Allocation for Social Good

Auditing Mechanisms for Utility Maximization

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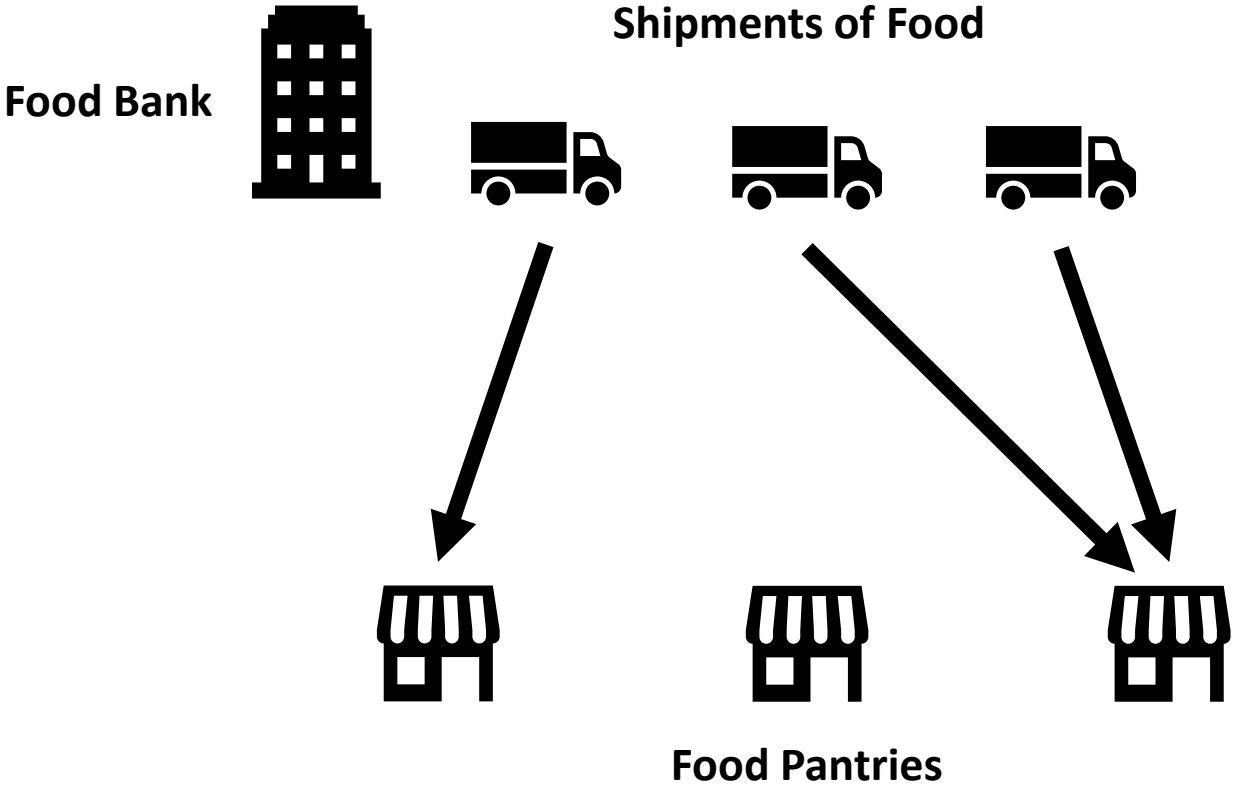
Food Banks and Food Pantries



Difficulties in this setting

- Private information
- Self-interest
- No monetary transfers
  - Can interfere with operating costs
  - More demand does not equal more money

Tools

Auditing

- Non-profits are often obligated to observe how their resources are being utilized
- We can use this information to help maintain accountability

Repeated Interactions

- Enforce incentives by withholding future allocations
- Reduce no money problem to a utility maximization problem (i.e. allocation minus payments)

Outline

- We show how auditing can be used to improve social utility.
  - Auditing can decrease the payments of existing auctions
  - Auditing can give rise to new optimal utility maximizing auctions
- We show how to reduce any repeated allocation problem without money to a single round social utility maximization problem.

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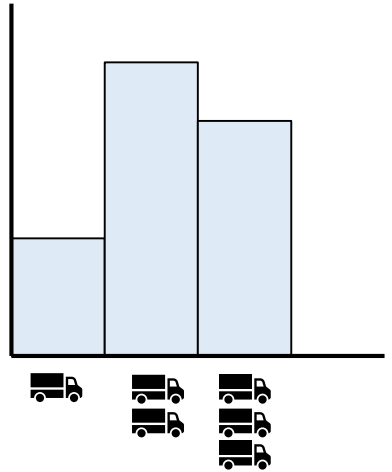
Preliminaries

$t_1 \sim G_1$ $t_2 \sim G_2$ . . . $t_N \sim G_N$

$t_i =$ 

$d_i \sim t_i$

Auditing Mechanism

1. Private types $t_i \sim G_i$ are realized.
2. Each agent reports a type $\hat{t}_i$ to the center.
3. The center makes an allocation $\boldsymbol{x}(\hat{\boldsymbol{t}}) = \big(x_1(\boldsymbol{t}), \ldots, x_N(\boldsymbol{t})\big)$.
4. Each agent i's demand $d_i \sim t_i$ is realized.
5. The center audits the agents and observes a level of consumption $d_{\text{obs}} := \min(d_i, x_i)$ for each agent.
6. The center charges a payment $p_i(\hat{\boldsymbol{t}}, d_{\text{obs}})$.

Food Pantry Utility

- **Value**: $\min(d_i, x_i)$
- **Payment**: $p_i(\hat{\boldsymbol{t}}, \min(d_i, x_i))$

Quasilinear utility

$$Utility = Value \ - payment$$

Interim: $u_i(\hat{t}_i, t_i)$

An auditing mechanism $\mathcal{M}$ is *Bayesian-Nash incentive compatible* (BIC) if it makes honest reporting a Bayesian Nash equilibrium, i.e. if under $\mathcal{M}$ we have $u_i(t_i, t_i) \geq u_i(\hat{t}_i, t_i)$ for all t_i.

Social Utility Objective

$$\max \sum_i u_i(\hat{t}_i, t_i)$$

$$s.t.$$

$$\forall\, t_i, \hat{t}_i \quad u_i(t_i, t_i) \geq u_i(\hat{t}_i, t_i)$$

$$\forall\, i, \boldsymbol{t}, d_{obs} \quad p_i(\boldsymbol{t}, d_{\text{obs}}) \geq 0$$

Maximize value minus payments

BIC constraints

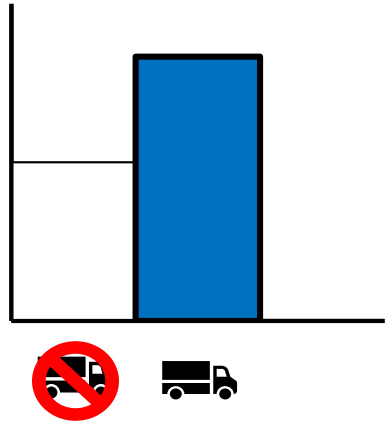
No negative payments

Difficult to solve for the general case

Unit Demand Setting

- Each agent either gets allocated one shipment or nothing.
- Usually unit demand is a simple setting to optimize using classical auction theory.
- Problem: two payment terms, one for each observed outcome.
- We show you only need to charge when the item goes unused when maximizing utility.

$t_i =$



Waste-not-Pay-not Mechanisms

Single Parameter with Auditing

Myerson's Lemma with Auditing

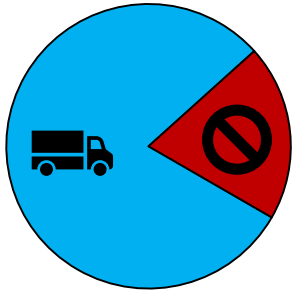
Every **waste-not-pay-not** *mechanism satisfies* **BIC** *constraints if and only if for each agent i, the following two conditions hold:*

1. *The interim allocation rule x_i is monotone non-decreasing.*
2. *The expected payment for reporting $\hat{t}_i$* ***when the observed demand is 0 is***

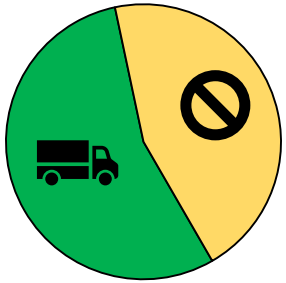
$$p_i(\hat{t}_i, d_{\text{obs}} = 0) = \frac{\hat{t}_i \cdot x_i(\hat{t}_i)}{(1 - \hat{t}_i)} - \int_0^{\hat{t}_i} \frac{x_i(v)}{(1-v)^2} dv$$

Audited Second Price Auction

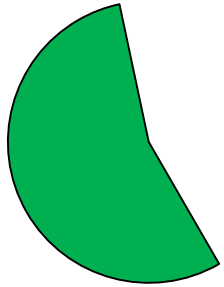
Agent 1



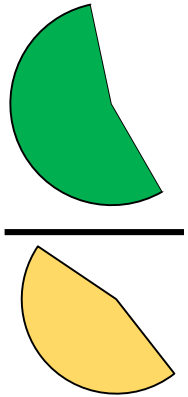
Agent 2



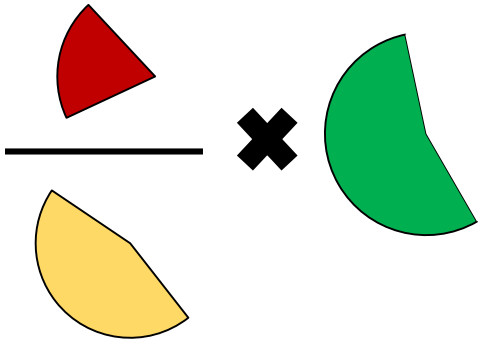
SPA Price



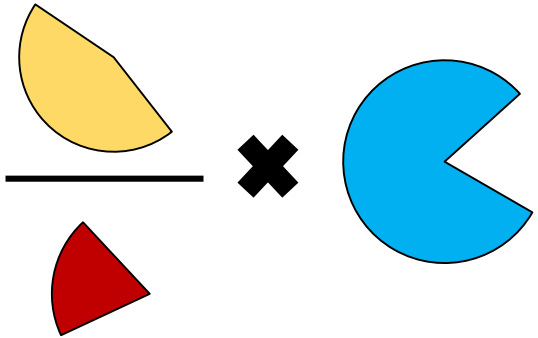
Audited SPA Payment



Audited SPA Expected Payment



If Agent 2 Deviated



Audited Second Price Auction

Payments are SPA payments scaled by:

$$\frac{1 - type}{1 - price} \leq 1 \quad \text{When type} \geq \text{price}$$

$$\frac{1 - type}{1 - price} \geq 1 \quad \text{When type} \leq \text{price}$$

Ex: Uniform Distribution

$\mathbf{E}[Winning\ type] = \frac{2}{3}$ and $\mathbf{E}[Price] = \frac{1}{3}$

Auditing cuts the expected payment in half

Auditing payments

- By changing payments we can increase the utility of the optimal social utility mechanism whenever it charges a payment.

- Does auditing have a different optimal social utility allocation rule?

- We can derive new optimal social utility allocation rules which can give larger gains than just altering the payment.

Beyond Unit Demand

- Optimal social utility mechanism is not characterized
- VCG can also be improved with auditing
- Optimal auditing payments depend on the typespace

Roadmap of auditing and debt mechanisms

- We show how auditing can be used to improve social utility.
  - Auditing can decrease the payments of existing auctions
  - Auditing can give rise to new optimal utility maximizing auctions

- We show how to reduce any repeated allocation problem without money to a social utility maximization problem.

Dynamic Mechanism Basics

- At each round k each agent realizes a new type from their prior distribution $t^k \sim G$
- Each agent decides which type to report to the mechanism using a strategy that depends on not only their current type but the history of their interactions.

Dynamic Mechanism Basics cont.

- An agent’s optimal strategy must take future interactions into account
- We assume an infinite time horizon without discounting
- We choose overtaking as our optimality criterion since it gives us resolution over finite deviations in strategy.

Debt Mechanisms

- Described by three components a static mechanism $\mathcal{M}$ and two constants: the allocation length l and the debt rate r

- Each round can be one of two types:

Allocation Rounds:
- Is allocated based on allocation rule x
- Payment p is added to an agent's debt
- Occur in consecutive batches of size l

Punishment Rounds:
- Agent is allocated nothing
- Debt is reduced by debt rate r
- When debt is 0 returns to allocation rounds

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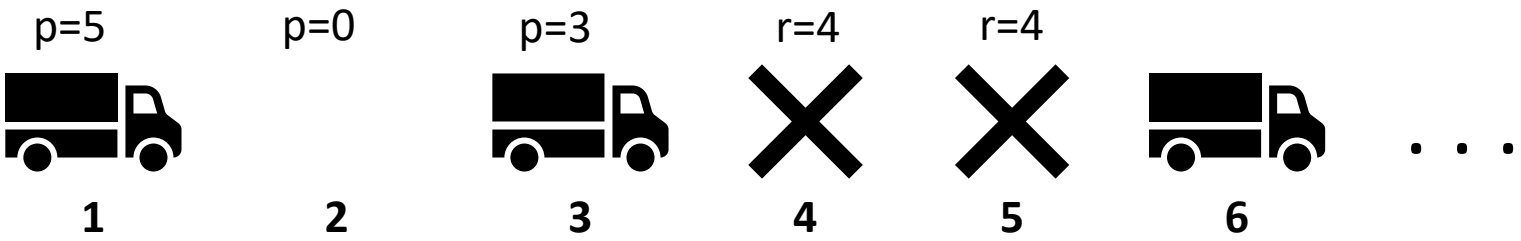
Ex:

Debt rate: r = 4
Allocation length =3

Debt = 0



Reduction to Utility Maximization

Given a debt mechanism $\mathcal{M}_D = (\mathcal{M}, r, l)$ if:

- Single round mechansim $\mathcal{M}$ satisfies BIC constraints
- $r = \mathbf{E}_t[t \cdot x(t) - p(t)]$

Average welfare $\mathcal{M}_D$ = Expected Utility $\mathcal{M}$

Related Work

Utility Maximization

Ruggiero Cavallo. Optimal decision-making with minimal waste: Strategyproof redistribution of vcg payments.

Jason D. Hartline and Tim Roughgarden. Optimal mechanism design and money burning.

Repeated allocation without money

Artur Gorokh, Siddhartha Banerjee, and Krishnamurthy Iyer. From monetary to non-monetary mechanism design via artificial currencies.

Mingyu Guo, Vincent Conitzer, and Daniel M. Reeves. Competitive repeated allocation with-out payments.

Santiago Balseiro, Huseyin Gurkan, and Peng Sun. Multi-agent mechanism design without money.

Auditing

Hongyao Ma, Reshef Meir, David C. Parkes, and James Zou. Contingent payment mechanisms to maximize resource utilization.

Robert G. Hansen. Auctions with contingent payments.

Summary

- Shown how to leverage auditing and repeated interactions to design efficient solutions to the food bank and food pantry problem.
- Payments can be lowered by using auditing
- Auditing can give rise to new optimal utility maximizing auctions
- Debt Mechanisms can reduce any repeated welfare maximization problem without money to a static utility maximization problem

Thanks!