Bidding Rings Revisited

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This talk is based on a paper with:

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The Collusion Problem

- The **setting**:
  - One **non-repeated** auction
  - A **single good**
  - IPV from an **arbitrary** (continuously differentiable) distribution
  - risk neutrality, no externalities

- Can agents **coordinate their bidding** in a way that causes them to gain, even when:
  - colluding agents can **deviate** from the protocol
    - they remain perfectly competitive
  - ring center is **unwilling to lose money** on expectation
  - non-colluding agents play **best responses** to the colluders
  - **more than one** bidding ring may exist
  - agents **aren't forced** to participate in the bidding ring

Second-Price Auctions

Collusion protocol [Graham & Marshall; 1990], [Mailath & Zemsky; 1991], …

1. **Knockout auction**: each agent i states valuation v_i to a ring center

2. Ring center places a bid of v_i for the **high bidder**, drops lower bidders

   let v_2 denote the valuation of the second-highest bidder in the auction, and
   let $v^r{}_2$ denote the valuation of the second-highest bidder in the bidding ring

3. If he **wins**, i pays v_2 to the auctioneer and
   $\max(v^r{}_2 - v_2, 0)$ to the ring center

   let c denote the ring center's expected profit, and
   let k denote the number of bidders in the bidding ring

4. The ring center makes a **payment to each bidder** of $c' \leq c/k$, regardless of the auction's outcome

Second-Price Auctions

Bidding Ring Checklist:

- ☐ agents gain from collusion
- ☐ colluding agents will not deviate from the protocol
- ☐ ring center doesn't lose money on expectation
- ☐ non-colluding agents play best responses to the colluders
- ☐ more than one bidding ring may exist
- ☐ agents are free to opt out of the bidding ring

Graham & Marshall protocol:

- ☑ agents gain c'
- ☑ truth telling is a dominant strategy for ring members
- ☑ ring center is budget-balanced or profitable on expectation, depending on c'
- ☑ truth telling is a dominant strategy for non-ring members
- ☑ doesn't affect the dominant strategy
- ☑ agents would gain nothing by declining participation

V. Krishna, Auction Theory, 2002, *Collusion in First-Price Auctions*, pp. 160-1:

The operation of bidding rings in first-price auctions introduces some new elements. *First, unlike in a second-price auction, the cartel agreement in a first-price auction is* not *self-enforcing and, hence, is somewhat fragile. To see this simply, consider an all-inclusive cartel. Assuming that the highest value exceeds the reserve price set by the seller, such a cartel will try to obtain the object at the reserve price by submitting only one bid at this level and ensuring that no other bid exceeds this amount. But now consider a bidder whose value is greater than the reserve price but is not the highest. Such a bidder has the* ***incentive to cheat*** *on the cartel agreement and, by submitting a bid that just exceeds the reserve price, win the object. This suggests that second-price auctions are more susceptible to collusive practices–they have a built-in enforcement mechanism–than are first-price auctions. [...]*

Second, even if the bidders are ex ante *symmetric, the operation of a cartel naturally introduces asymmetries among bidders. While this did not affect bidding behavior in second-price auctions–it was still a dominant strategy to bid one's value–it does affect behavior in first-price auctions. In particular,* ***bidders not in the cartel*** *face a different decision problem if there is a cartel in operation than if there is not. We have already seen that the analysis of bidding behavior in asymmetric first-price auctions is more problematic; as a result,* ***our understanding of bidding rings in this context is more limited***.

First-Price Auctions

Collusion possible in restricted settings: [McAfee & McMillan; 1993]

1. All bidders belong to the cartel
   a) Weak cartels (no side payments permitted): all agents bid same amount, using auctioneer's tie-breaking rule to select a winner
   b) Strong cartels: knockout auction drops all but one bidder, who bids the reserve price; he redistributes his winnings through side payments

2. One bidding ring exists; some bidders bid as singletons
   a) non-cartel bidders bid as though collusion is impossible: their strategies depend on the number of bidders in the auction, so their bid is lower after some bidders have been dropped
   b) each bidder has valuation $\in \{0, 1\}$

First-Price Auctions

Bidding Ring Checklist:	McAfee & McMillan protocol:
☐ agents gain from collusion	☑ agents gain under all protocols
☐ colluding agents will not deviate from the protocol	☑ an equilibrium exists for strong cartels only
☐ ring center doesn't lose money on expectation	☑ ring center never pays or receives any money
☐ non-colluding agents play best responses to the colluders	☒ no (except when $v_i \in \{0, 1\}$, which violates arbitrary distribution)
☐ more than one bidding ring may exist	☒ no
☐ agents are free to opt out of the bidding ring	☒ no (though M&M do discuss "cartel formation games")

What makes collusion in first-price auctions so much harder?

- number of bidders matters to agents' equilibrium strategies
- We need to think carefully about information structure...

What do Agents Know About the Number of Bidders?

- Classical first-price auction: n bidders

$$b^e(v_i,n) = \frac{n-1}{n} v_i$$

$$b^e(v_i,n) = v_i - F(v_i)^{-(n-1)} \int_0^{v_i} F(u)^{n-1} du$$

(valuations drawn from PDF f, CDF F)

- First-price auctions with number of participants drawn from PDF p [McAfee & McMillan; 1987], [Harstad, Kagel, Levin; 1990]

$$b^e(v_i,p) = \sum_j \frac{F^{j-1}(v_i)p_j}{\sum_k F^{k-1}(v_i)p_k} b^e(v_i,j)$$

- First-price auctions with number of participants drawn from PDF p; participation revelation
  1. Bidders declare intention to bid
  2. Auctioneer announces number of declarations
  3. Declared bidders submit bids

$$b^e(v_i,n) = v_i - F(v_i)^{-(n-1)} \int_0^{v_i} F(u)^{n-1} du$$

Bidding Ring Economic Environment

- Determine number of agents and agent-ring center relationships so that distributions over numbers of bidders in different rings are independent:
  - draw number of ring centers from γ_c (support on $\{2, \ldots\}$)
  - for each ring, draw number of invited agents from γ_a (support on $\{1, \ldots\}$)
    - if only one agent invited, keep the agent but dissolve the bidding ring

$$p = \sum_{n=2}^{\infty} \gamma_C(n) \left(\bigotimes_n \gamma_A \right)$$

where:

$$(d * d')(m) \equiv \sum_{j=0}^{\infty} d(m-j)\, d'(j)$$

$$\bigotimes_n d \equiv \overbrace{d * d * d * \ldots * d}^{d \text{ repeated } n \text{ times}}$$

- Each agent's type is (v_i, s_i)
  - v_i drawn independently from continuously differentiable CDF F, support on $[0,1]$
  - s_i is the number of bidders in i's bidding ring

$$p_{s_i} = \sum_{n=2}^{\infty} \gamma_C(n) \left(\bigotimes_{n-1} \gamma_A \right) * \delta_{s_i}$$

$$p_{n,s_i} = \left(\bigotimes_{n-1} \gamma_A \right) * \delta_{s_i}$$

where:

$$\delta_m(j) \equiv \begin{cases} 1 \text{ if } j = m; \\ 0 \text{ otherwise.} \end{cases}$$

Symmetrizing Auctions

Denote the bid of agent i as $\mu_i \in \mathbb{R}^+ \cup \{\emptyset\}$;
Denote the set of bids from all agents as $\pi \in \Pi$;
Denote an auction's transfer function for agent i as $t_i : \Pi \to \mathbb{R}$.

Protocol for First-Price Auctions

1. Each agent i **sends a message** μ_i to the ring center

2. If at least one agent **declined to participate**:
   - the ring center registers in the main auction for every agent who accepted the invitation to the bidding ring
   - for bidder i, the ring center submits a bid of $b^e(\mu_i, p_{n\text{-}k+1,k})$

3. If all k agents **accepted the invitation**:
   - for the bidder with the highest reported valuation (bidder h), the ring center places a bid of $b^e(\mu_h, p_{n,1})$
   - all other bidders are dropped
   - the ring center pays c to *all* agents in the bidding ring

4. If **bidder h wins** in the main auction he must pay:
   - $b^e(\mu_h, p_{n,1})$ to the center
   - $b^e(\mu_h, p_{n,k})$ - $b^e(\mu_h, p_{n,1})$ to the ring center

Equilibrium Analysis

Theorem 1 *It is a* ***Bayes-Nash equilibrium*** *for all bidding ring members to choose to participate and to truthfully declare their valuations to their respective ring centers, and for all non-bidding ring members to participate in the main auction with a bid of* $b^e(v, p_{n,1})$.

Partition strategies:

- Non-ring agent strategies: (¬P|¬R), (P¬T|¬R), (PT|¬R)
- Ring agent strategies: (¬P|R), (P¬T|R), (PT|R)

Overview of proof:

- Assume that all agents but i play (PT|R) or (PT|¬R)
- Participation: (PT|R) > (¬P|R); (PT|¬R) > (¬P|¬R)
- Truth-telling: (PT|R) > (P¬T|R); (PT|¬R) > (P¬T|¬R)

Participation: (PT|R) > (¬P|R); (PT|¬R) > (¬P|¬R)

- Non-Ring: (PT|¬R) > (¬P|¬R) is very straightforward
  - no participation cost, nonzero probability of winning
- Ring: (PT|R) > (¬P|R) is much less straightforward
  - if agents decline the ring invitation, they still know the number of other agents in their bidding ring: useful asymmetric information!
    - but the protocol "retaliates" when an agent declines participation…
  - consider auction (★), a FPA with stochastic number of bidders distributed according to $\boldsymbol{p_{n,s_i}}$
    - i has the same expected utility in the equilibrium of (★) as following (PT|R), because in both cases the auction allocates the good to the high bidder, both have the same distribution of bidders, and both make i pay $\boldsymbol{b^e(v_i, p_{n,s_i})}$ if he wins
    - thus show that playing a best response after declining the ring invitation gives i lower expected utility than the equilibrium of (★)
  - if i declines participation, auctioneer announces $n + s_i - 1$ participants
    - the $s_i - 1$ bidders from i's ring will bid $\boldsymbol{b^e(v, p_{n,s_i})}$
    - the $n - 1$ other bidders/rings will bid $\boldsymbol{b^e(v, p_{n+s_i-1,1})}$
    - for $s_i \geq 2$, we can show $\boldsymbol{b^e(v, p_{n+s_i-1,1}) > b^e(v, p_{n,s_i})}$ ** surprisingly nontrivial, see paper
    - thus the $n - 1$ bidders bid more than in the equilibrium of (★)
      - this reduces i's chance of winning without increasing his utility when he does win, as compared to the equilibrium of (★)

Truth-Telling: (PT|R) > (P¬T|R); (PT|¬R) > (P¬T|¬R)

- One-stage mechanism M:
  - center announces n, number of participants
  - each bidder i submits a bid μ_i to the center
  - the bidder i with the highest bid gets the good, pays $b^e(v_i, p_{n,s_i})$
  - all bidders with $s_i \geq 2$ are paid c_{n,s_i}

- Redefine (PT|¬R) as truth-telling
  - now show that truth-telling is an equilibrium in M for all bidders

- Proof sketch:
  - M always allocates the good to the high bidder
  - i's payment in M is taken from an auction aligned with his signal s_i
    - the payment c_{n,s_i} has no strategic impact
  - therefore M is symmetrizing, and so truth-telling is an equilibrium

∎

Do Ring Centers Gain?

Theorem 2 *The ring center* ***gains on expectation*** *if it pays agents* $c_{n,k} = \frac{1}{k}(g_{n,k} - c'_{n,k})$ *with* $0 < c'_{n,k} \leq g_{n,k}$ *and*

$$g_{n,k} = k \int_0^\infty f(v_i) \sum_{j=2}^{\infty} p_{n,k}(j) F^{j-1}(v_i) \left(b^e(v_i, p_{n,k}) - b^e(v_i, p_{n,1})\right) dv_i,$$

and is ***budget-balanced on expectation*** *when* $c'_{n,k} = 0$.

Proof Sketch:

- when ring member wins, center gets $b^e(v_i, p_{n,k}) - b^e(v_i, p_{n,1})$
- for $k \geq 2$, $b^e(v_i, p_{n,k}) > b^e(v_i, p_{n,1})$ **application of same (non-trivial) trick as in Theorem 1
  - therefore expected gain $g_{n,k} > 0$
- center can divide all or part of this expected gain among the ring members to budget-balance or gain on expectation

Do Bidding Rings Help Agents?

Three ways of asking this question:

1. Could any agent gain by deviating from the protocol?
2. Would any agent be better off if his bidding ring did not exist?
3. Would any agent would be better off in an economic environment that did not include bidding rings at all?

Theorem 3 *An agent i has higher expected utility in a bidding ring of size k bidding as described in Theorem 1 than he does if the bidding ring does not exist and k additional agents (including i) participate directly in the main auction as singleton bidders, again bidding as described in Theorem 1, for $c_{n,k} \geq 0$.*

Proof Sketch:

- The auctioneer announces n bidders when the bidding ring exists, and announces $n - k + 1$ bidders when the bidding ring doesn't exist
- Both cases result in economically efficient allocation
- i's payment: $b^e(v_i, p_{n,k})$ if the ring exists, $b^e(v_i, p_{n+k-1,1})$ if the ring doesn't exist
- As argued in Theorem 1, $b^e(v_i, p_{n,k}) < b^e(v_i, p_{n+k-1,1})$

∎

Do Bidding Rings Help Agents?

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1. Could any agent gain by deviating from the protocol?
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Corollary 1 *In the equilibrium described in Theorem 1, singleton bidders and members of other bidding clubs have higher expected utility when $k \geq 2$ agents form a bidding club than when k additional agents participate directly in the main auction as singleton bidders.*

Proof Sketch:

- The auctioneer announces n bidders when the bidding ring exists, and announces $n - k + 1$ bidders when the bidding ring doesn't exist
- Both cases result in economically efficient allocation
- i's payment: $b^e(v_i, p_{n,1})$ if the ring exists, $b^e(v_i, p_{n+k\text{-}1,1})$ if the ring doesn't exist
- As argued in Theorem 1, $b^e(v_i, p_{n,k}) < b^e(v_i, p_{n+k\text{-}1,1})$
- As argued in Theorem 2, $b^e(v_i, p_{n,1}) < b^e(v_i, p_{n,k})$
- Thus $b^e(v_i, p_{n,1}) < b^e(v_i, p_{n+k\text{-}1,1})$

$\blacksquare$

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1. Could any agent gain by deviating from the protocol?
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Theorem 4 *(ex-post ring) For all $\tau_i \in \mathcal{T}$, for all $k \geq 2$, for all $n \geq 2$, for all $c_{n,k} > 0$, agent i obtains greater expected utility by:*

1. participating in a bidding ring of size k in E_{br} and following the equilibrium from Theorem 1; than by

2. participating in a first-price auction with participation revelation in E_s with number of bidders distributed according to $p_{n,k}$; or by

3. participating in a first-price auction with a stochastic number of participants in E_s with number of bidders distributed according to $p_{n,k}$.

When $c_{n,k} = 0$, agent i obtains the same expected utility in all three cases.

- Proven using algebraic manipulation of expressions for expected utility.

Do Bidding Rings Help Agents?

Three ways of asking this question:

1. Could any agent gain by deviating from the protocol?
2. Would any agent be better off if his bidding ring did not exist?
3. Would any agent would be better off in an economic environment that did not include bidding rings at all?

Corollary 2 *(ex-post singleton) For all $\tau_i \in \mathcal{T}$, for all $n \geq 2$, agent i obtains the* ***same expected utility*** *by:*

1. *participating as a* ***singleton bidder*** *in E_{br} and following the equilibrium from Theorem 1; as by*

2. *participating in a* ***first-price auction with participation revelation*** *in E_s with number of bidders distributed according to $p_{n,1}$; and by*

3. *participating in a* ***first-price auction with a stochastic number of participants*** *in E_s with number of bidders distributed according to $p_{n,k}$.*

Do Bidding Rings Help Agents?

Three ways of asking this question:

1. Could any agent gain by deviating from the protocol?
2. Would any agent be better off if his bidding ring did not exist?
3. Would any agent would be better off in an economic environment that did not include bidding rings at all?

Corollary 3 *(ex-ante) For all $n \geq 2$, as long as $\exists n$, $\exists k$, $\gamma_c(n) > 0$ and $\gamma_a(k) > 0$ and $c_{n,k} > 0$, agent i obtains greater expected utility by:*

1. *participating in E_{br} and following the equilibrium from Theorem 1; than by*
2. *participating in a first-price auction with participation revelation in E_s with number of bidders distributed according to p; or by*
3. *participating in a first-price auction with a stochastic number of bidders in E_s with number of bidders distributed according to p.*

When $\forall n$, $\forall k$, $c_{n,k} = 0$, agent i obtains the same expected utility in both cases.

Another Equilibrium

Proposition 4 *It is a Bayes-Nash equilibrium for each bidding ring invitee to* ***decline his bidding ring invitation****, and for each agent i to bid $b^e(v_i, n)$.*

Sketch of Proof:

- if one agent declines, others are no worse off declining too
- no agents accept, so signals contain no useful information
- thus agents bid as in the standard equilibrium of auctions with stochastic number of bidders; participation revelation

Corollary 4 *When $c_{n,k} > 0$,* ***all bidders prefer*** *the equilibrium from Theorem 1 to the equilibrium from Proposition 4 ex-ante; ex post* ***bidding ring invitees prefer*** *the equilibrium from Theorem 1 to the equilibrium from Proposition 4, while* ***singleton bidders are indifferent*** *between the equilibria. When $c_{n,k} = 0$,* ***all bidders are indifferent*** *between the equilibria both ex ante and ex post.*

- follows immediately from Proposition 4, Theorem 4, Corollaries 2, 3
- trivially, ring centers also prefer the equilibrium from Theorem 1
- thus, auctioneers prefer the equilibrium from Proposition 4

Conclusions

Bidding Ring Checklist:

- ☐ agents gain from collusion
- ☐ colluding agents will not deviate from the protocol
- ☐ ring center doesn't lose money on expectation
- ☐ non-colluding agents play best responses to the colluders
- ☐ more than one bidding ring may exist
- ☐ agents are free to opt out of the bidding ring

Our Protocol for First-Price:

- ☑ yes (compared to (i) a world without one ring; (ii) a world without any rings)
- ☑ colluding agents always play best responses
- ☑ ring center is budget-balanced or profitable on expectation, depending on c'
- ☑ yes
- ☑ yes
- ☑ yes (Theorem 1 shows that agents lose by opting out)

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