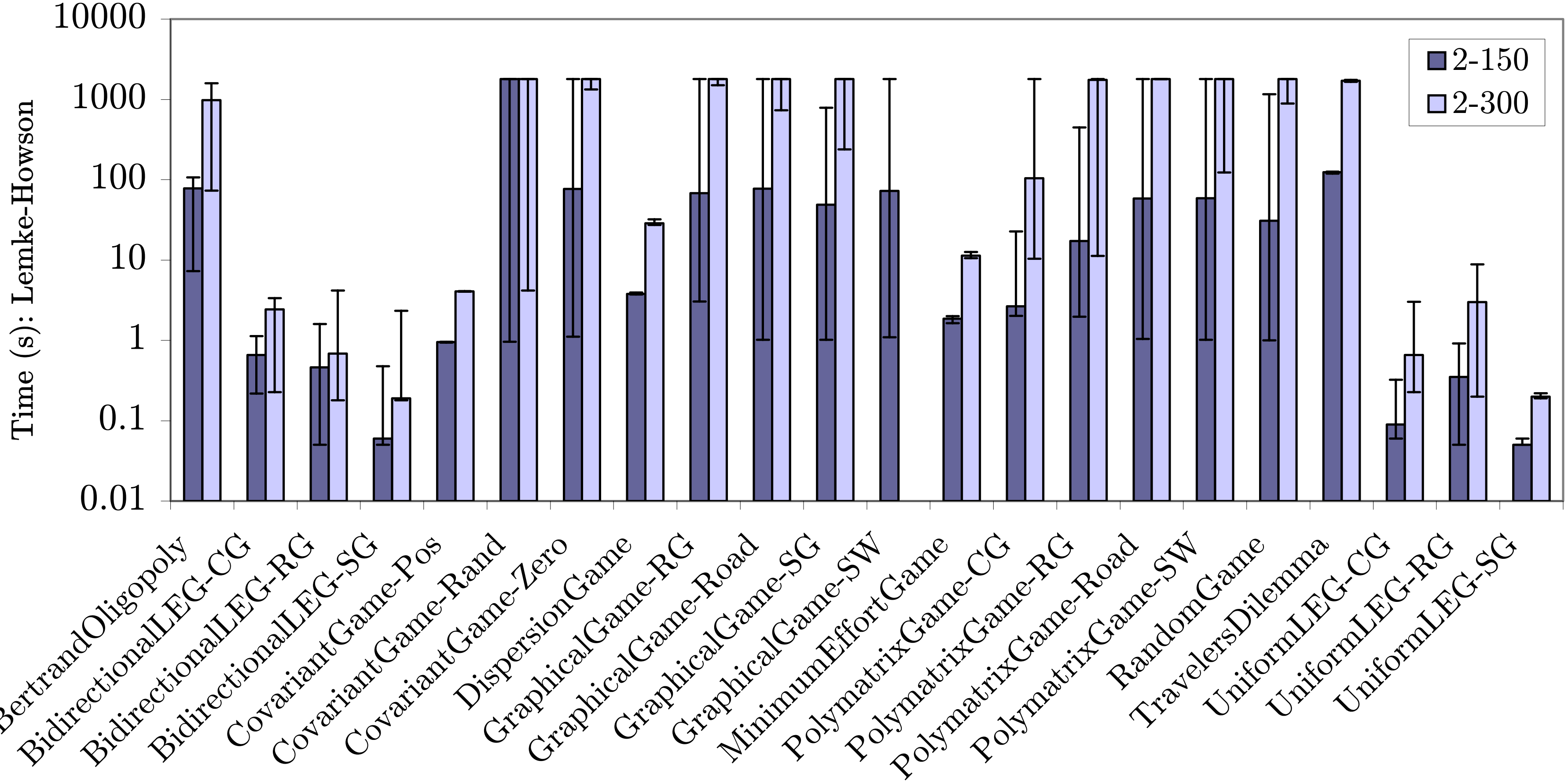
# Computing Sample Nash Equilibrium

- Algorithms tend to be very complex
  - Gambit: a comprehensive software package
    - Lemke-Howson (2-player games)
    - Simplicial Subdivision (n-player)
    - both use iterated removal of dominated strategies
  - Govindan-Wilson
    - a new path-following approach
- All have worst-case exponential lower-bounds
  - not known how tight these bounds actually are
  - complexity class for the problem is unknown

# Experimental Setup

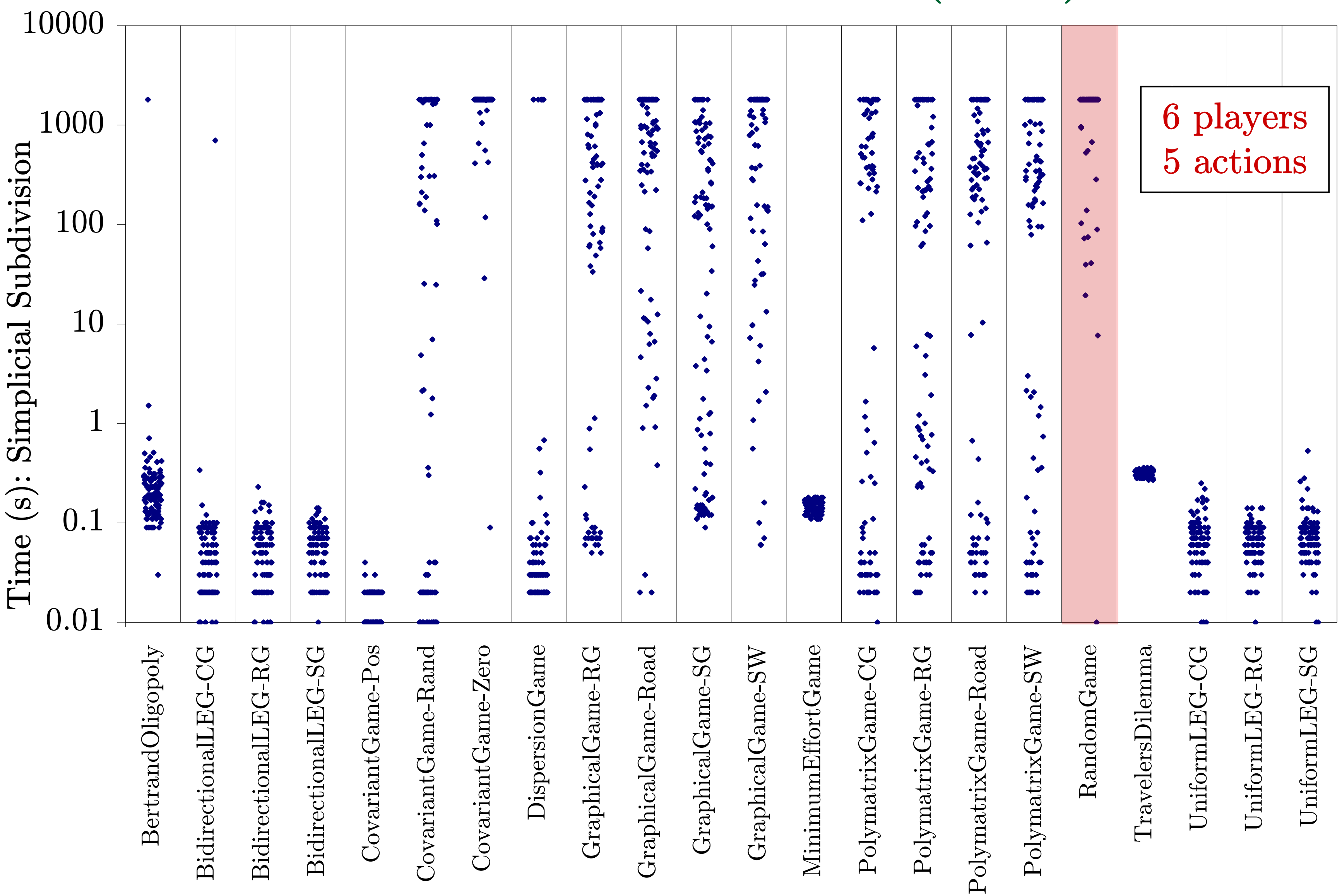
- Four fixed size datasets:
  - focus on differences due to structure
  - 2 players: with 150 and 300 actions
  - 6 players, 5 actions
  - 18 players, 2 actions
- 22 different distributions from GAMUT
  - many, but not nearly all, of GAMUT distributions
- 100 instances for each size/distribution

# Effect of Problem Size (LH)

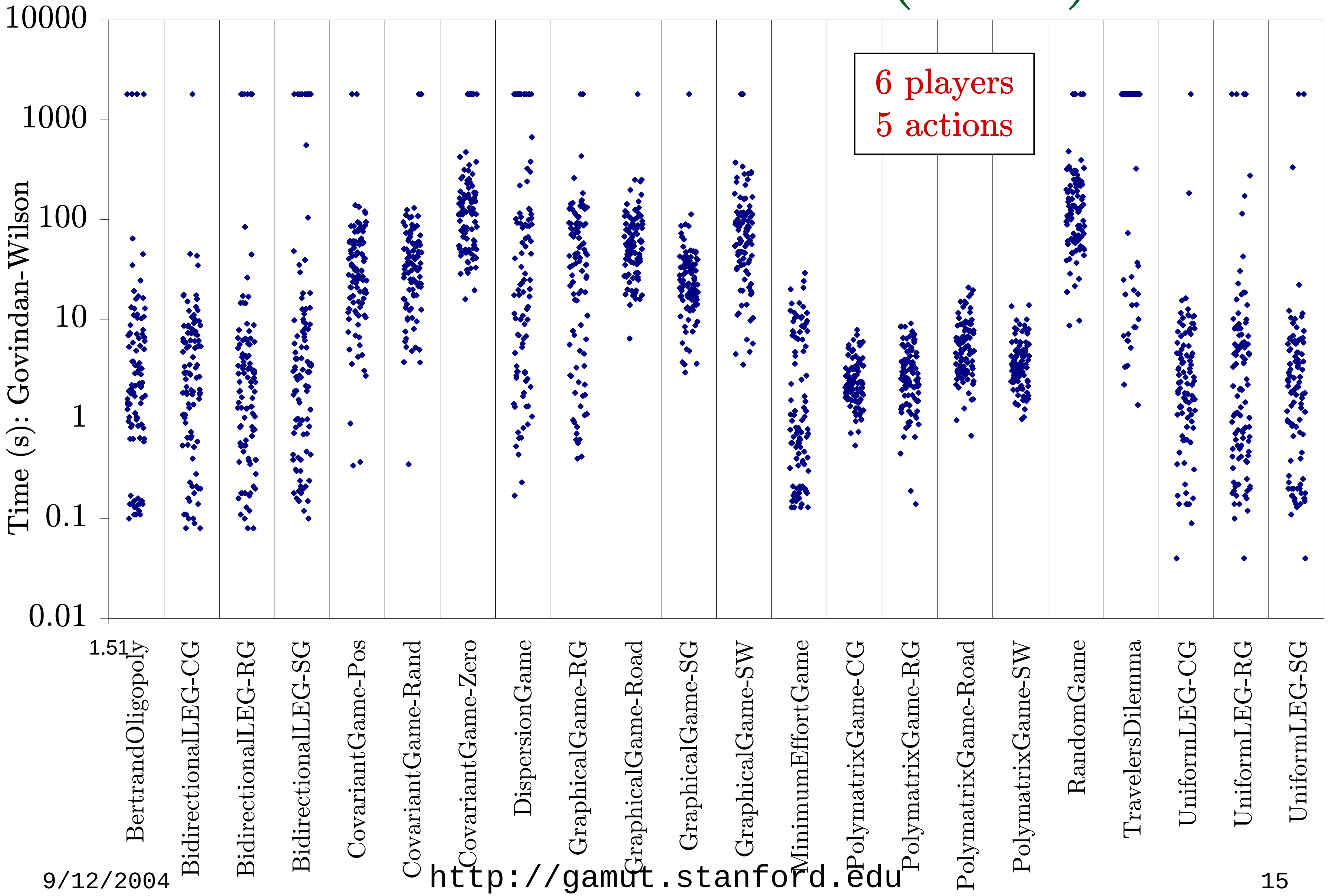



- Behavior varies across distributions
- Such variation occurs at different problem sizes

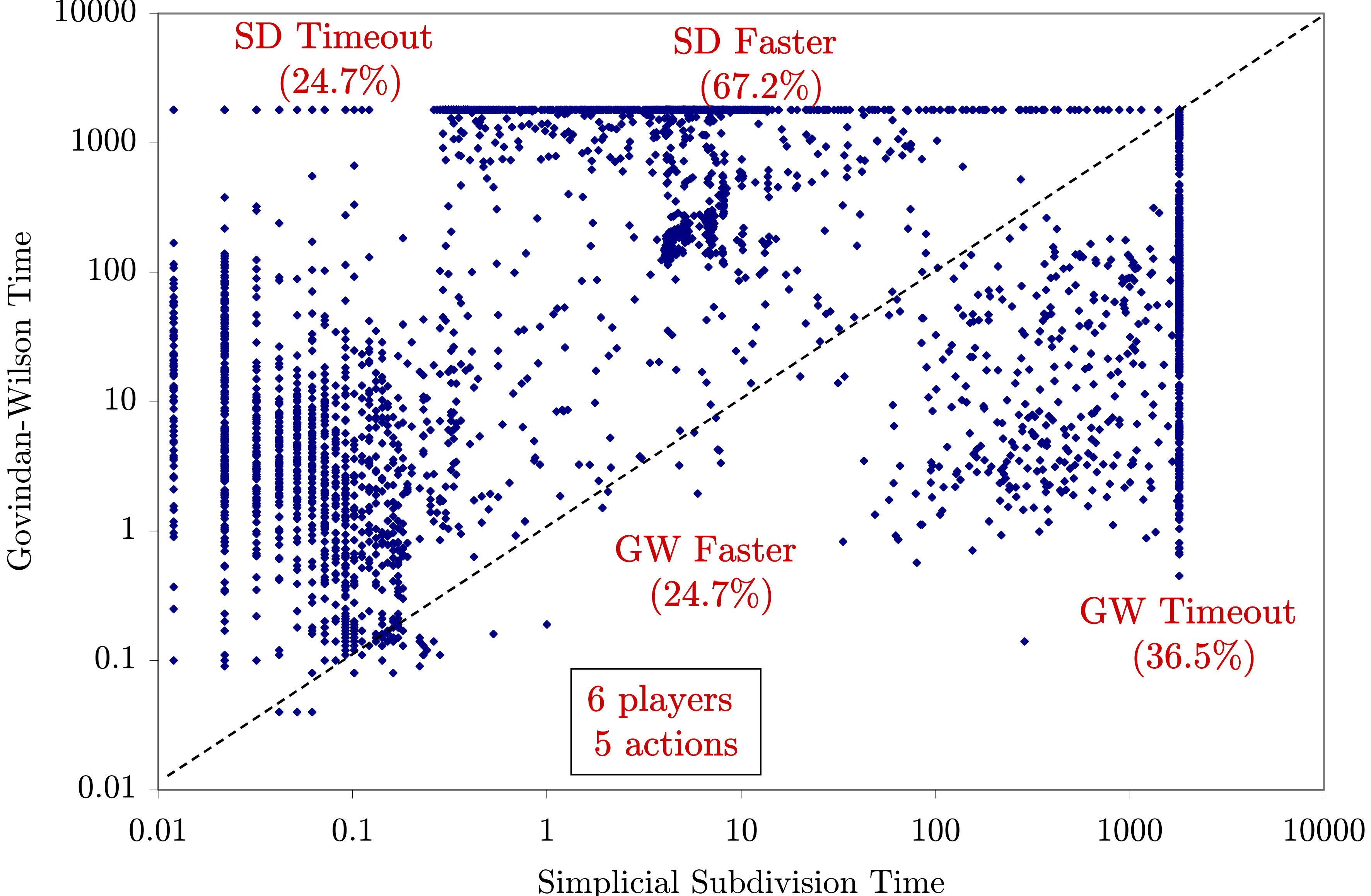
# Runtime Distributions (SD)



# Runtime Distributions (GW)



# Algorithm Correlation



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# Multiagent Adaptation

- Active, yet young area
  - not always clear what the goals are
  - our goal is to show the importance of distribution choice; *not* to evaluate algorithms
- Gathered data using three algorithms:
  - Minimax-Q [Littman,1994]
    - safety level guarantee
  - WoLF [Bowling, Veloso, 2001]
    - converges to best response
  - SingleAgent-Q [Watkins, Dayan, 1992]
    - ignores strategic aspects and opponent adaptation

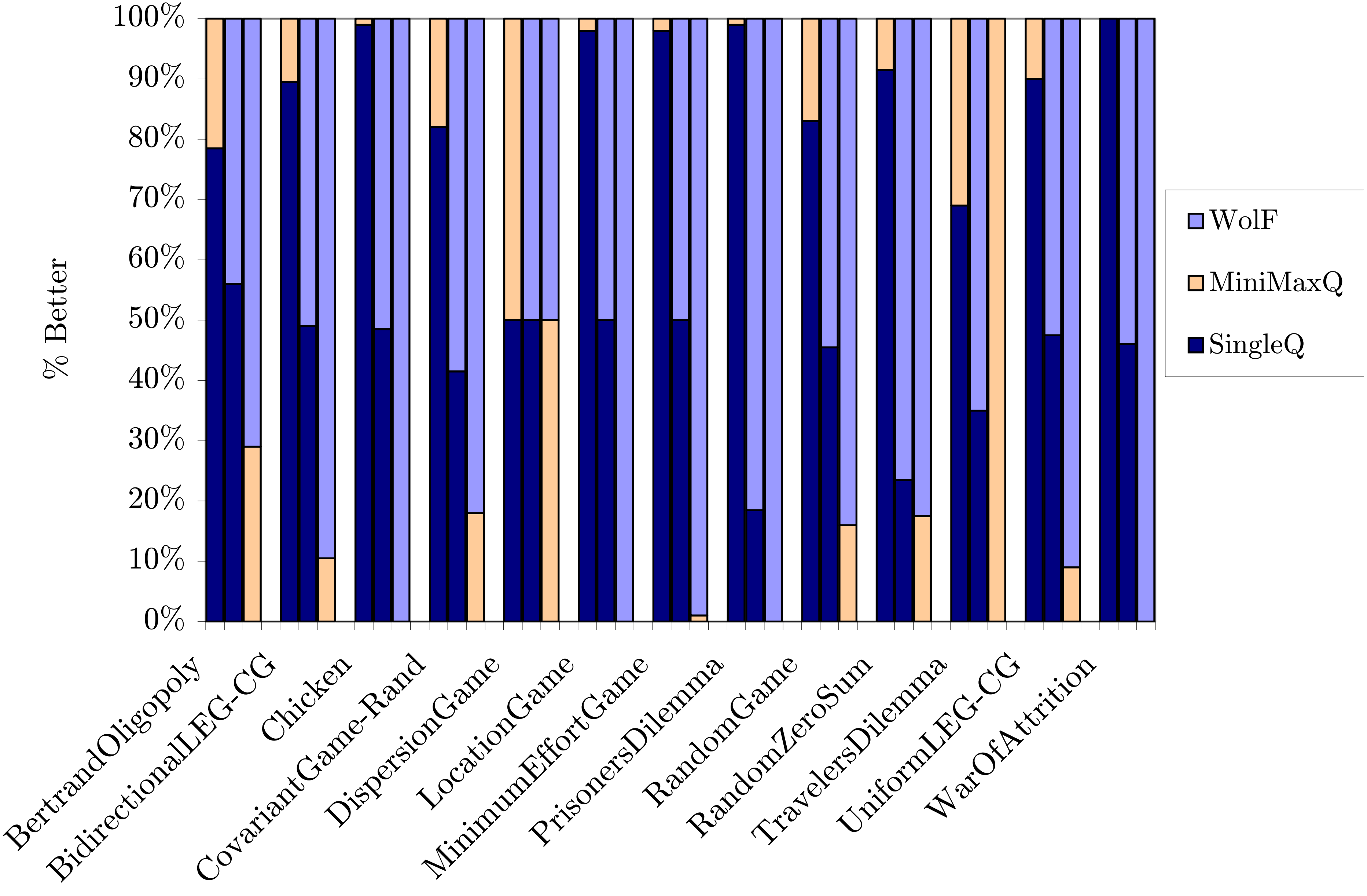
# Experimental Setup

- Repeated 2x2 game setting
  - 100,000 rounds played
  - report average payoffs over the final 10,000 rounds
- 100 instances from 13 distributions
- 9 pairings
  - all pairings (including self); as both players
  - averaged over 10 runs for each pairing
- Algorithm parameters fixed
  - tried to match those reported in literature

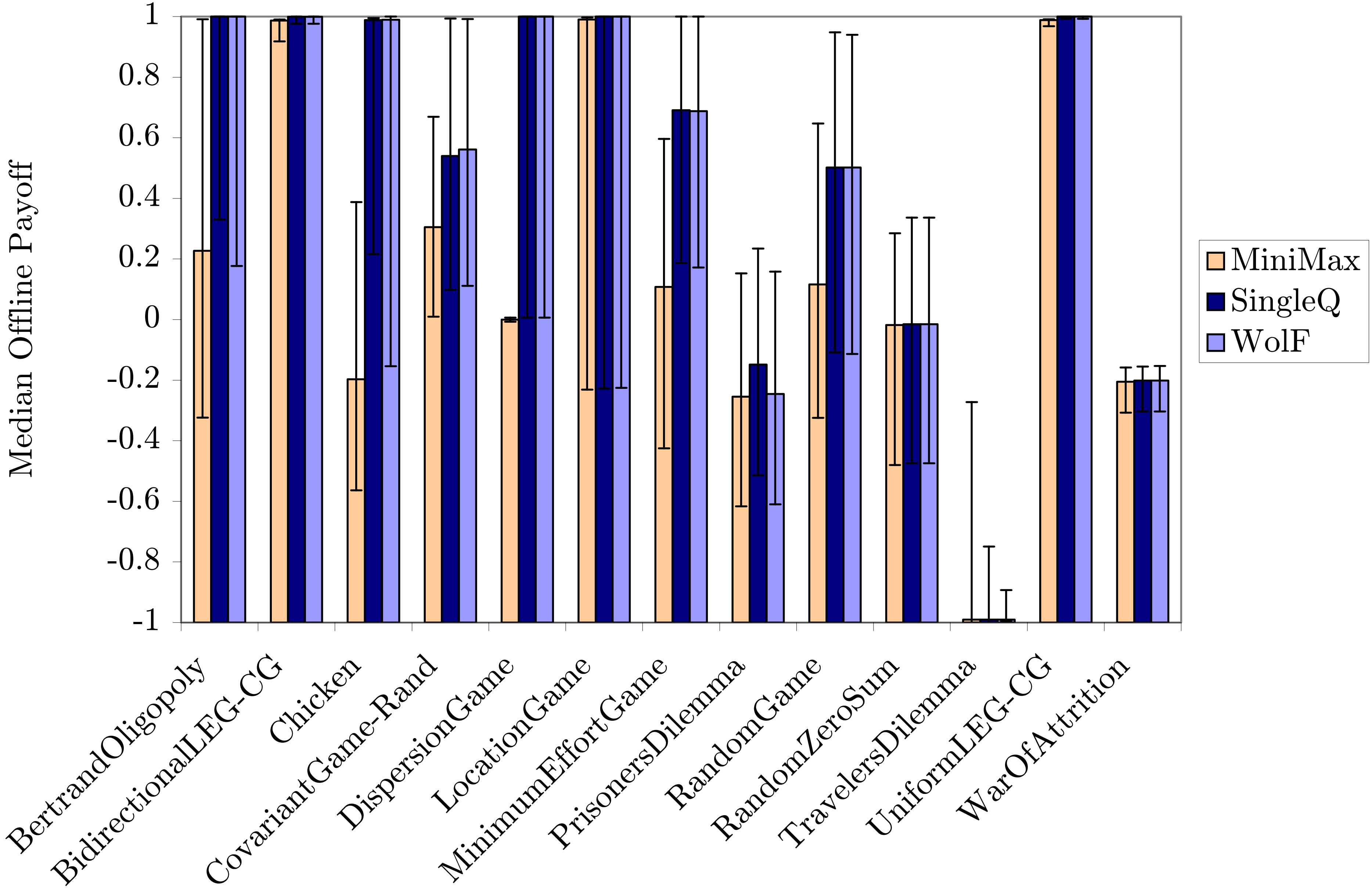
# Result Evaluation

- Tons of data
- Lots of possible metrics could be considered
- We focus on just two:
  - pairwise: fraction of time one algorithm is better than another
  - median payoff obtained as player 1
    - payoffs are normalized between [-1,1]
    - not always comparable across distributions
  - in both metrics, results differ across distributions!

# Pairwise Performance



# Median Payoff Performance



# Conclusion

- GAMUT is a comprehensive test suite
  - based on extensive literature survey
  - capable of generating games from many classes
  - extensible
- Choice of test data is extremely important
  - experiments show high runtime variation across different classes of games for several state-of-the-art algorithms and two computational problems
- Behavior of game-theoretic algorithms is still poorly understood
  - we hope GAMUT will be used to address this and more!

http://gamut.stanford.edu

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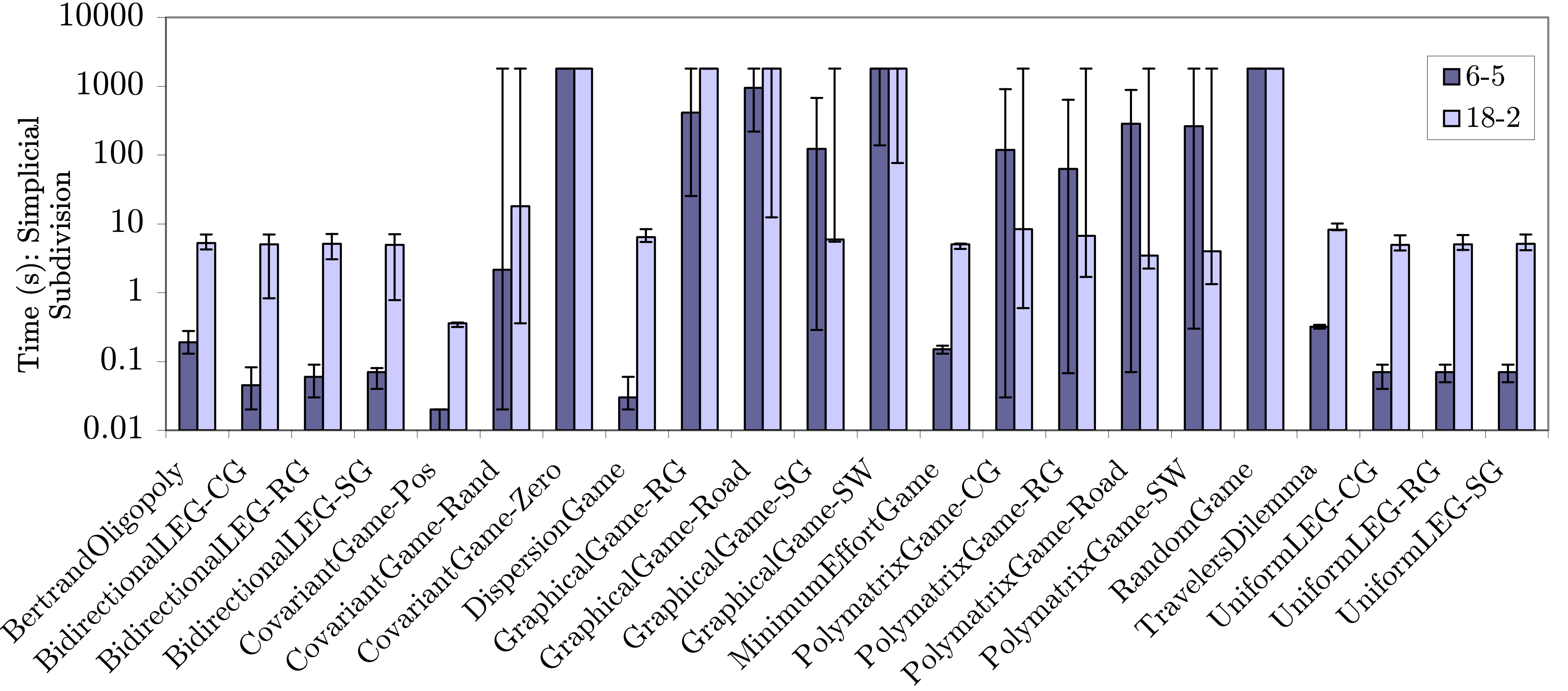
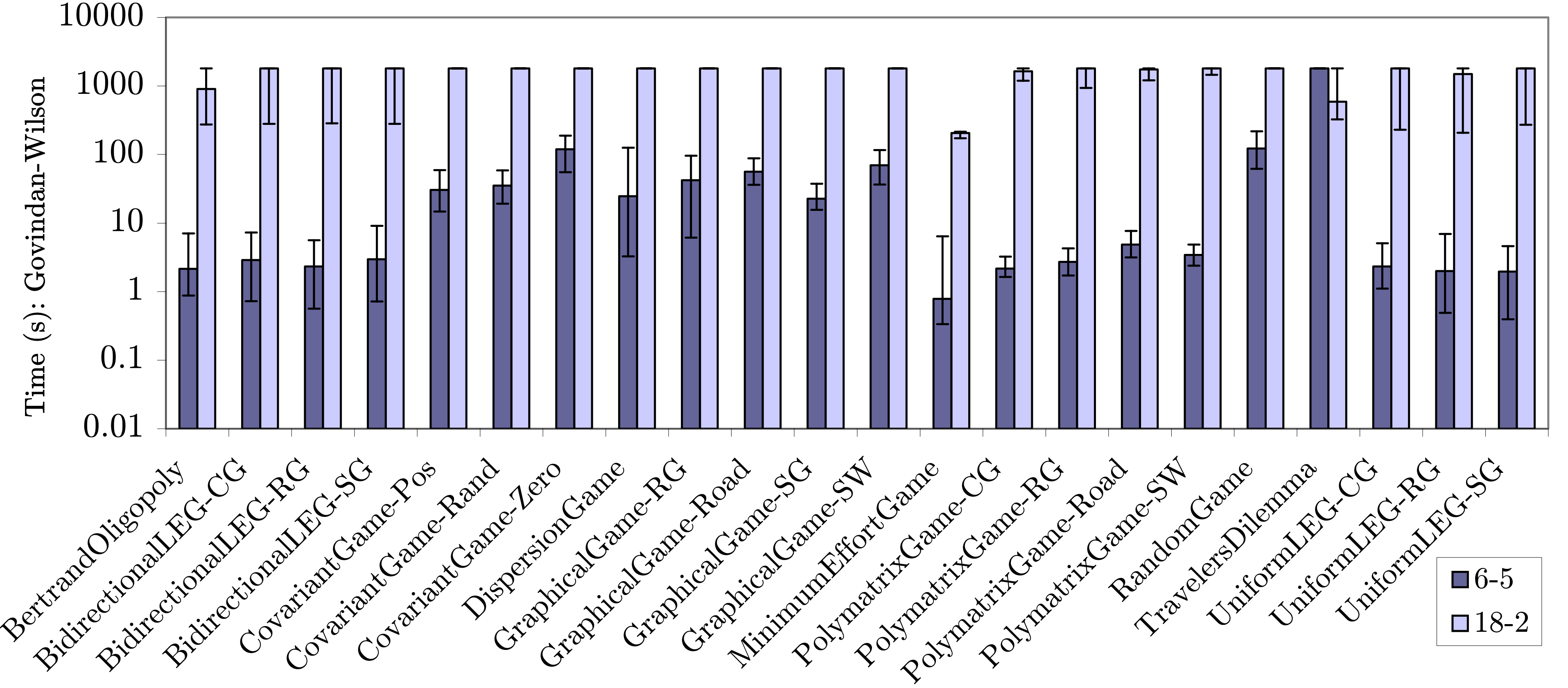


# Effect of Problem Size (GW)



# Runtime Distributions: Summary

- Distribution of runtimes varies significantly across inputs
  – cannot be inferred based on knowing algorithm and input size
- Some games solved in preprocessing
- Games taxonomically related seem to have more similar distributions
- Algorithms appear to be significantly different from each other
  – runtime variation is not specific to any single algorithm
- Shows why GAMUT is needed

# So, what is a game?

- In order to generate and perform computation on games we must be clear about:

1. Semantics:
   - a game is defined by (players, actions, payoffs)
2. Syntax:
   - *representations* may include Normal Form, Extensive Form, Graphical Games, etc.
   - can be *compact, complete*

- Not uncontroversial in GT
  - less controversial for computational purposes

- In GAMUT 1.0 we focus on games representable compactly in normal form