Learning the Empirical Hardness of Combinatorial Auctions

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Introduction

- Recent trend: study of average/empirical hardness as opposed to the worst-case complexity ($\mathcal{NP}$-Hardness) [Cheeseman et al.; Selman et al.]
- Our proposal: predict the running time of an algorithm on a particular instance based on features of that instance
- Today:
  - a methodology for doing this
  - its application to the combinatorial auction winner determination problem (WDP)

Why?

- Predict running time
  - for its own sake
  - build algorithm portfolios
- Theoretical understanding of hardness
  - tune distributions for hardness
  - improve algorithms
- Problem specific
  - WDP: design bidding rules

Related Work

- Decision problems:
  - phase transitions in solvability, corresponding to hardness spike [Cheeseman et al.; Selman et al.]
  - solution invariants: e.g., backbone [Gomes et al.]
- Optimization problems:
  - experimental:
    - reduce to decision problem [Zhang et al.]
    - introduce backbone concepts [Walsh et al.]
  - theoretical:
    - polynomial/exponential transition in search algorithms [Zhang]
    - predict A* nodes expanded for problem distribution [Korf, Reid]
- Learning
  - dynamic restart policies [Kautz et al.]

Combinatorial Auctions

- Auctioneer sells a set of non-homogeneous items
- Bidders often have complex *valuations*
  - *complementarities*
    - *e.g.* $V(TV \,\&\, VCR) > V(TV) + V(VCR)$
  - *substitutabilities*
    - $V(TV_1 \,\&\, TV_2) < V(TV_1) + V(TV_2)$
- Solution: allow bids on *bundles* of goods
  - achieves a higher revenue and social welfare than separate auctions
- Two hard problems:
  - Expressing valuations
  - Determining optimal allocation

Winner Determination Problem

- Equivalent to weighted set packing
- Input: *m bids* $< S_i, p_i >, S_i \subseteq \{1 \ldots N\}$
- Objective: find revenue-maximizing non-conflicting allocation

$$\text{maximize:} \quad \sum_{i=1}^{m} x_i p_i$$

$$\text{subject to:} \quad \sum_{i|g \in S_i} x_i \leq 1 \qquad \forall g$$

$$x_i \in \{0, 1\} \qquad \forall i$$

- Even constant factor approximation is $\mathcal{NP}$-Hard
- Square-root approximation known
- Polynomial in the number of bids

WDP Case Study

- Difficulty: highly parameterized, complex distributions
- Hard to analyze theoretically
  - large variation in edge costs and branching factors throughout the search tree [Korf, Reid, Zhang]
- Too many parameters to vary systematically [Walsh et al., Gomes et. al.]
- Parameters affect expected optimum: difficult to transform to decision problem [Zhang et al.]

Methodology

1. Select algorithm
2. Select set of input distributions
3. Factor out known sources of hardness
4. Choose features
5. Generate instances
6. Compute running time, features
7. Learn a model of running time

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WDP Distributions

- Legacy (7 distributions)
  - sample bid sizes/prices independently from simple statistical distributions
- Combinatorial Auctions Test Suite (CATS)
  - Attempted to model bidder valuations to provide more motivated CA distributions
    1. **regions**: real estate
    2. **arbitrary**: complementarity described by weighted graph
    3. **matching**: FAA take-off & landing auctions
    4. **scheduling**: single resource, multiple deadlines for each agent [Wellman]

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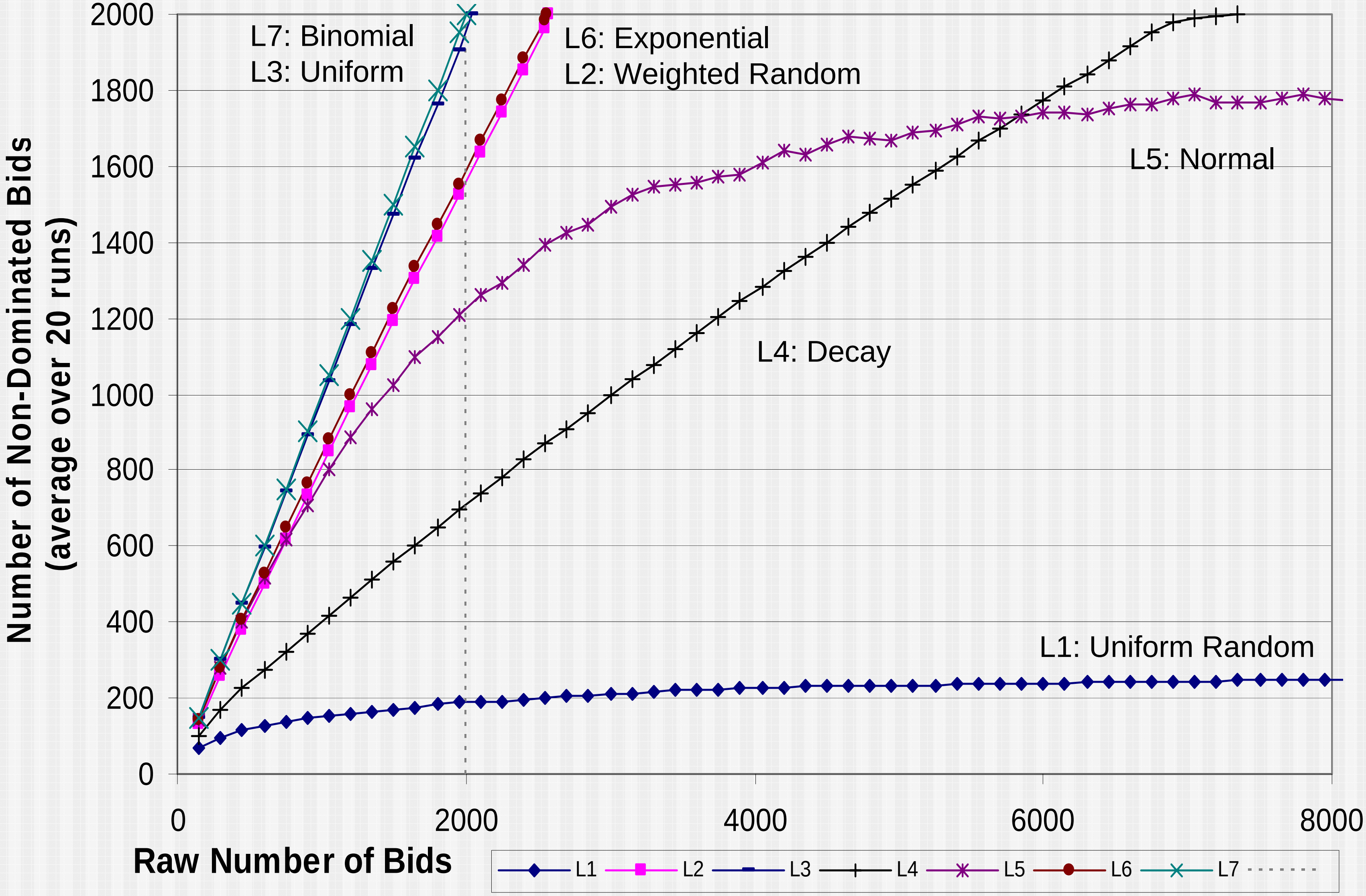
Problem Size

- Some sources of hardness well-understood
  - hold these constant to focus on unknown sources of hardness
- Common: input size
- Problem size is affected by preprocessing techniques! (e.g. arc-consistency)
- WDP: *dominated bids* can be removed
- *(raw #bids, #goods)* is a **very misleading** measure of size for legacy distributions
  - we fix size as (#non-dominated bids, #goods)

Raw vs. Non-Dominated Bids

(64 goods, target of 2000 non-dominated bids)



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Features

- No automatic way to construct features
  - must come from domain knowledge
- We require features to be:
  - polynomial-time computable
  - distribution-independent
- We identified 35 features
  - after using various statistical feature selection techniques, we were left with 25

Features

- Bid Good Graph (BGG)
  1. Bid node degree stats
  2. Good node degree stats
- Price-based features
  9. std. deviation
  10. stdev price/#goods
  11. stdev price/ √#goods
- Bid Graph (BG)
  3. node degree stats
  4. edge density
  5. clustering coef. and deviation
  6. avg. min. path. length
  7. ratio of 5 & 6
  8. node eccentricity stats
- LP Relaxation
  12. L_1, L_2, L_∞ norms of integer slack vector

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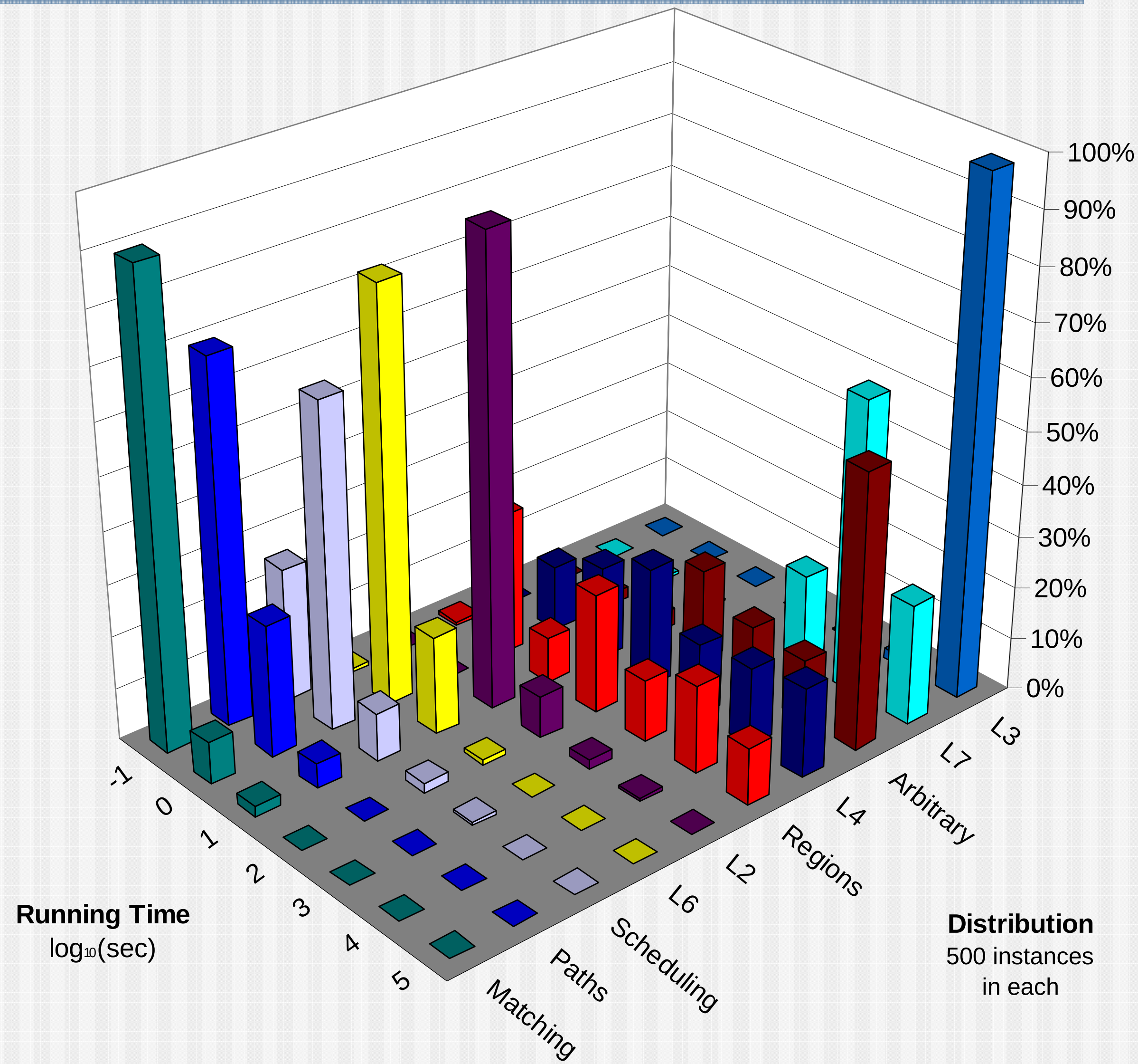
Experimental Setup

- **Sample** parameters uniformly from range of acceptable values
- 3 separate datasets:
  - 256 goods, 1000 non-dominated bids
  - 144 goods, 1000 non-dominated bids
  - 64 goods, 2000 non-dominated bids
- 4500 instances/dataset, from 9 distributions
- Collecting data took approximately **3 years** of CPU time! (550 MHz Xeons, Linux 2.12)
- Running times varied from 0.01 sec to 22 hours (CPLEX capped at 130000 nodes)

Gross Hardness (256 goods, 1000 bids)



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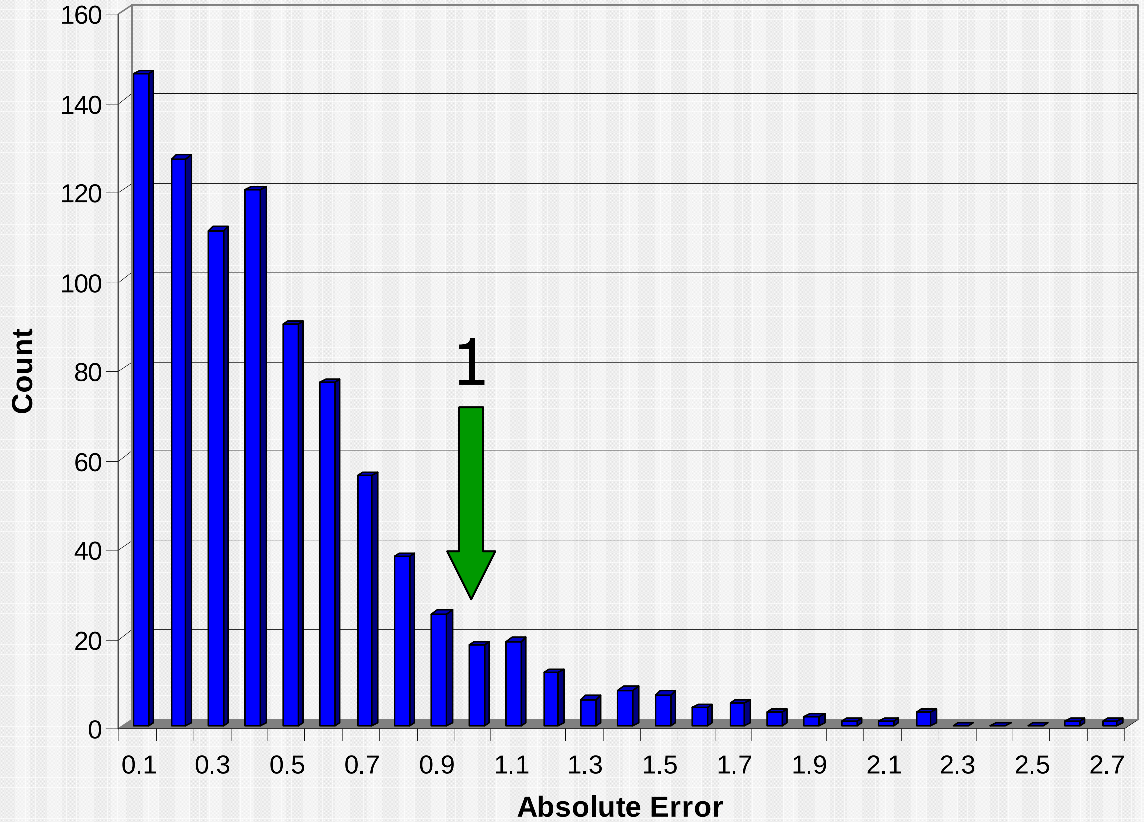
Learning

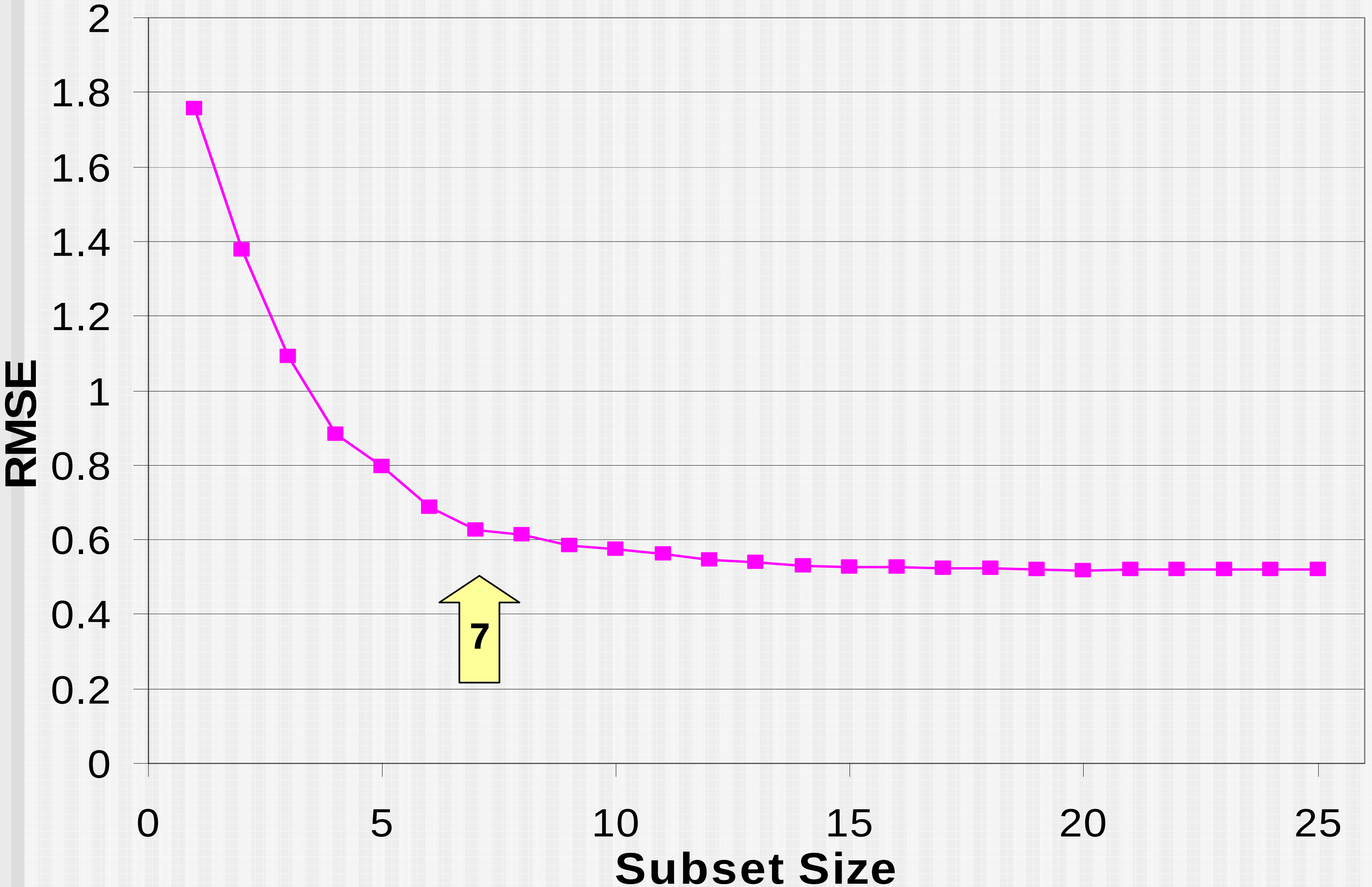
- Classification: misleading error measure
- Statistical regression: learn a continuous function of features that predicts **log of running time**
- Supervised learning: data broken into 80% training set, 20% test set
- Started with simplest technique: linear regression
  - find a hyperplane that minimizes root mean squared error (RMSE) on training data
- Linear regression is useful:
  - as a (surprisingly good) baseline
  - yields a very interpretable model with understandable variables

LR: Error

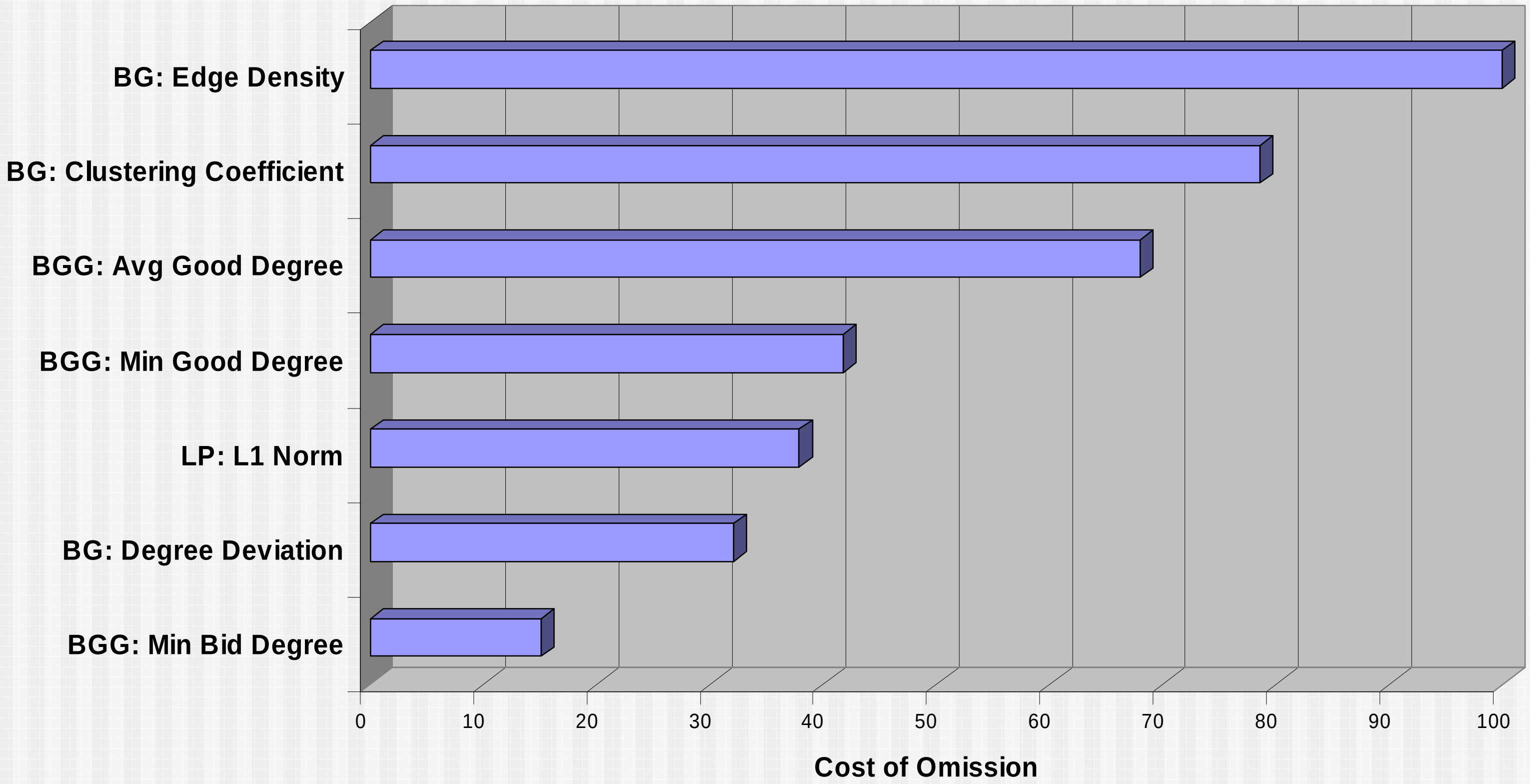
Dataset	RMSE	MAE
1000 Bids, 256 Goods	0.581	0.436



LR: Subset Selection



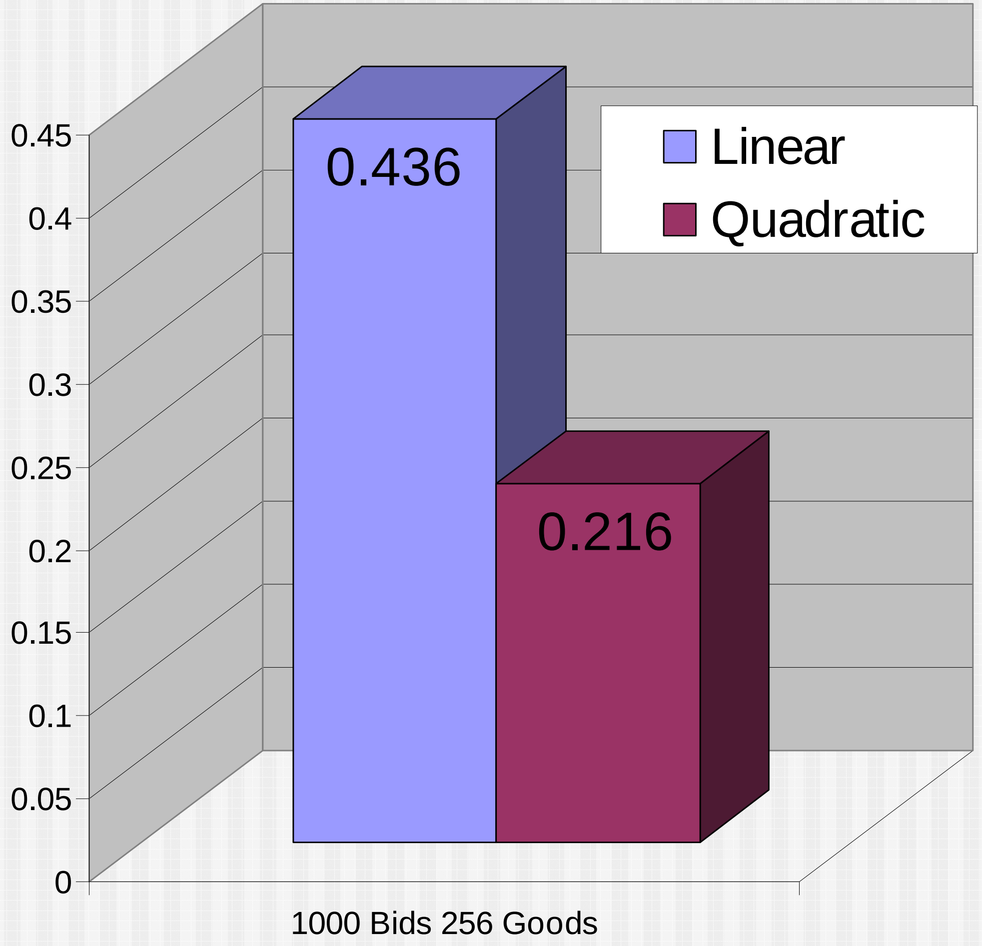
LR: Cost of Omission (subset size 7)



Non-Linear Approaches

- Linear regression doesn't consider interactions between variables; likely to underfit data
- Consider 2nd degree polynomials
- Variables = pairwise products of original features
  - total of 325 variables
  - (cf. clauses/variables)
- More predictability, less interpretability

Quadratic vs Linear Regression



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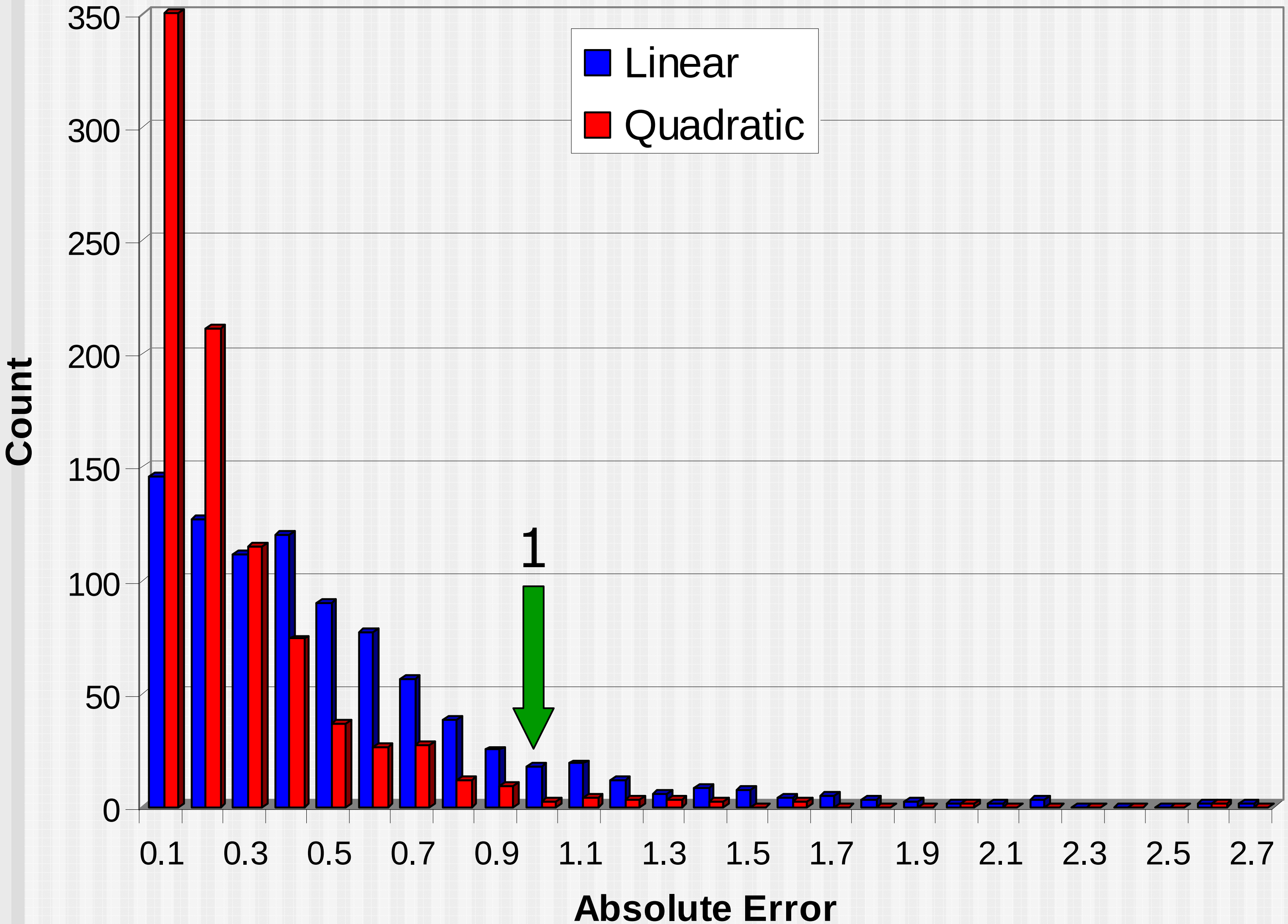
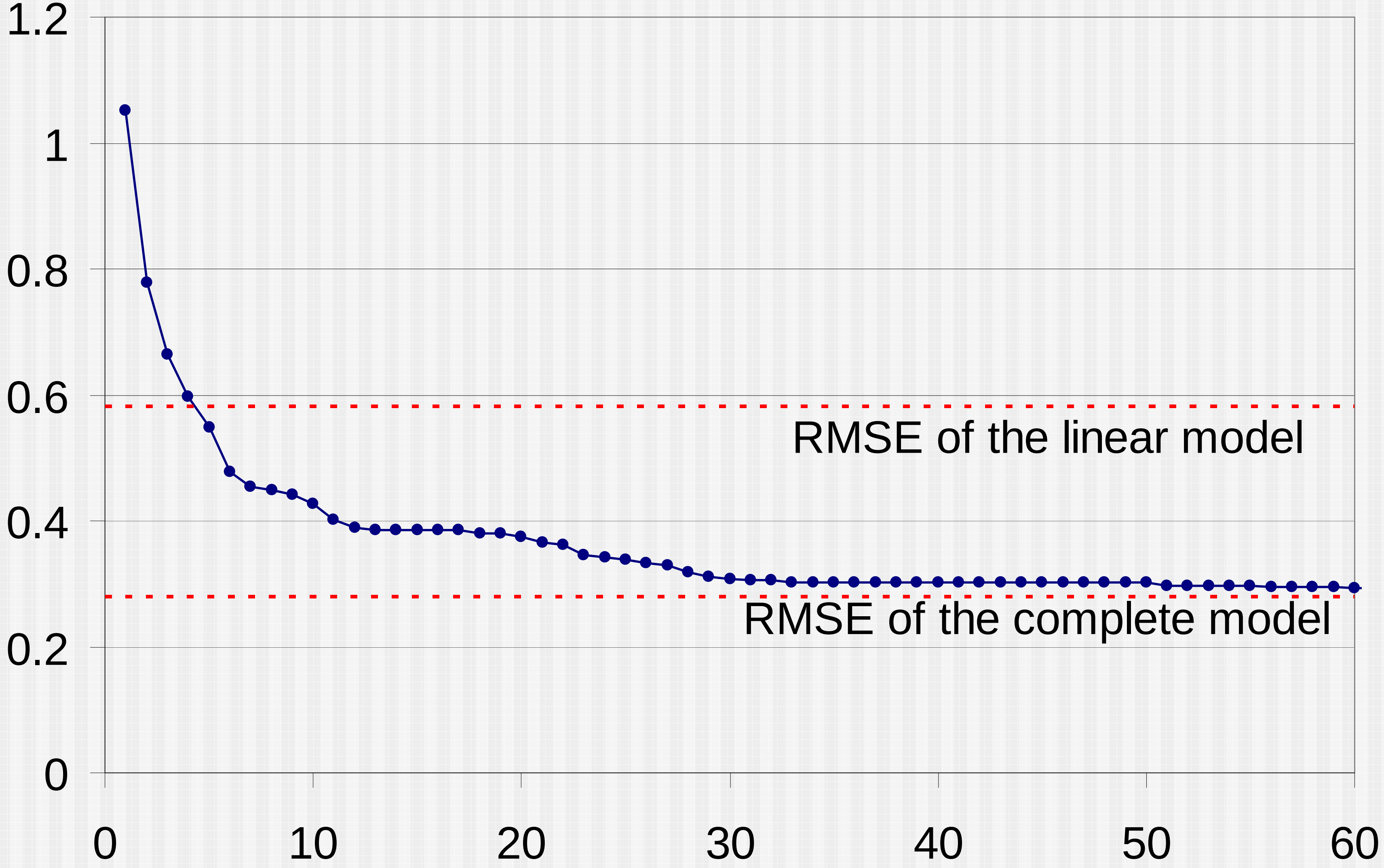
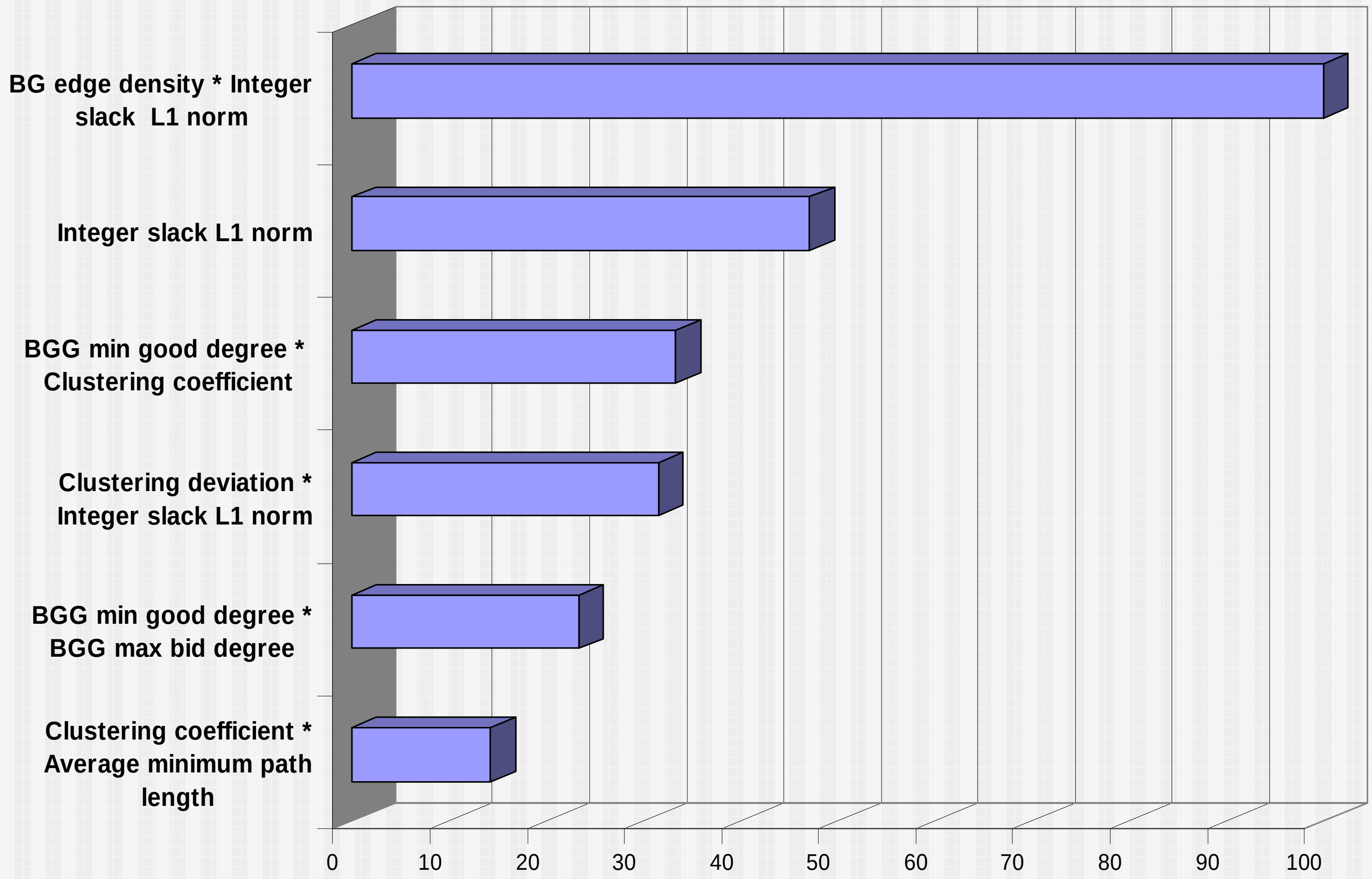


QR: RMSE vs. Subset Size



Cost of Omission (subset size 6)



What's Next?

- Constructing algorithm portfolios
  - combine several uncorrelated algorithms
  - good initial results for WDP
- Tuning distributions for hardness
  - releasing new version of CATS

Summary

- Algorithms are predictable
  - Empirical hardness can be studied in a disciplined way
- Once again: Structure matters!
  - Uniform distributions aren’t the best testbeds
  - Constraint graphs are very useful
  - Hypothesis: good heuristics make good features (e.g. LP)
- Our methodology is general and can be applied to other problems!