Pragmatic Algorithmic Game Theory

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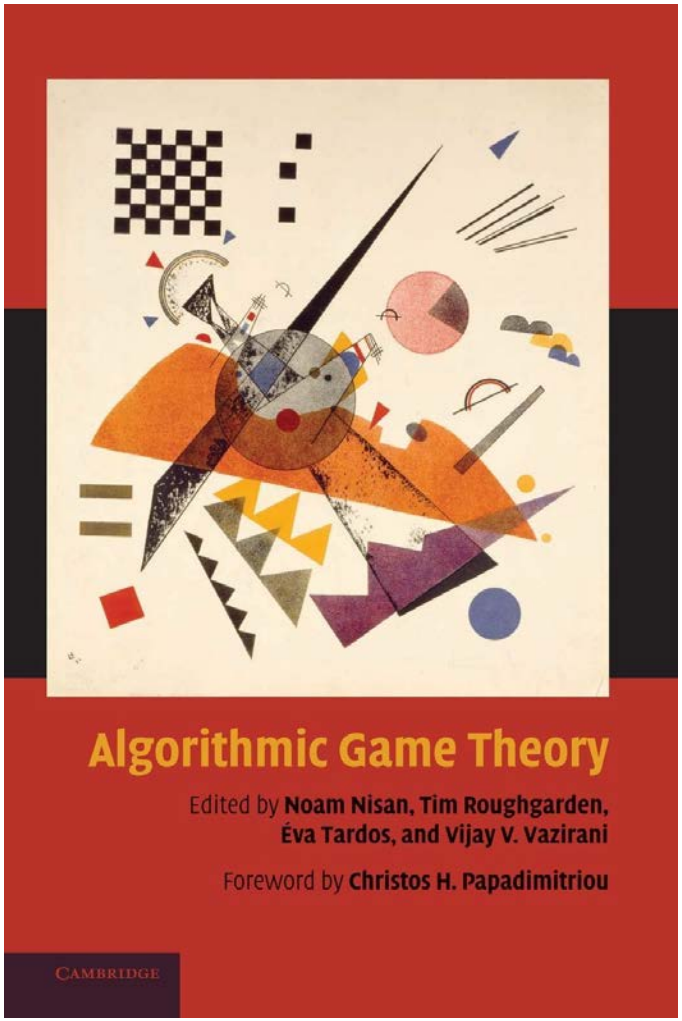
Algorithmic Game Theory

- "Research at the interface of CS, game theory, and economic theory, largely motivated by the Internet."
- Particular topics of concern
  - Design of new mechanisms, esp. auctions and market algorithms
  - Analysis of existing mechanisms
  - Equilibrium computation
- Typically, **very general settings** attacked using **theoretical tools**
- This approach has yielded impressive
  - impossibility
  - optimality
  - approximation results.
- However, sometimes it is very difficult to obtain clean theoretical results that address **complex, realistic problems**



Pragmatic AGT

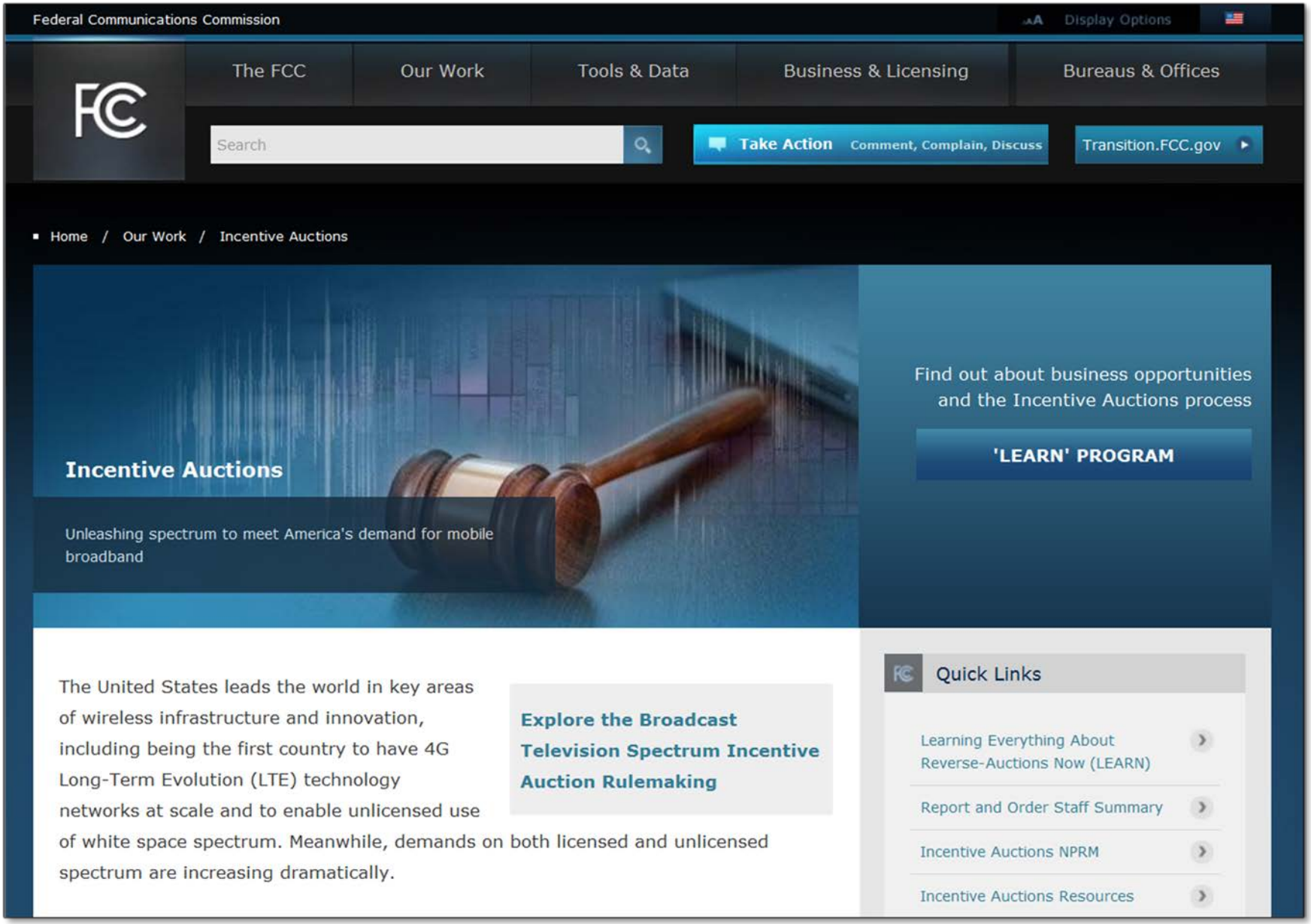
- More traction on realistic problems by leveraging one or both of the following forms of pragmatism:
  1. Aiming to achieve good performance **only on problems of interest**, rather than in relatively unconstrained settings
  2. Adopting **statistical rather than analytical methods**, thereby defining problems of interest implicitly via a dataset and/or appealing to data-driven measures of performance.
- I'll describe work that attacks **"core AGT" problems** but is pragmatic in both senses
  - I'm far from the only one working in this vein
  - Indeed, EC is a great place for interaction between those who adopt "pure" and "pragmatic" approaches

Repurposing Radio Spectrum via an "Incentive Auction"

[Frechette & LB, unpublished]

Also draws on [Hutter, Hoos & LB, 2011]; [Xu, Hutter, Hoos & LB, 2009]

FCC's "Incentive Auction"



Approved and Ready to Go

Before the
Federal Communications Commission
Washington, D.C. 20554

In the Matter of	)	
	)	
Expanding the Economic and Innovation	)	GN Docket No. 12-268
Opportunities of Spectrum Through Incentive	)	
Auctions	)	

REPORT AND ORDER

Adopted: May 15, 2014 **Released: June 2, 2014**

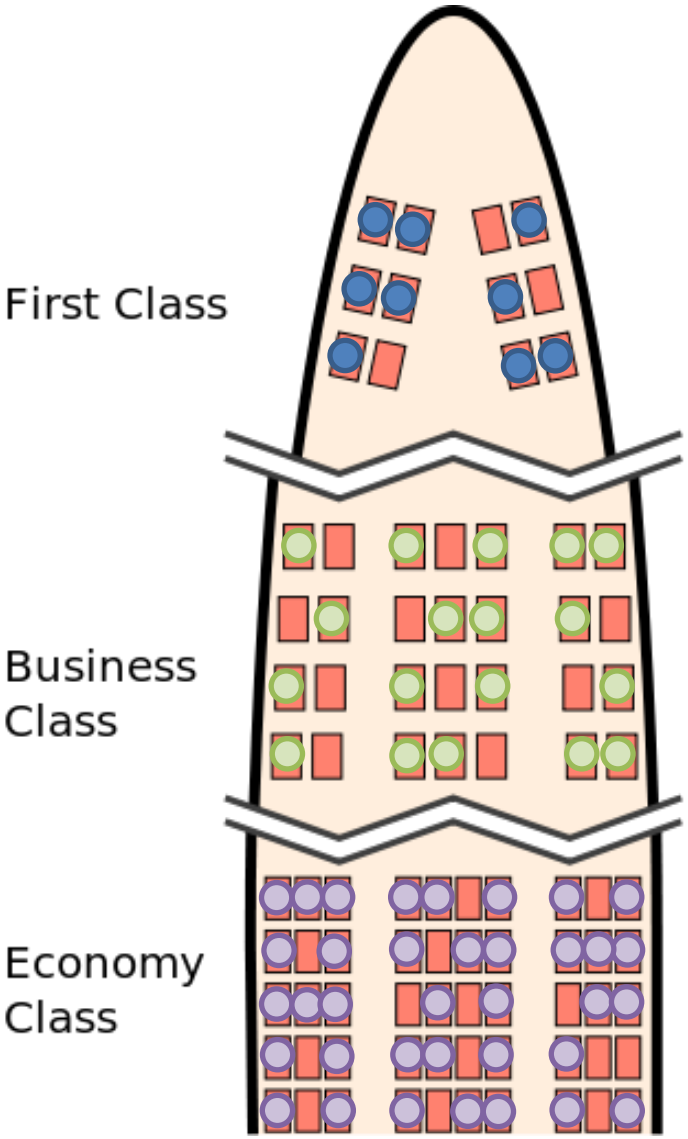
By the Commission: Chairman Wheeler and Commissioners Clyburn and Rosenworcel issuing separate statements; Commissioners Pai and O'Rielly dissenting and issuing separate statements.

TABLE OF CONTENTS

The FCC's "Incentive Auction"

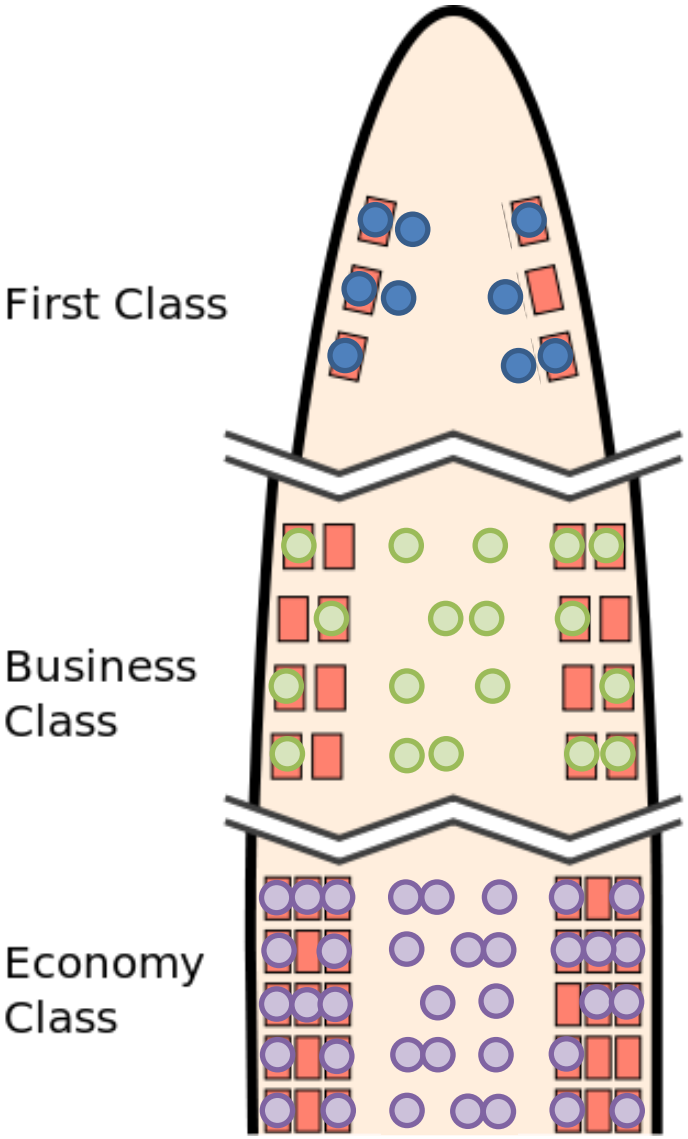
- **Reverse (descending-price) auction** for broadcasters
  - stations declare they're willing to stop broadcasting at a given, initially high, price
  - price descends as long as stations can feasibly be "repacked" into the reduced band, given interference constraints
- **Forward (ascending-price) auction** for telecom firms
  - prices in each region increase while demand exceeds supply
- **Auctions linked** to ensure revenue target is met
  - if not, clearing target reduced and the auctions continue

How Does the Reverse Auction Work?



- Let's consider the example of **airline overbooking**, where passengers either fly in their assigned cabin or are compensated to give up their seat
- Thus, the **feasibility constraint** is (# passengers in cabin) $\leq$ (# seats)
- We'll use a **descending clock** auction to set compensations
- Let's start with a plane big enough to hold everyone…

Reverse Auction: Descending Clock



The airline substitutes a smaller plane and offers compensation

$1,000

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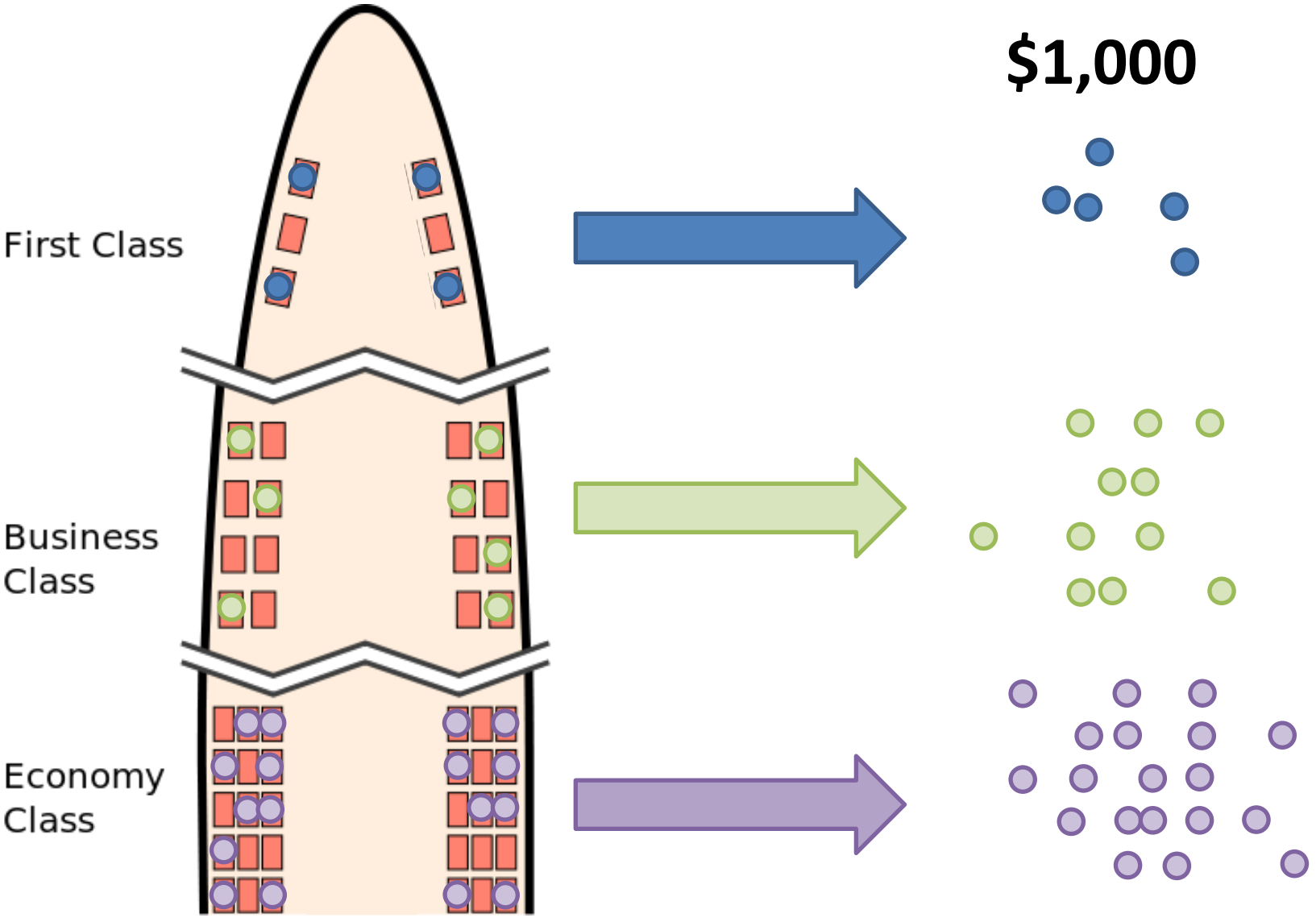

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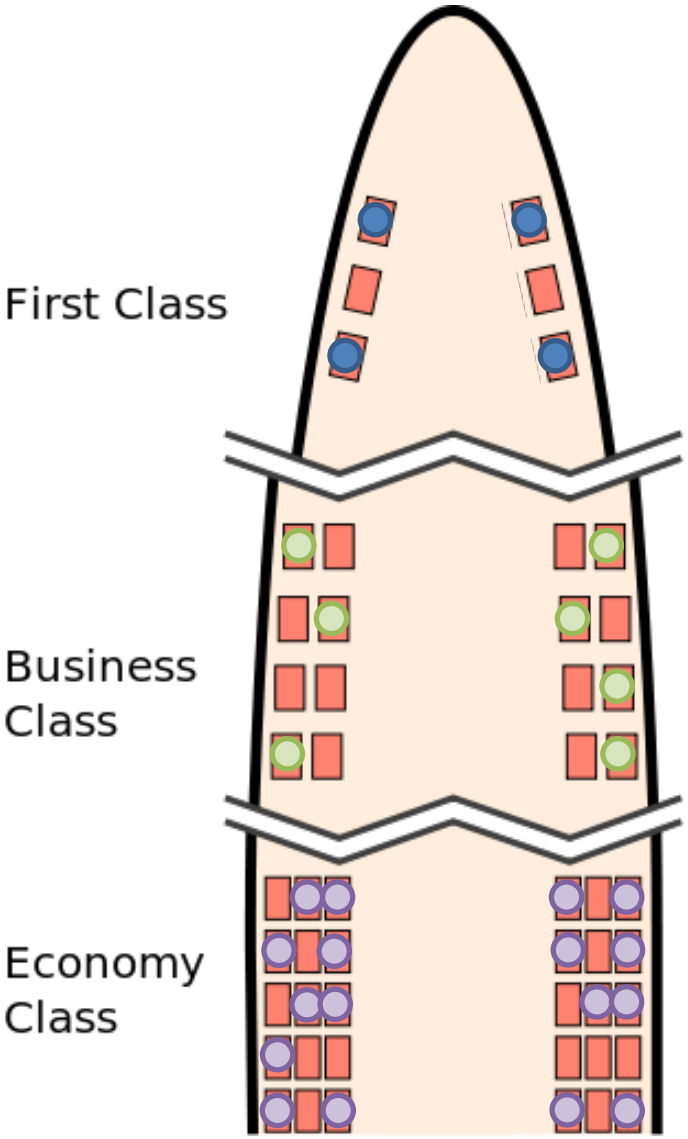

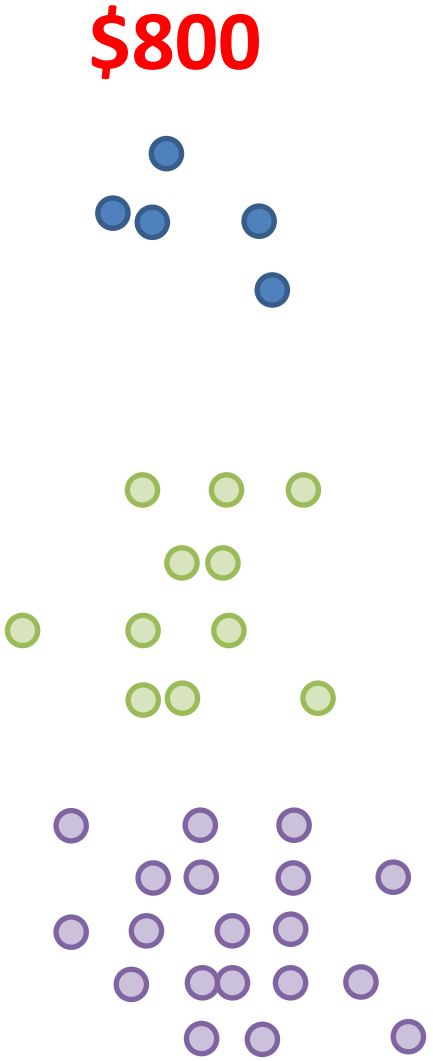

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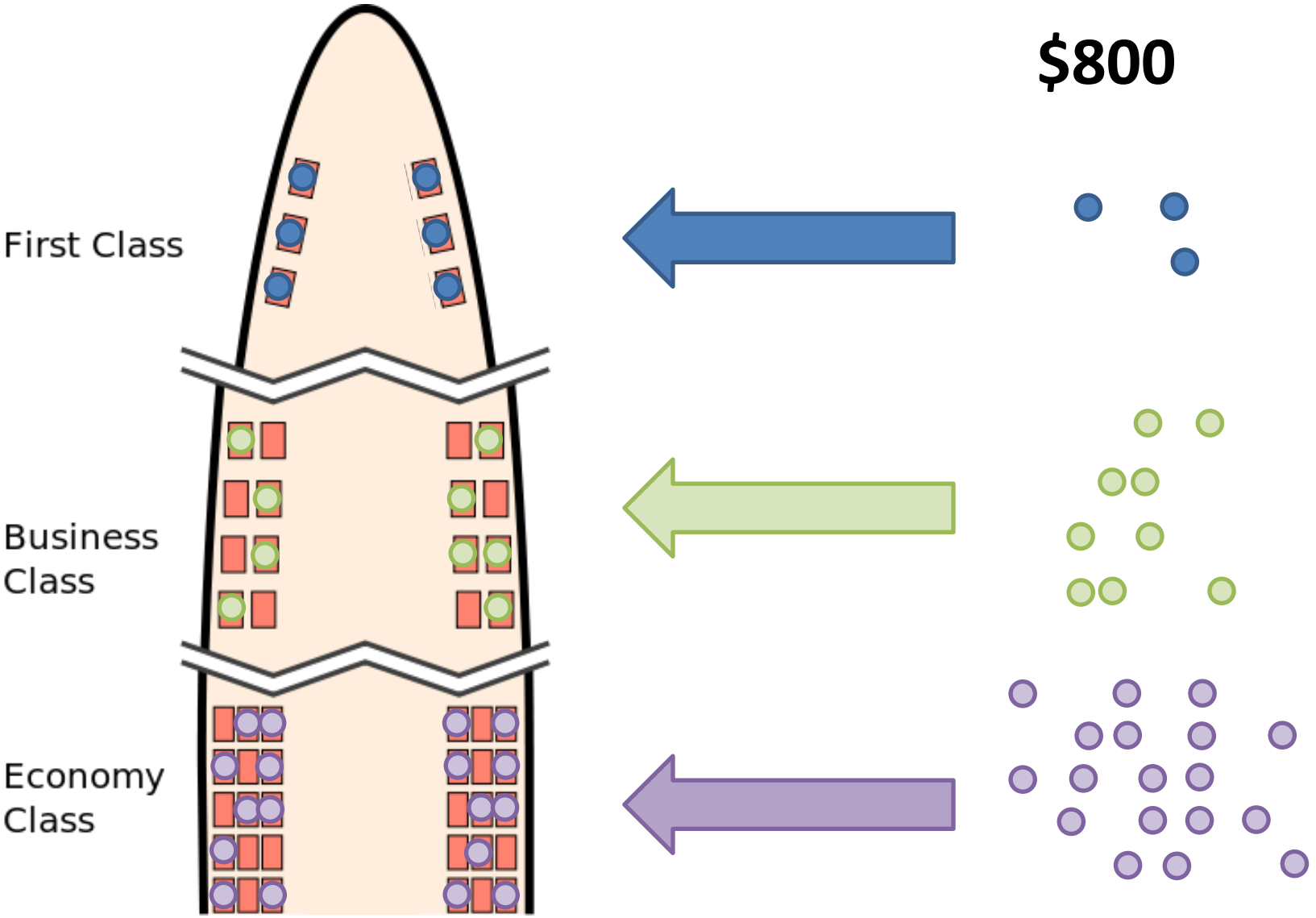

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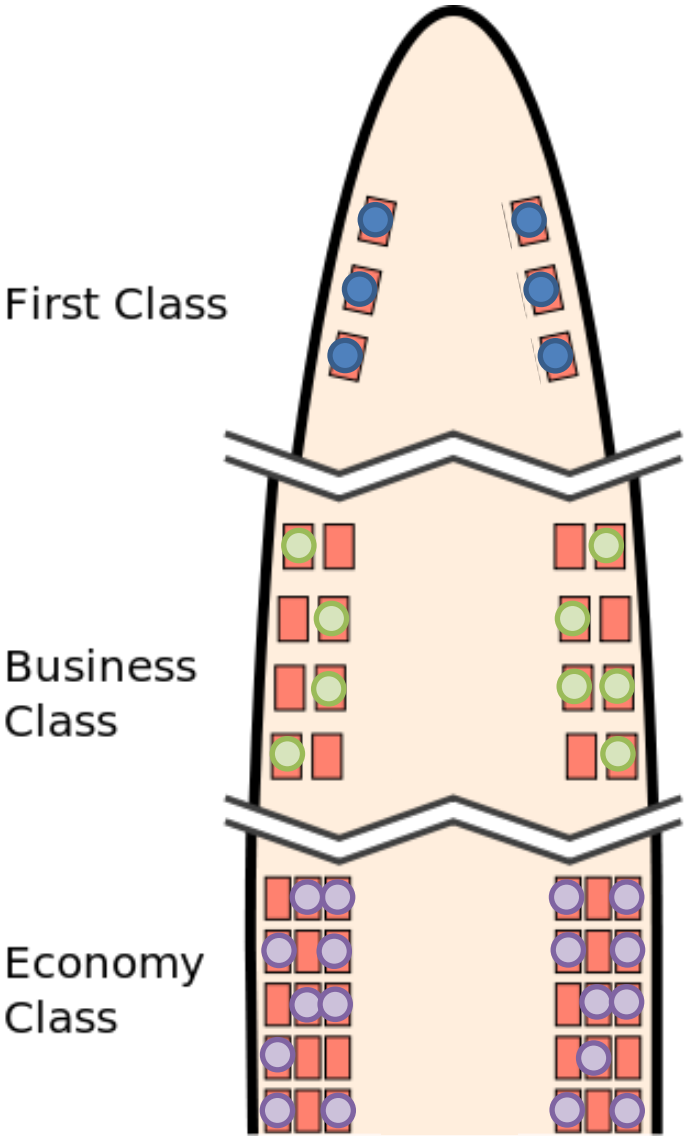

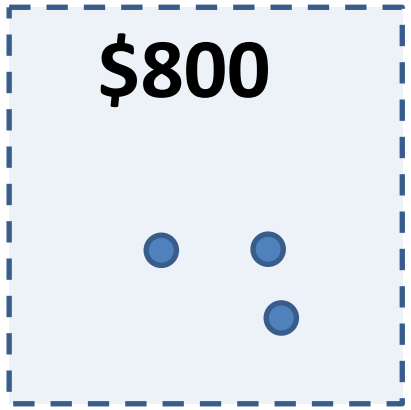

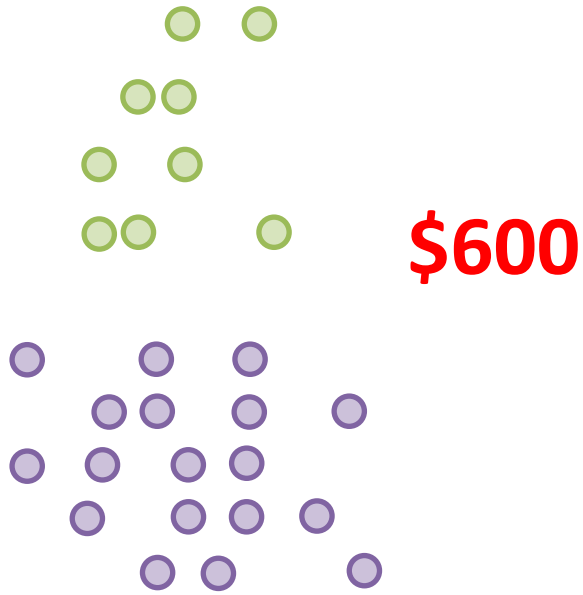

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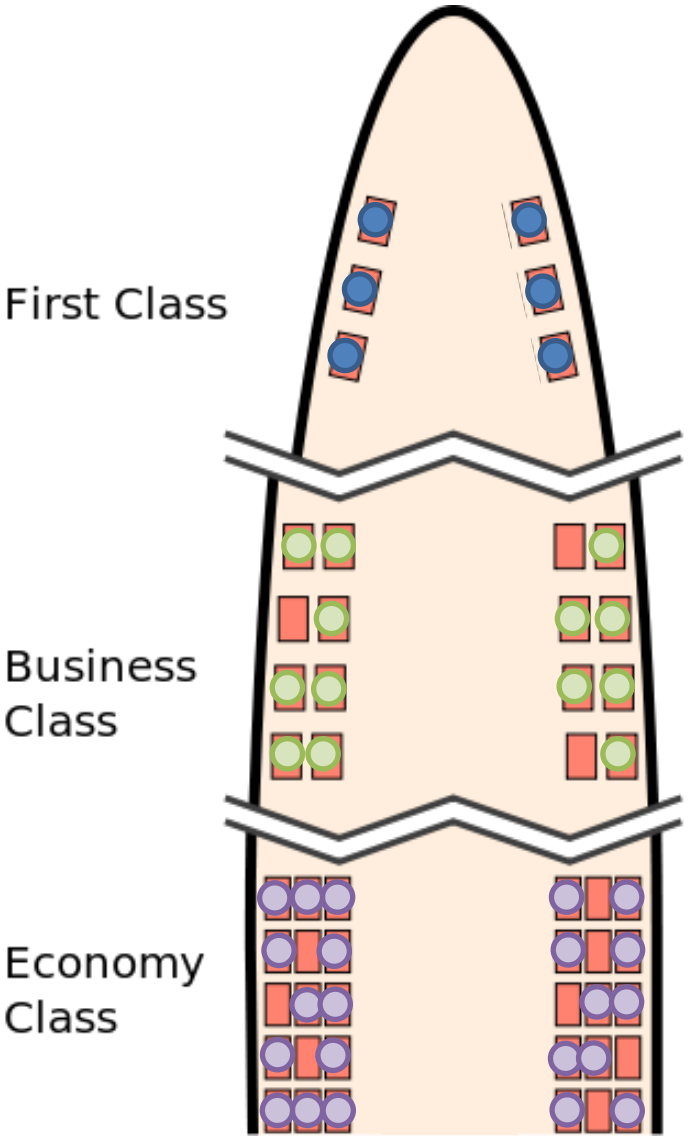

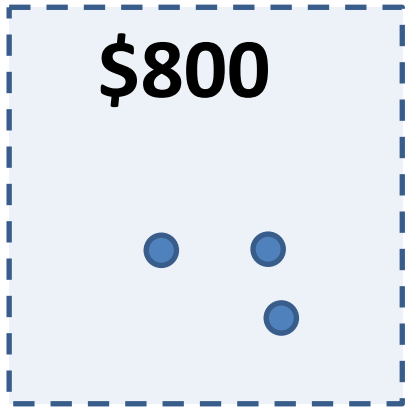

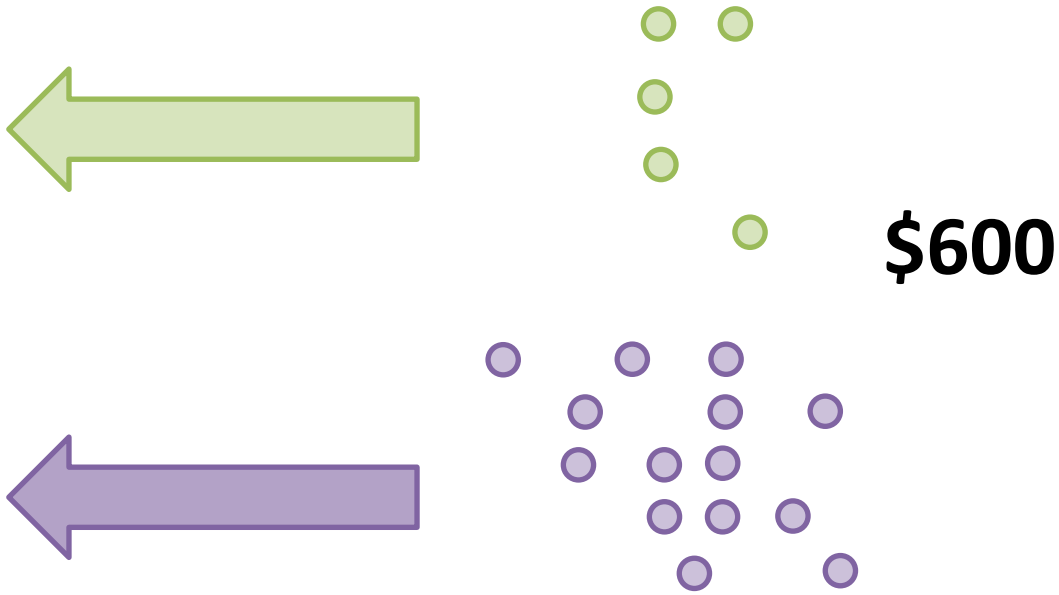

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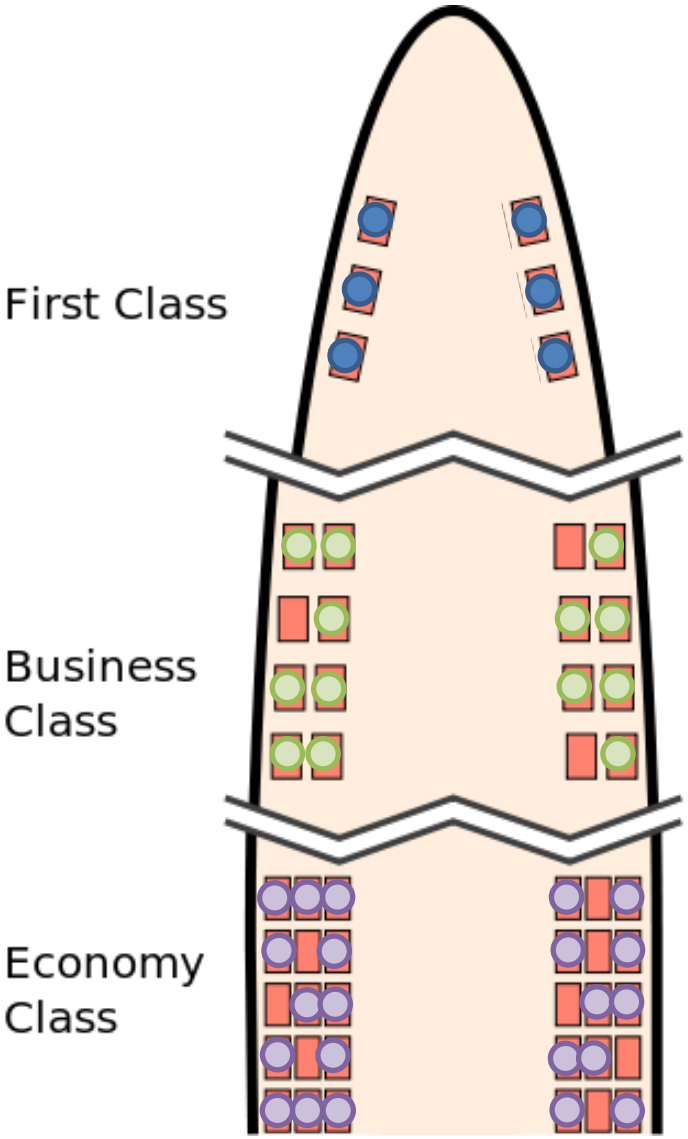

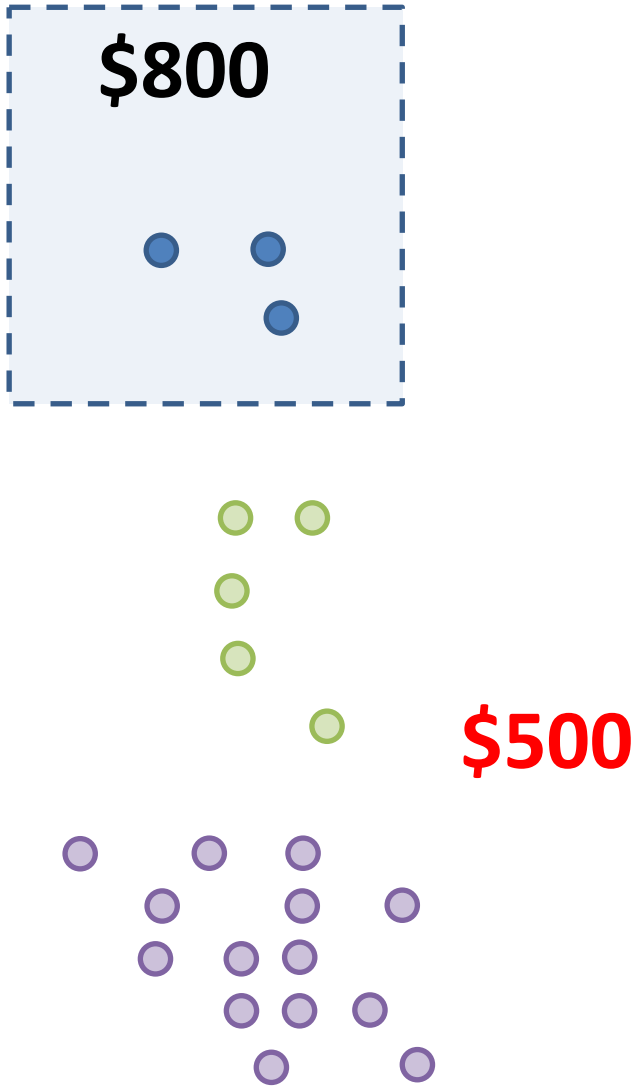

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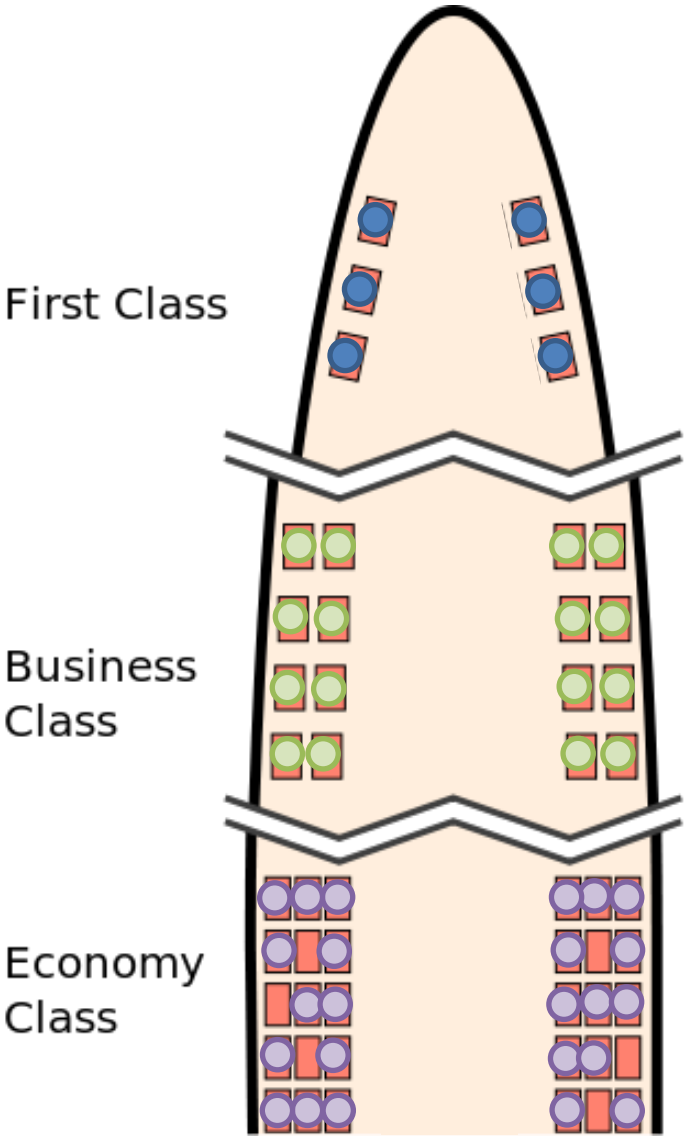

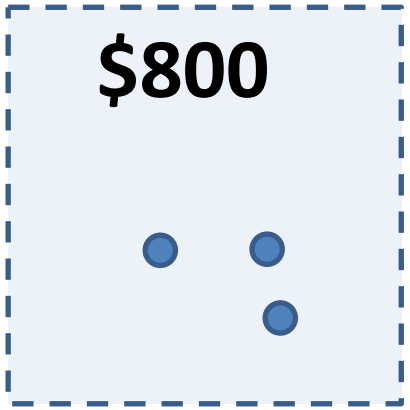

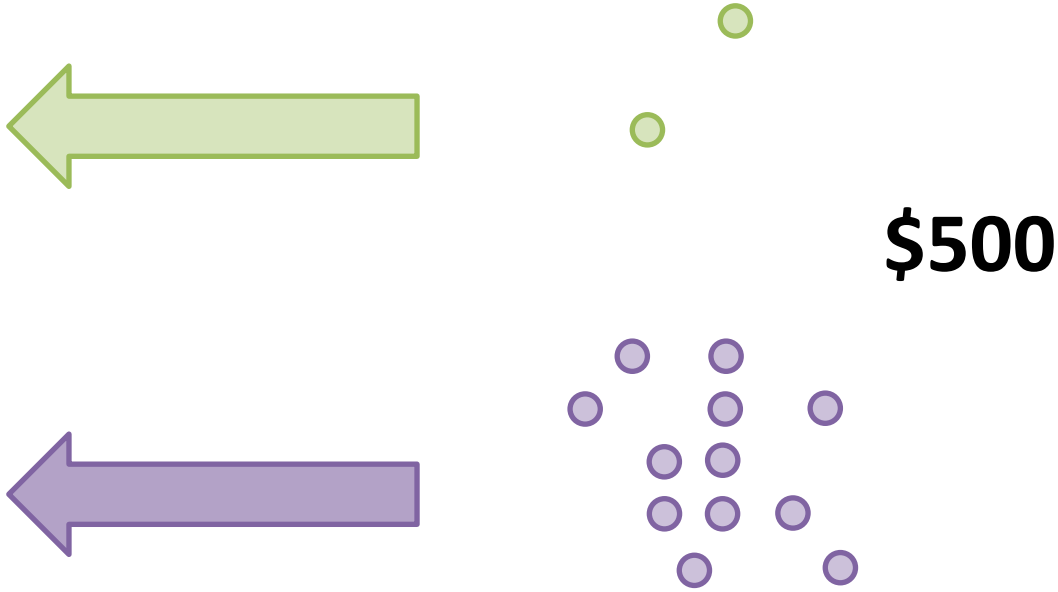

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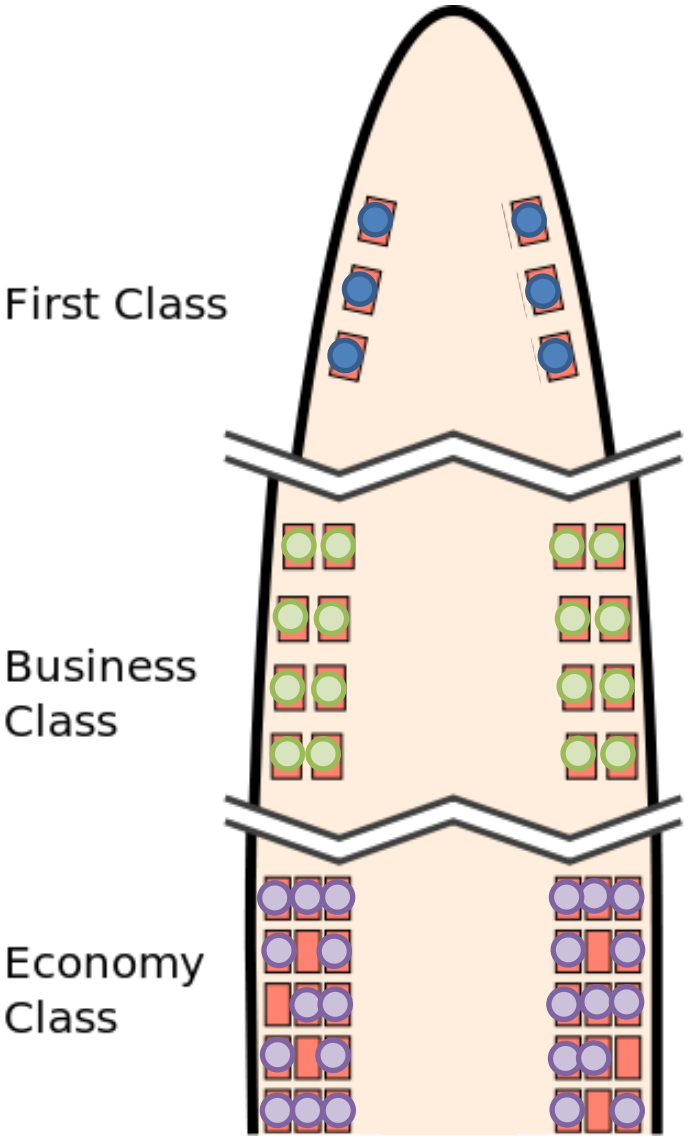

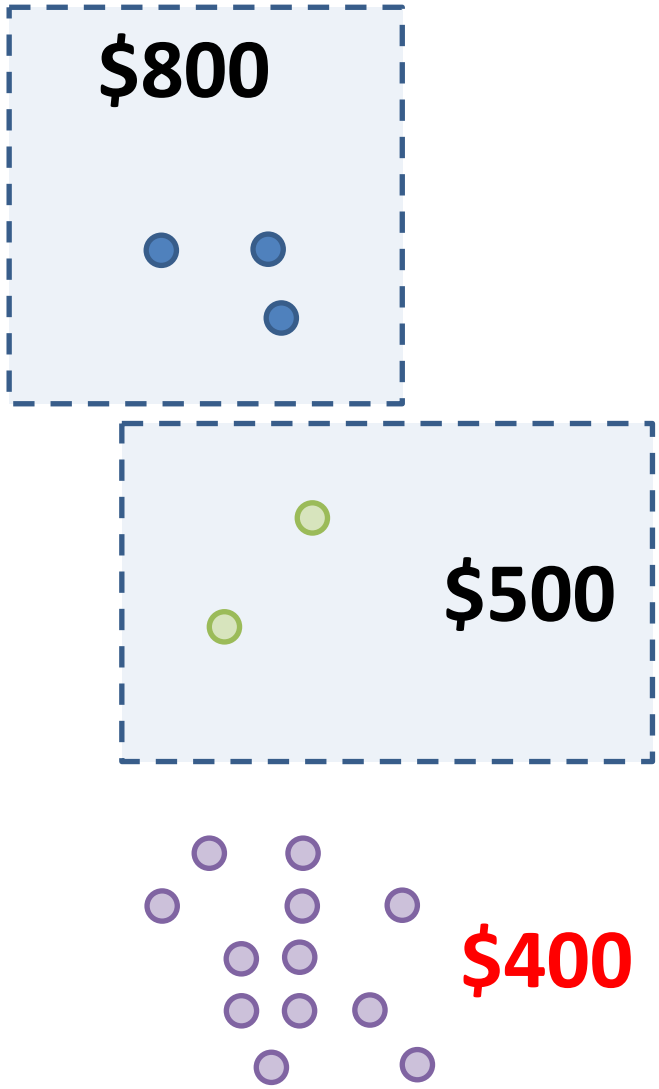

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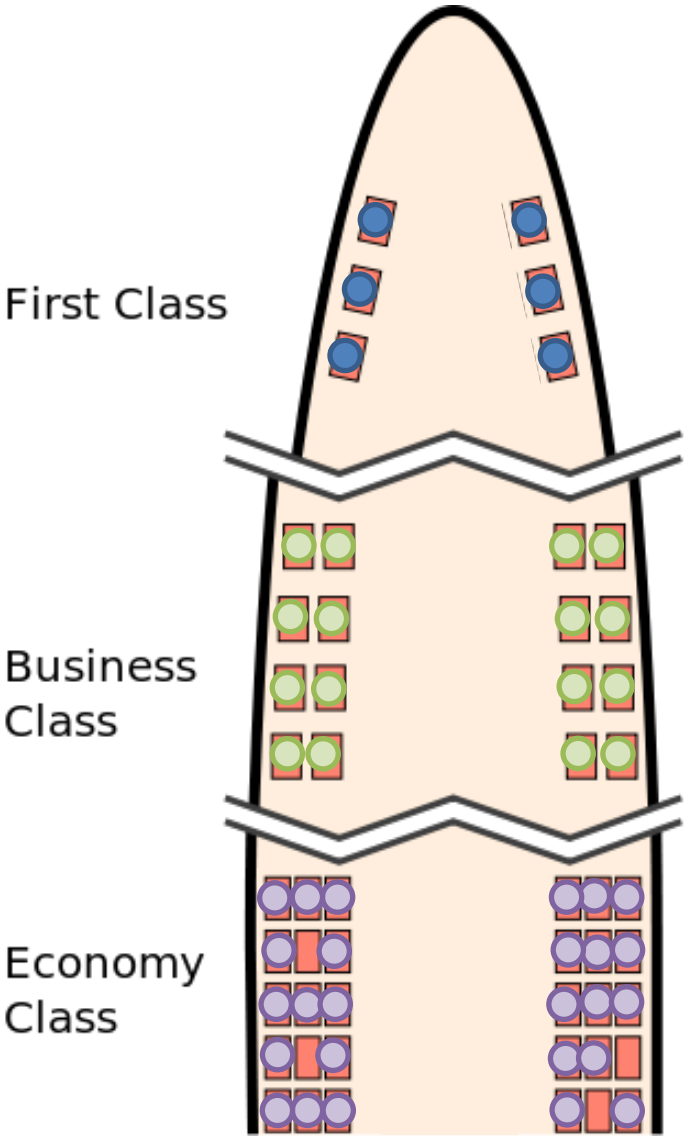

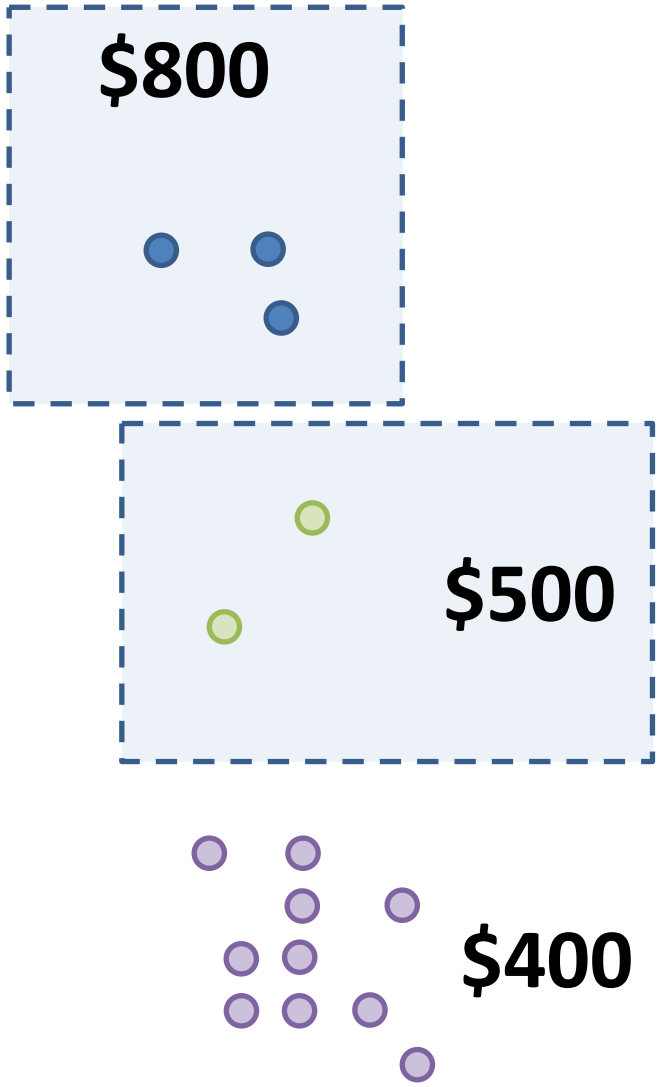

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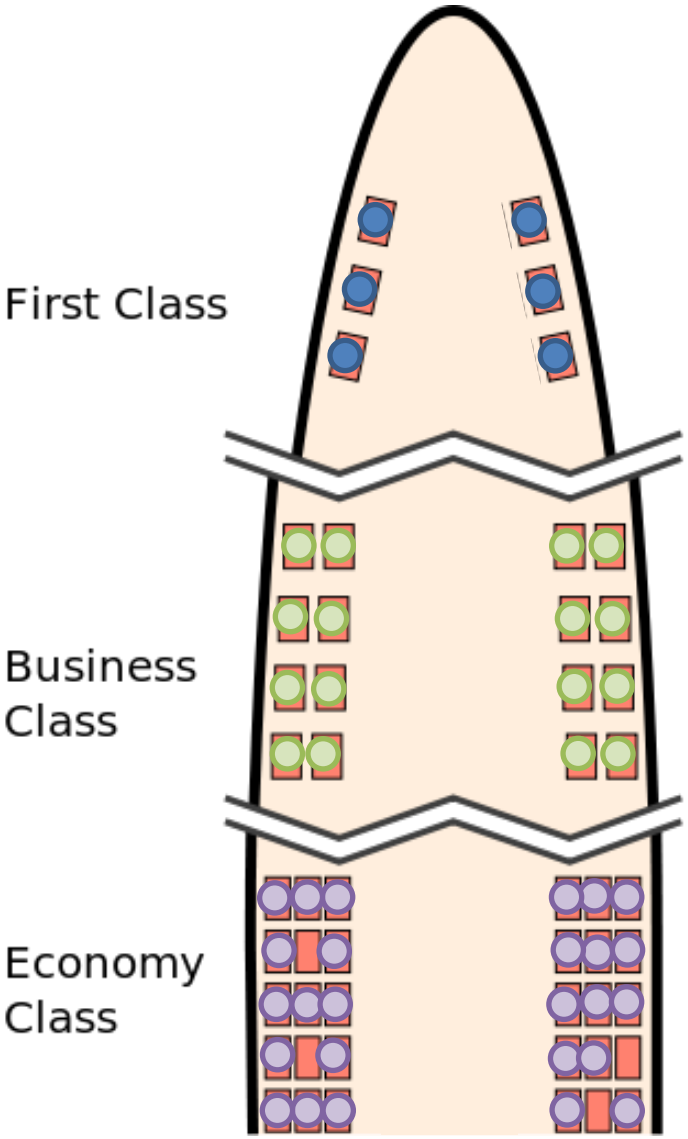

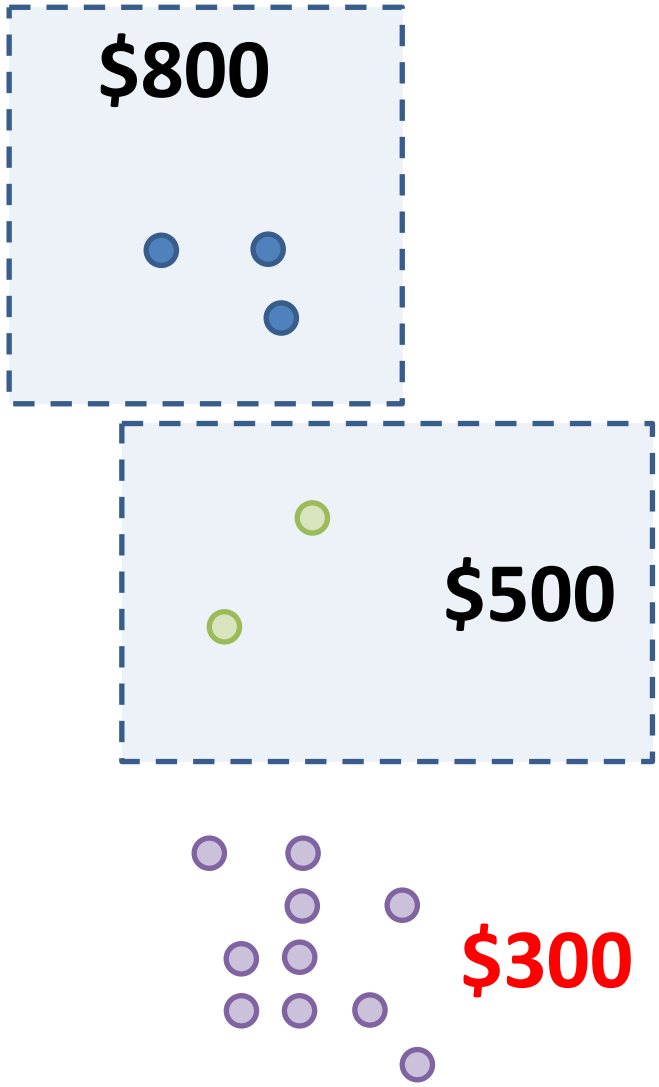

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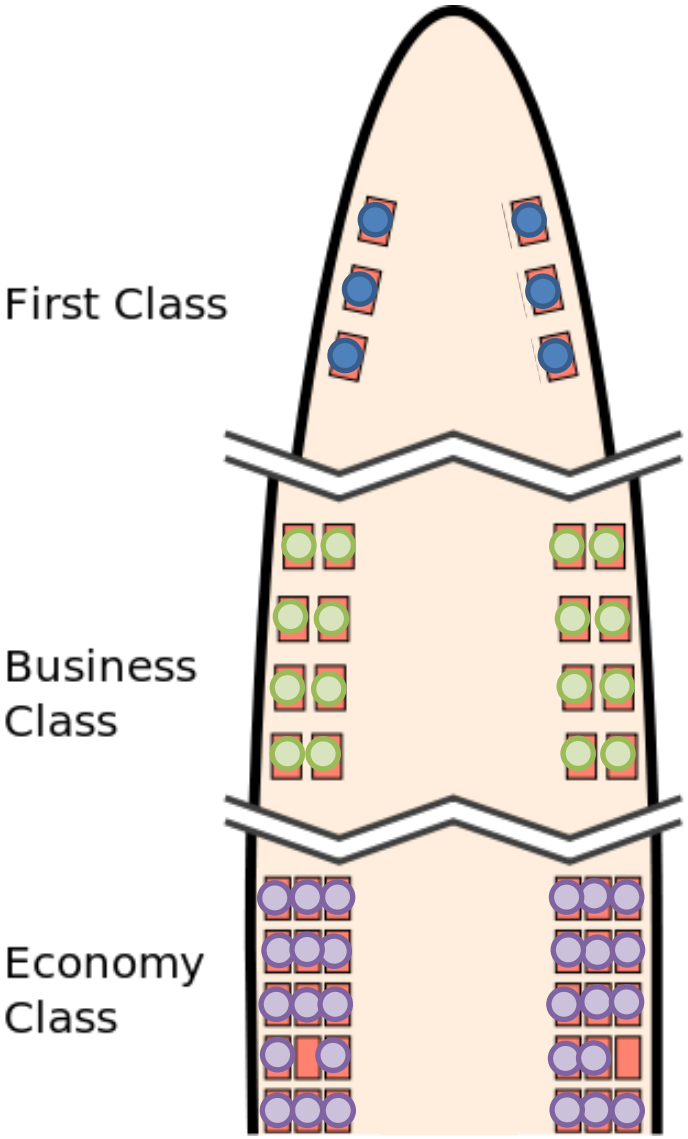

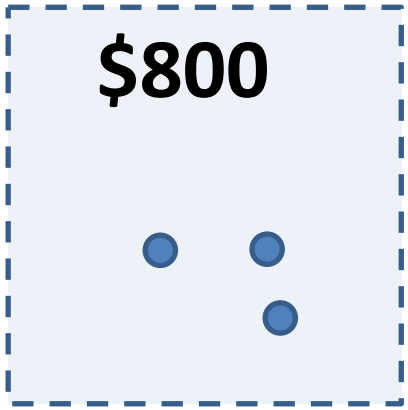

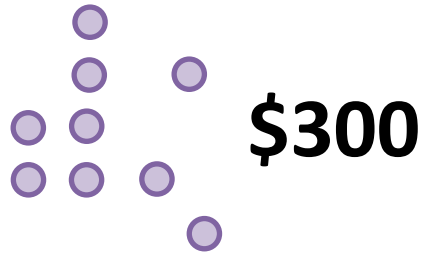

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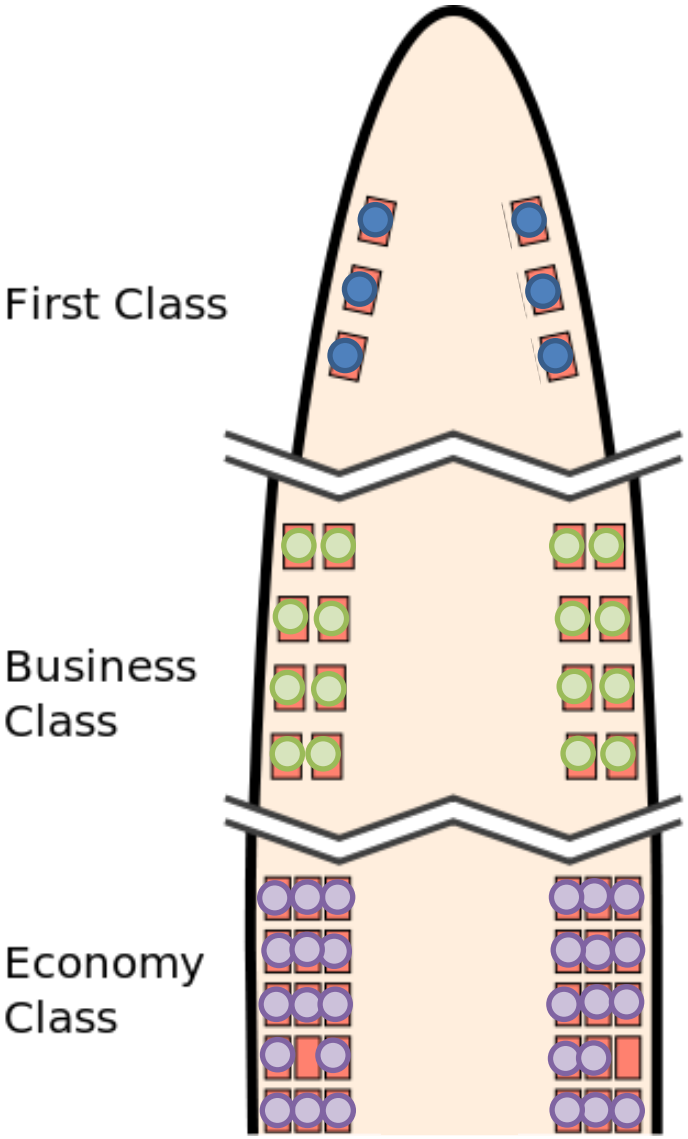

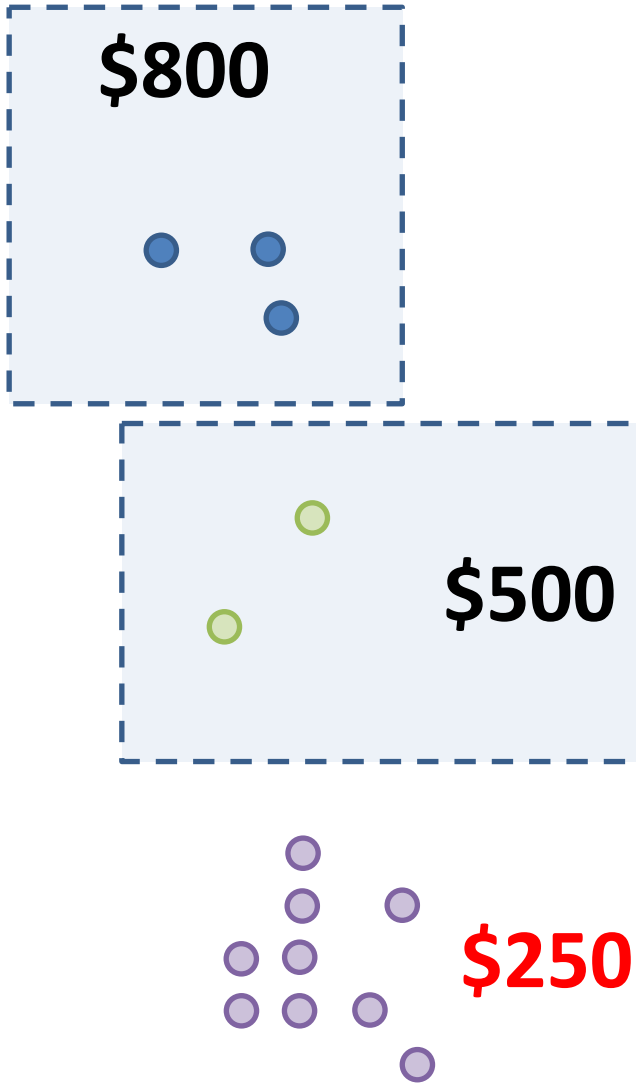

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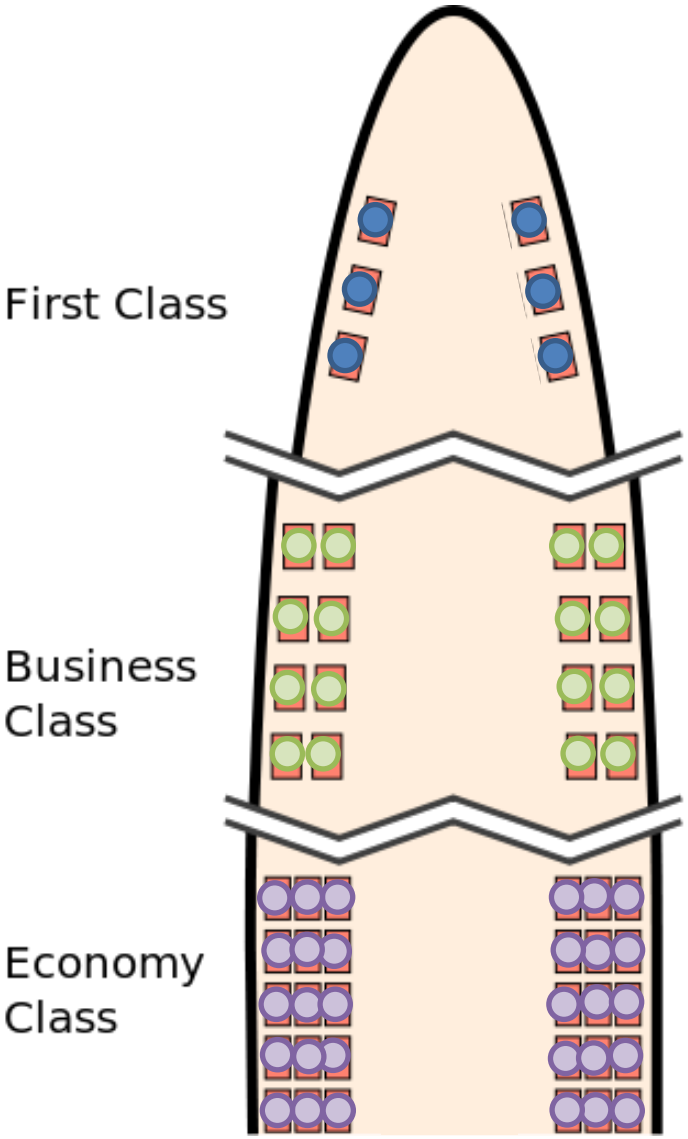

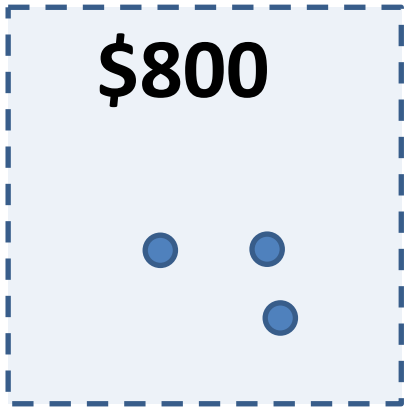

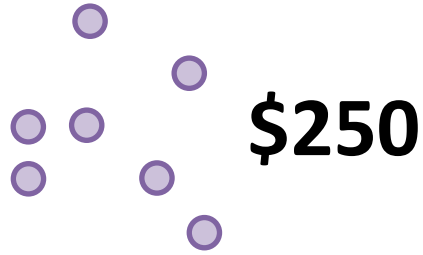

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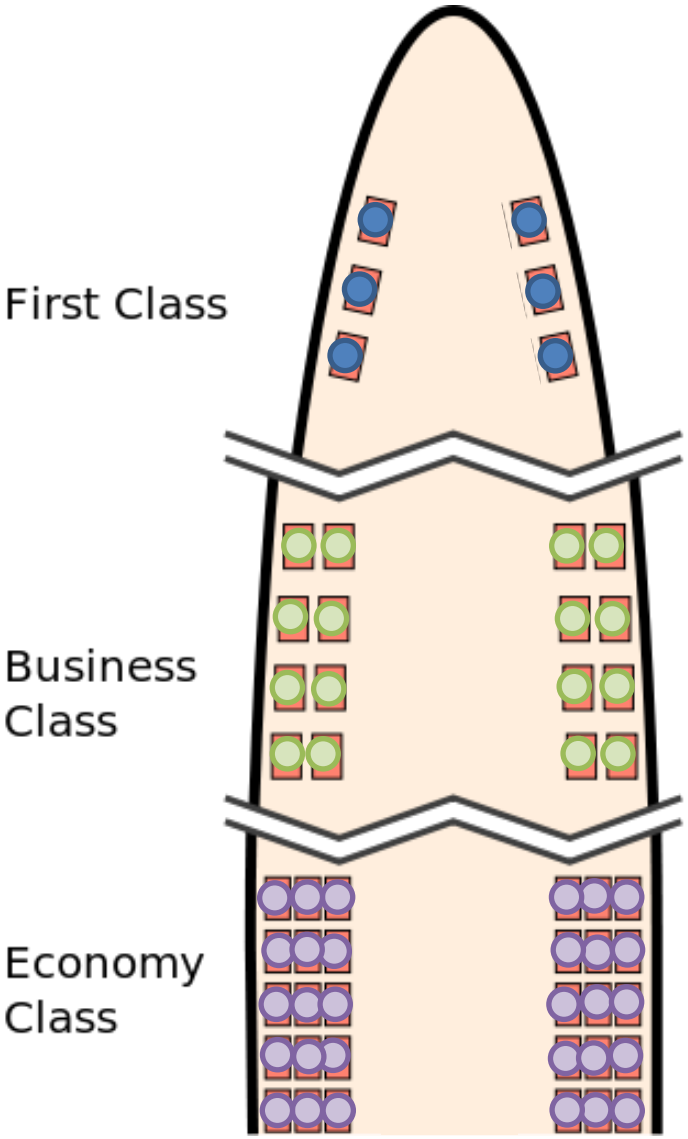

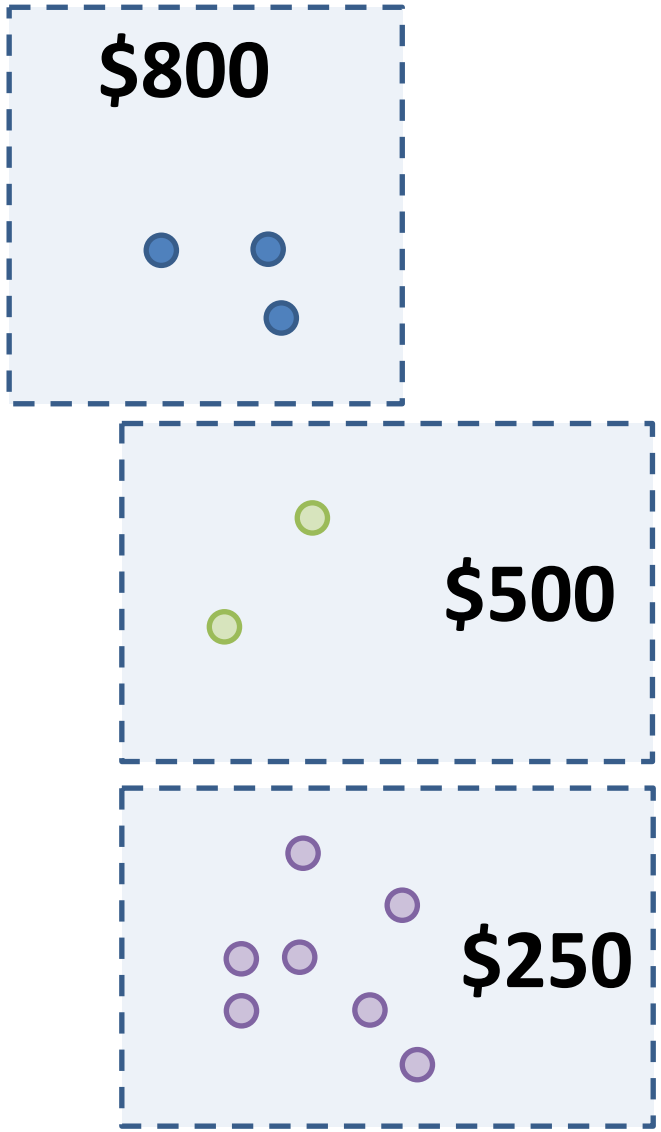

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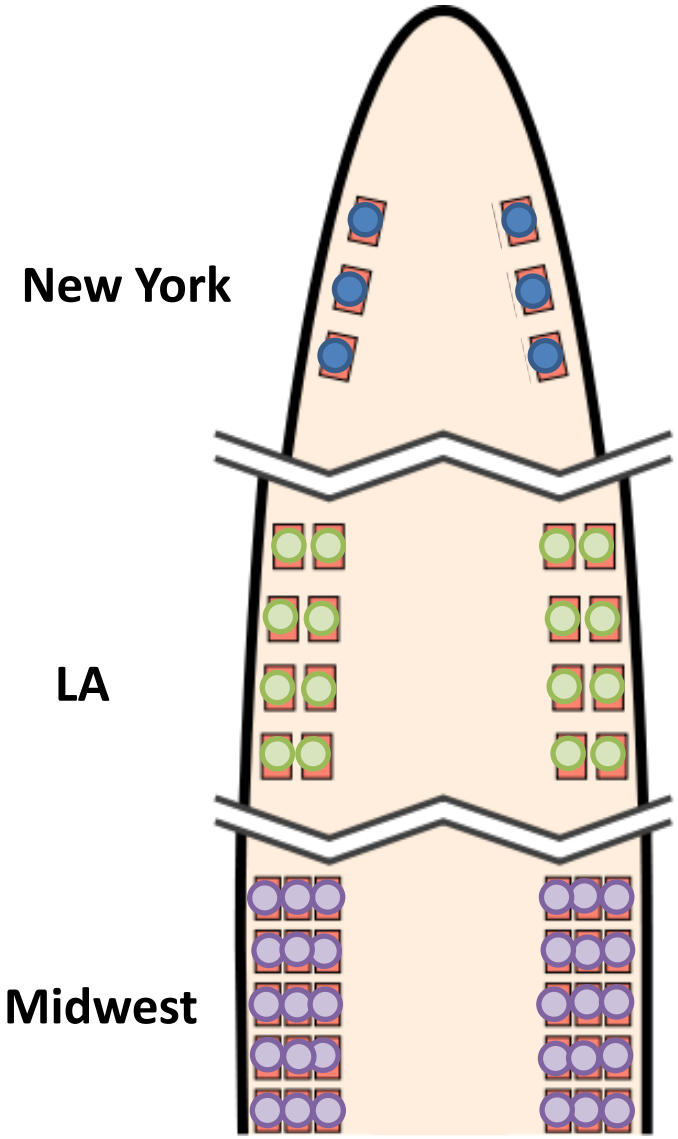
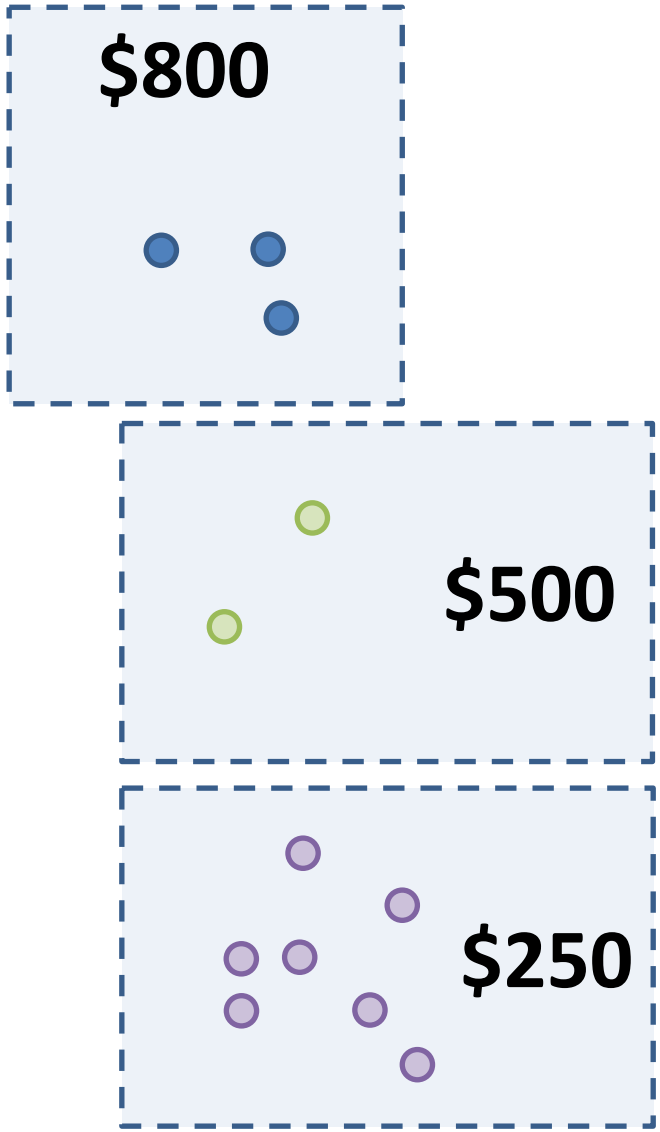




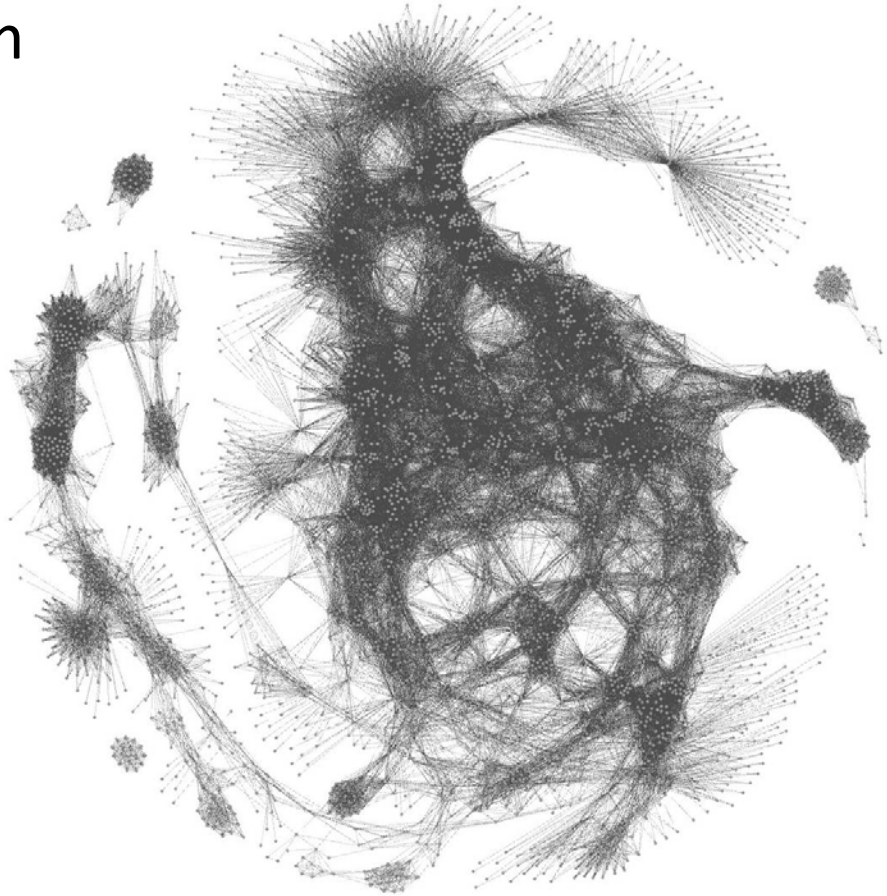
Real Constraints are Messier

- The **feasibility constraints** are not uniform
  - nearby stations can **freeze at different times**

Feasibility Testing

Key computational problem: testing the feasibility of a given repacking, based on interference constraints

- Basis of "frozen test": millions per auction
- A hard **graph-colouring** problem
  - 130,000 constraints
  - Initial skepticism about whether this problem could be solved exactly at a national scale
- We're doing it, using tools from **empirical algorithmics**
  - SAT encoding
  - automatic algorithm configuration
  - algorithm portfolios

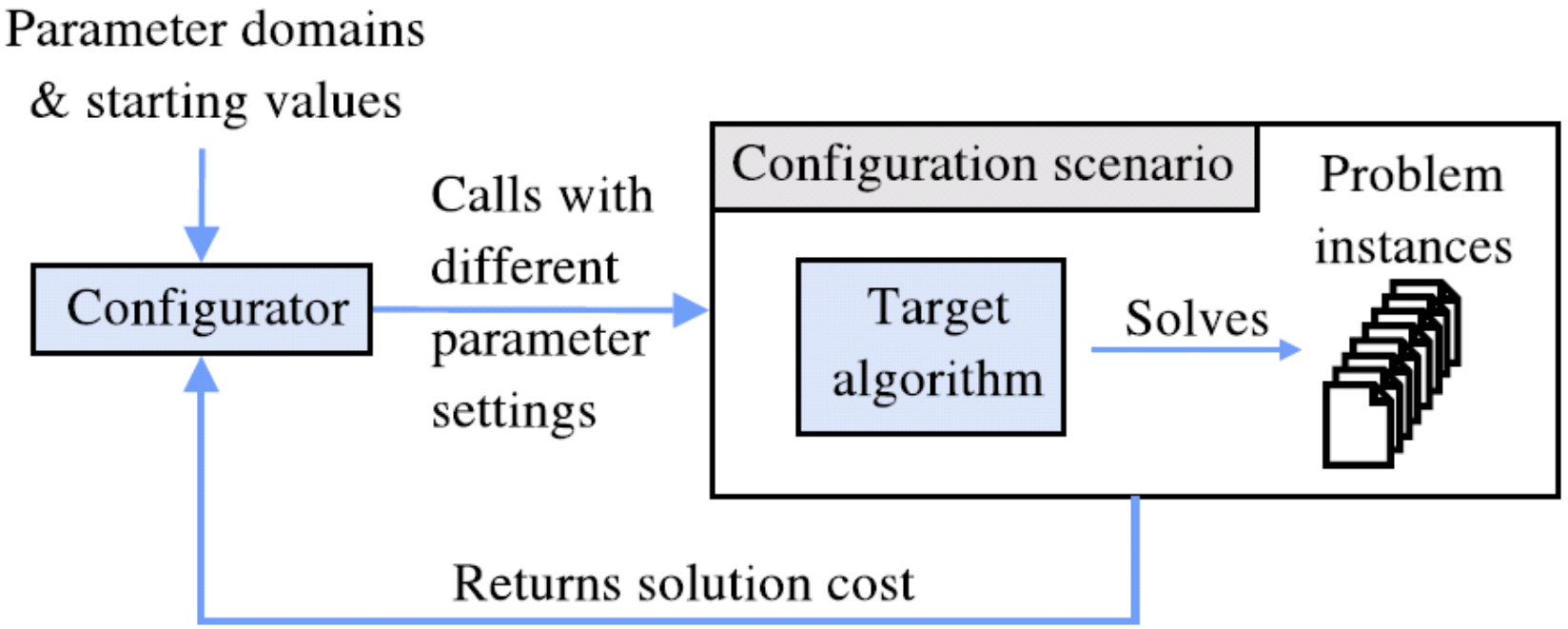


SAT Encoding

- $x_{s,c}$: the proposition that **station s is assigned to channel c**
  - one such variable for every station s and channel c
- Station s **must broadcast on one** of its allowable channels
  - For every station s and set of allowable channels $\{c_1, \ldots, c_n\}$, create a clause $(x_{s,c_1} \vee \cdots \vee x_{s,c_n})$
- Station s **may broadcast on at most one** of these channels
  - For every pair of channels c_1 and c_2 allowed for station s, create a clause $(\neg x_{s,c_1} \vee \neg x_{s,c_2})$
- The repacking **does not cause harmful interference**
  - For every interference rule stating that s_1 cannot broadcast on c_1 while s_2 broadcasts on c_2, create a clause $(\neg x_{s_1,c_1} \vee \neg x_{s_2,c_2})$
- Note: mostly 2-clauses
  - good for unit propagation: implies clique constraints

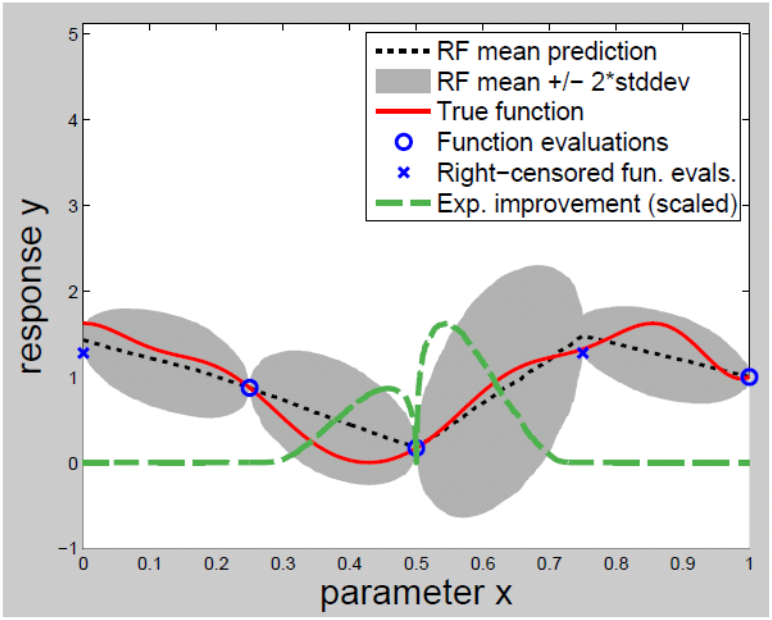
Algorithm Configuration

- High-performance solvers for NP-complete problems like SAT are typically **parameterized**
  - which branching heuristic, variable ordering, preprocessing strategy, clause learning technique, …
- Address with **algorithm configuration**



Sequential Model-based Algorithm Configuration (SMAC)

[Hutter, Hoos & LB, 2011]



Initialize with a single run for the default configuration

repeat

- Learn a random forest model $m : \Theta \times \Pi \to \mathbb{R}$ from data so far
- Marginalize out instance features: $f(\theta) = \mathbb{E}_\pi[m(\theta, \pi)]$
- Find θ that maximizes expected improvement in $f(\theta)$ over incumbent
- Compare θ to the incumbent, updating if it's better.

until *time budget exhausted*

Algorithm Portfolios

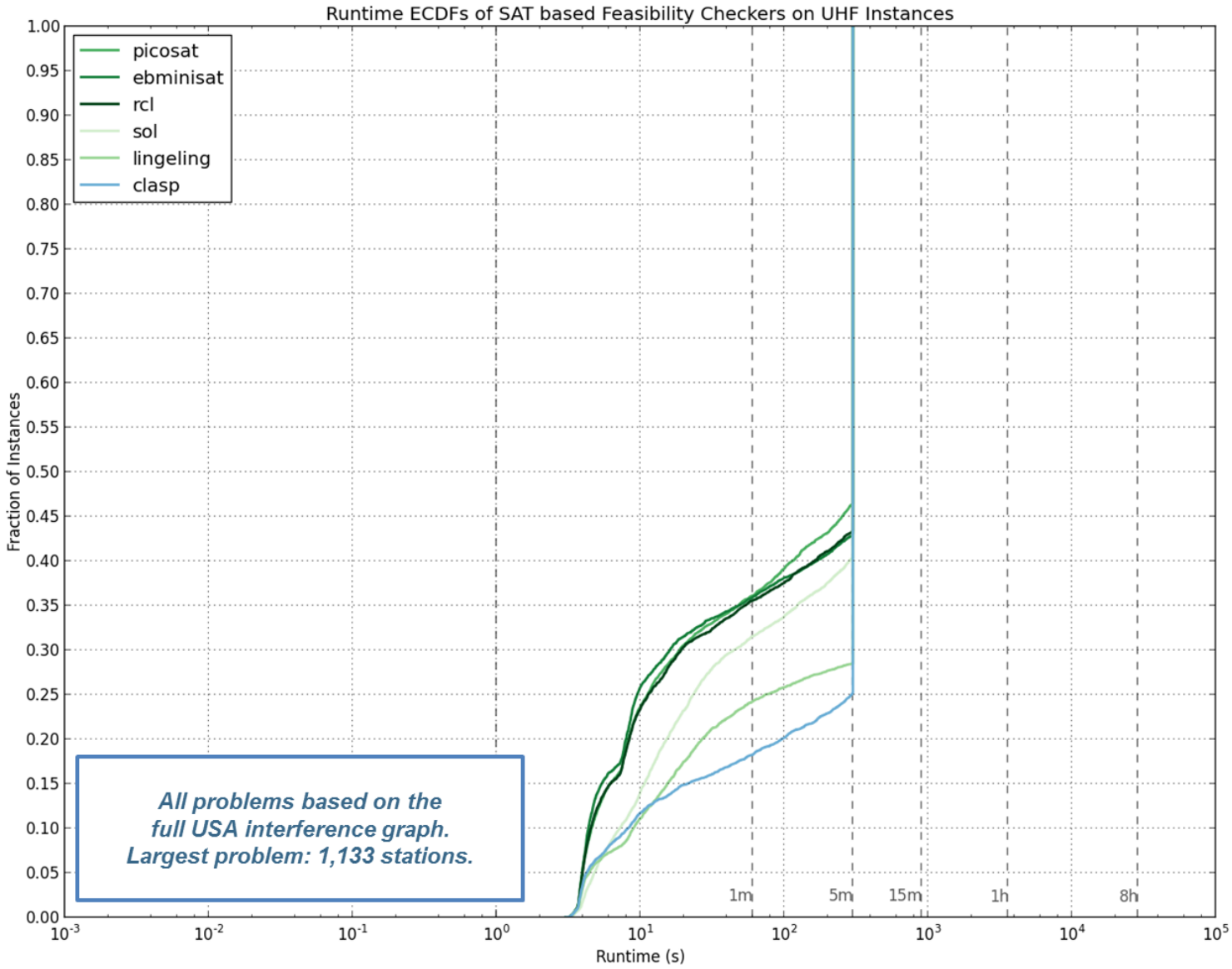
- Often different solvers perform well on different problem instances
- Idea: build an **algorithm portfolio**, consisting of different algorithms that can work together to solve a problem
- **SATzilla**: state-of-the-art portfolio developed by my group (2003-2013)
- some key ideas:
  - **presolver scheduling**
  - machine learning to **choose algorithm** on a per-instance basis
  - constituent solvers can be **automatically configured** ("Hydra")

How These Techniques are Pragmatic

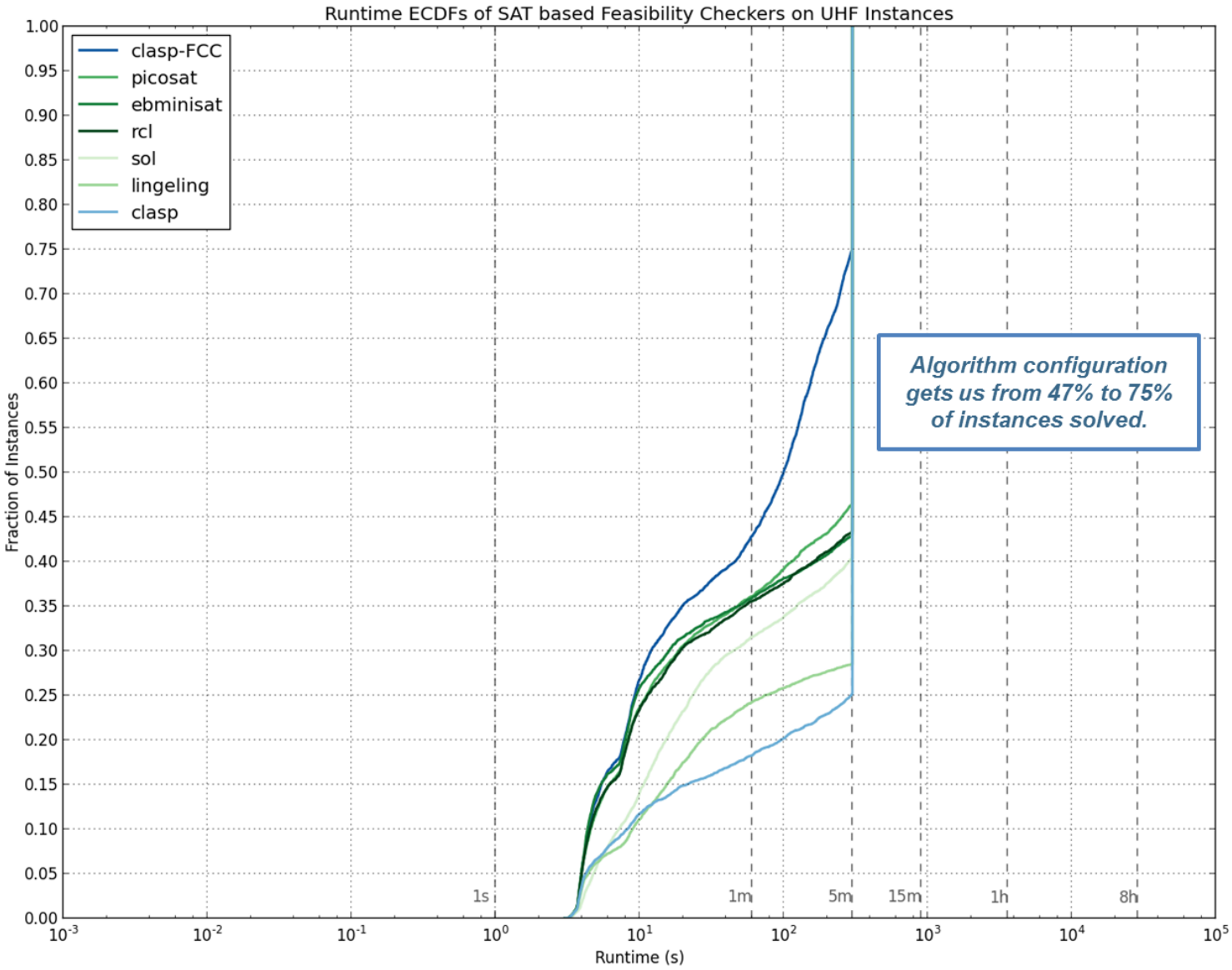
- Target distributions of interference graphs that **arise in practice**
  - subsets of the whole-country graph

- Achieve good performance **most of the time**, but tolerate occasional failures
  - treat timeouts as UNSAT

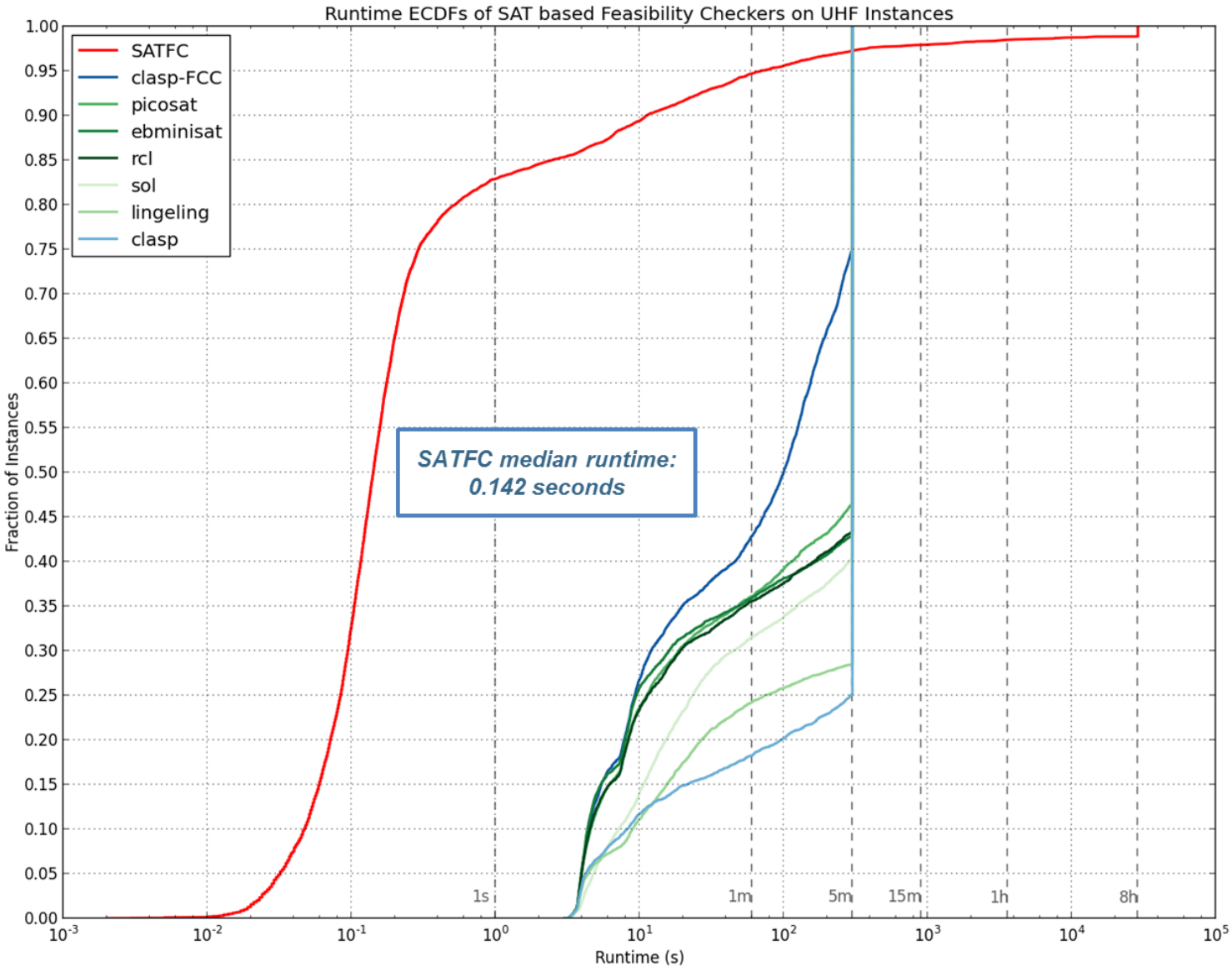
Comparing off-the-shelf SAT solvers (5 min cutoff)



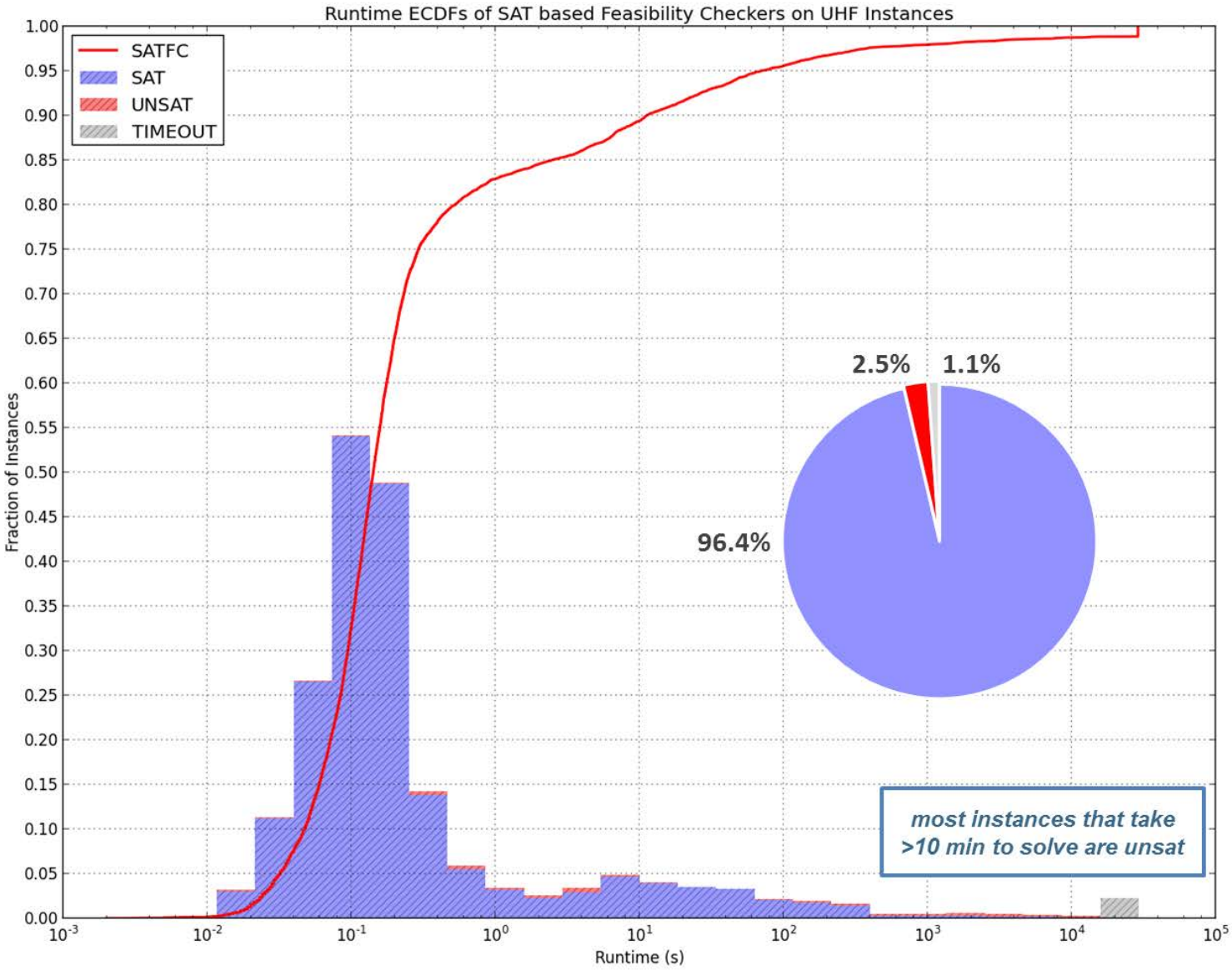
Adding our specially-configured version of clasp



Adding presolvers, other optimizations (8h cutoff)



SATFC performance and SAT/UNSAT breakdown



Including VHF Bands

- So far we've considered mechanisms for compensating stations for going off air
  - what if UHF stations can also be paid to **move to a lower spectrum band** (VHF)?
  - we now potentially face **multi-minded bidders**
    - willing to go to VHF at one price; off air at another
  - theory gives us less guidance here
    - An area of active study; multiple proposals have been made

Equilibrium Computation and Computational Mechanism Analysis

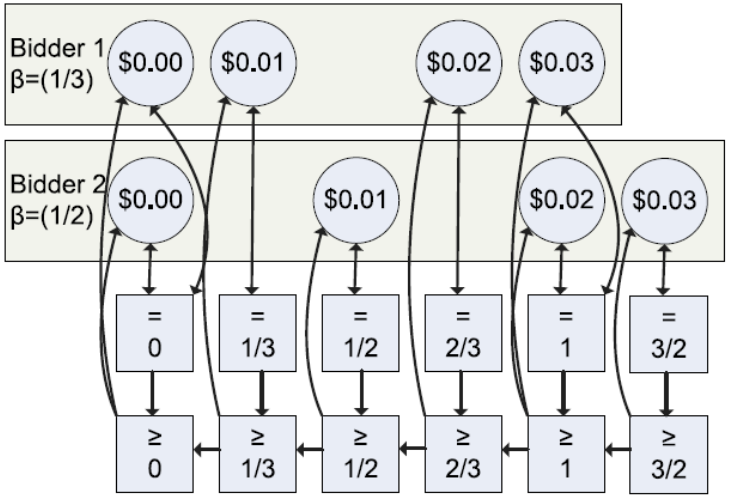
[Jiang & LB, 2011]

[Thompson & LB 2009; 2013; unpublished]

Action-Graph Games

- **Compactly represent games** exhibiting context-specific independence, anonymity or additivity [Jiang & LB, 2011]
  - the sort of structure exhibited by typical market settings
  - **pragmatic:** target typical case, not worst case

- **Fast algorithms** for computing quantities of interest
  - Nash equilibrium [Jiang & LB, 2011], correlated eq [Jiang & LB 2013], pure-strategy eq [Jiang & LB, 2010], all equilibria [Thompson & LB, 2011], ϵ-equilibrium [Daskalakis, Schoenebeck, Valiant & Valiant, 2009]



Computational Mechanism Analysis

- What happens **in equilibrium of real-world mechanisms**, under given valuation distributions?
  - go **beyond theoretical analysis** of mechanism properties
  - answer **quantitative questions** (e.g., "which gives higher revenue?")
  - gives answers even in **complex domains**
    (reserve prices; messy valuation distributions; general eqm concepts)

- How it works:
  - repeatedly **sample games** from the valuation distribution
  - represent these games as **AGGs**
  - **solve them** using general AGG solvers
  - obtain statistics on **economic quantities** of interest

- ***Pragmatic**: statistical, data-driven*

CMA Application: Ad Auction Evolution

[Thompson & LB, 2009]

- Search engines used **different auctions** over the years
  - GFP: Yahoo! and Overture 1997-2002
  - uGSP: Yahoo! 2002-2007
  - wGSP: Google, Microsoft, Yahoo! 2007-present

Question: **Is wGSP better** than GFP and uGSP?

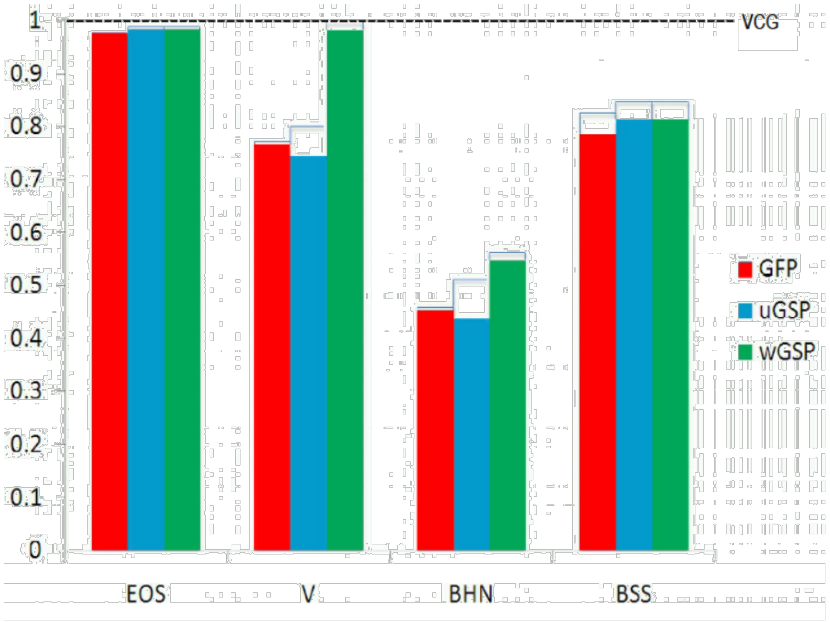
- better revenue?
- better efficiency?

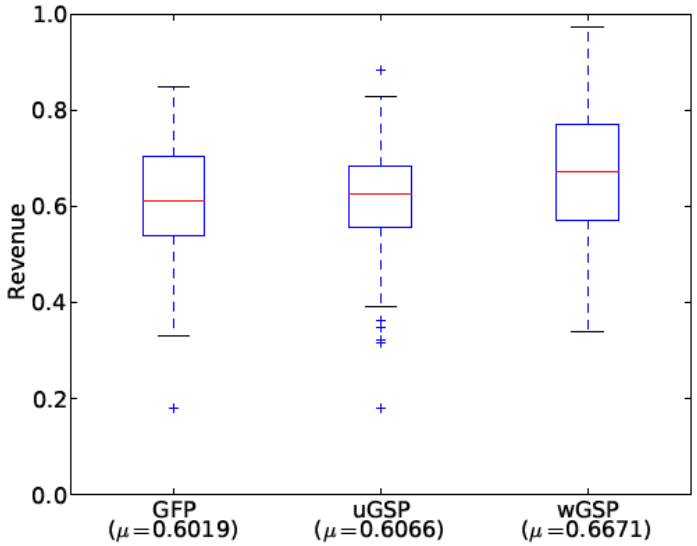
Approach:

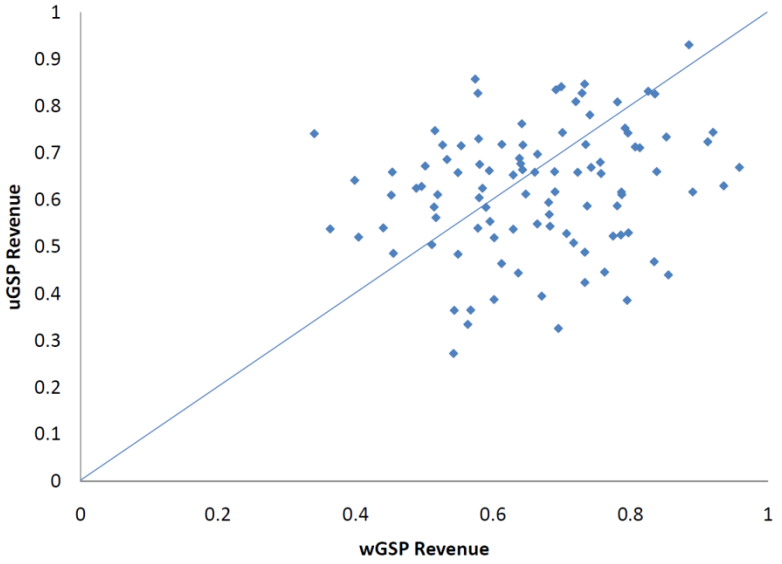
- Construct **perfect information AGGs** sampled from widely studied valuation distributions
- Compute revenue/welfare optimal/pessimal **equilibria**

Analyzing Ad Auctions: Efficiency, Revenue

[Thompson & LB, 2009]







How should reserve prices be set in GSP?

[Thompson & LB, 2013]

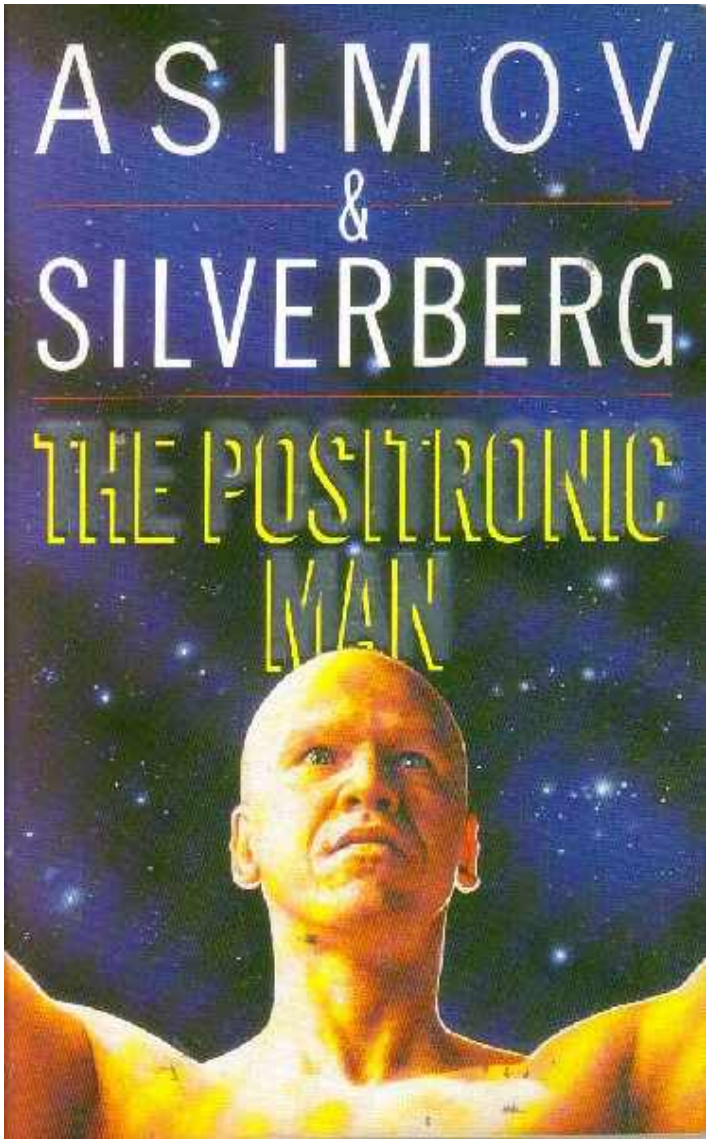
- **Goal: maximize (short term) revenue in GSP.**
  - 6 GSP variants; parameters control the way reserve prices are set
- Represent as AGG, varying parameters on a fine grid
- Find all equilibria
- Find parameters that optimize best/worst equilibrium revenue

Revenue-Pessimal Equilibrium			**Revenue-Optimal Equilibrium**		
Auction	Revenue	Parameter(s)	Auction	Revenue	Parameter(s)
Vanilla GSP	3.814	—	Vanilla GSP	9.911	—
Squashing	4.247	$s = 0.4$	QWR	10.820	$r = 5.0$
QWR	9.369	$r = 9.0$	Squashing	11.534	$s = 0.2$
Anchoring	10.212	$r = 13.0$	UWR	11.686	$r = 11.0$
QWR+Sq	10.217	$r = 15.0, s = 0.2$	**Anchoring**	**12.464**	$r = 11.0$
UWR	**11.024**	$r = 15.0$	**QWR+Sq**	**12.627**	$r = 7.0, s = 0.2$
UWR+Sq	**11.032**	$r = 15.0, s = 0.6$	**UWR+Sq**	**12.745**	$r = 9.0, s = 0.2$

Takeaway: unweighted reserves are both robust and outperform weighted reserves.

Positronic Economist

- AGGs provide general tools for equilibrium computation
  - but not straightforward to encode domains of interest
- **Positronic Economist**
  [Thompson & LB, unpublished]
  - A general tool for taking manual effort out of mechanism analysis
  - Leverages existing algorithms for computing (general; pure-strategy; all; Bayes-Nash) equilibria
  - **Generates compact AGGs** from natural descriptions of mechanisms and settings.



Example: Independent Private Values

- Preferences (in math):

$$u_i(x, p_i) = \begin{cases} v_i - p_i & x = i \\ -p_i & \text{otherwise} \end{cases}$$

- Preferences (in posec):

```
def u(i, v, o, a_i):
    if o.i_win:
        return v[i]-o.my_payment
    return -o.my_payment
```

- Note: we can study more complex settings too
  - budgets, common values, etc.

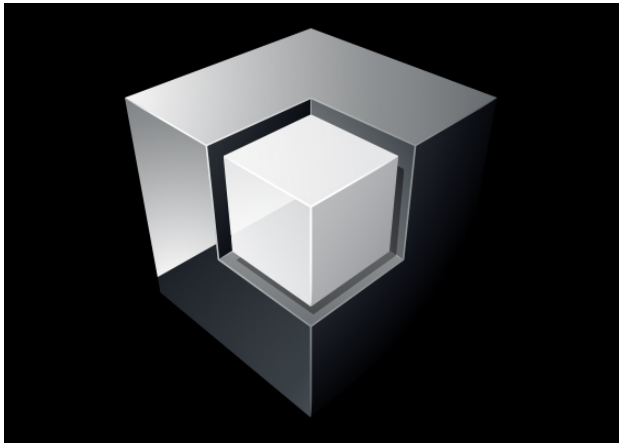
Example: Single-Unit Auction

- Mechanism:
  - First-price auction with bids {0, …, 9}
  - uniform random tie-breaking
- Mechanism (in posec):

```
def M(setting,i,theta_i,a_N):
    # i loses if anyone bids more than him
    if a_N.any([a for a in a_N.actions if a>a_N[i]]):
        return ProjectedOutcome(i_win=False,my_payment=0)

     # i wins in a t-way tie (where t can be 1)
    t = a_N.count(a_N[i])
    O = [ProjectedOutcome(i_win=True,my_payment=a_N[i]),
       ProjectedOutcome(i_win=False,my_payment=0.0)]
return PosEc.Distribution(O,[1.0/t,1-1.0/t])
```

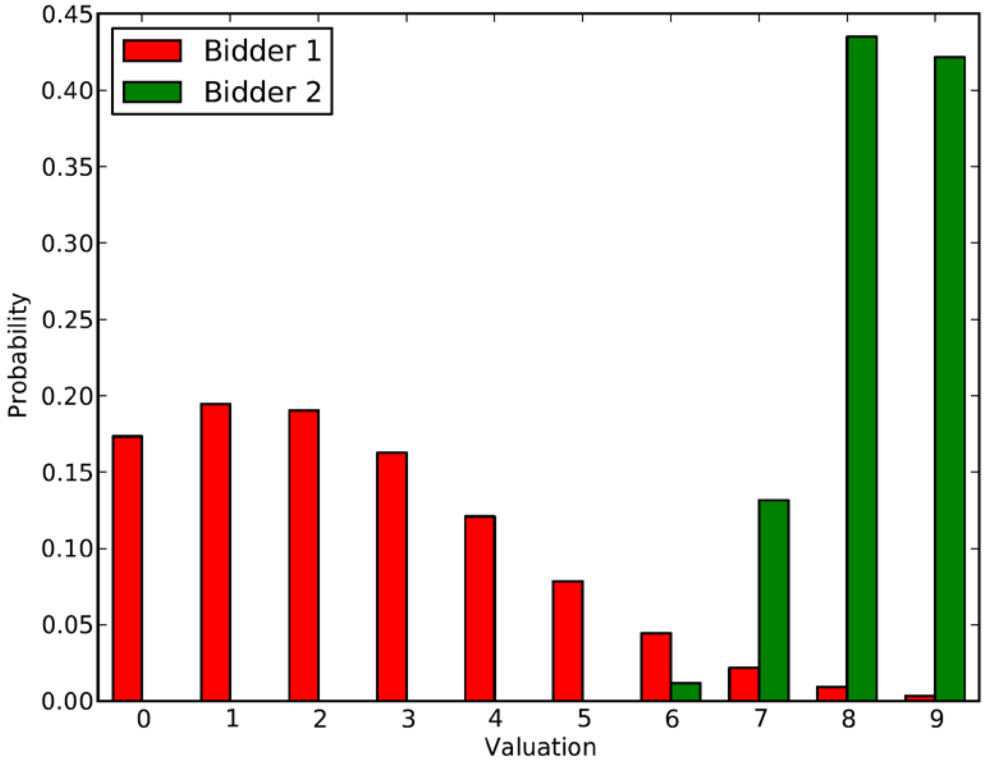
posec can build an AGG in two ways



- **Black-box** structure inference
  - Probe every joint action × joint type in the simultaneous-move Bayesian game.
- **White-box** structure inference
  - Reason from the structure of the mechanism definition to detect independencies without probing everything.
  - Facilitated via Python's operator overloading and Positronic Economist's accessor functions (e.g., count, distribution).

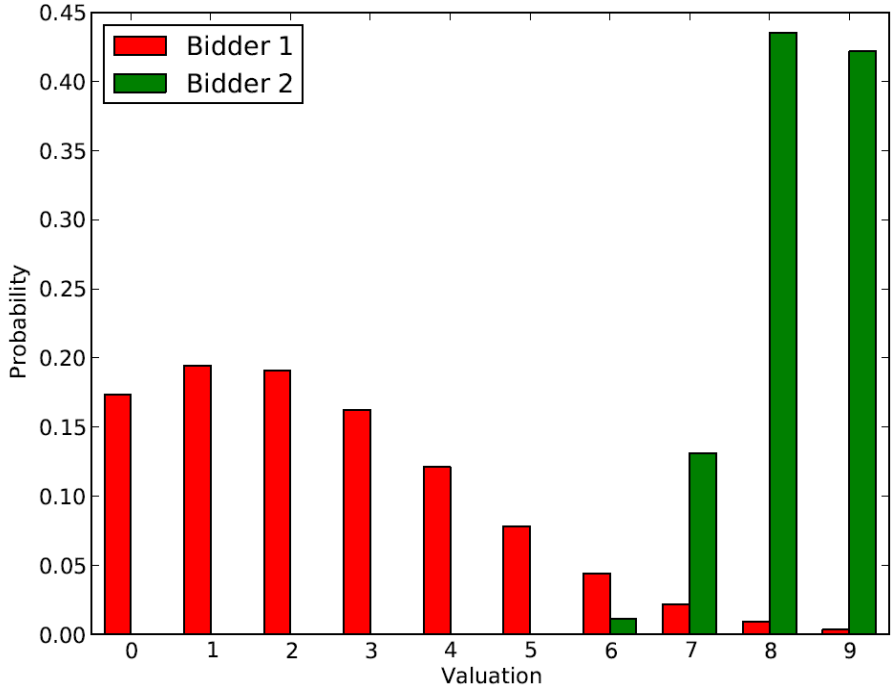
posec Performance

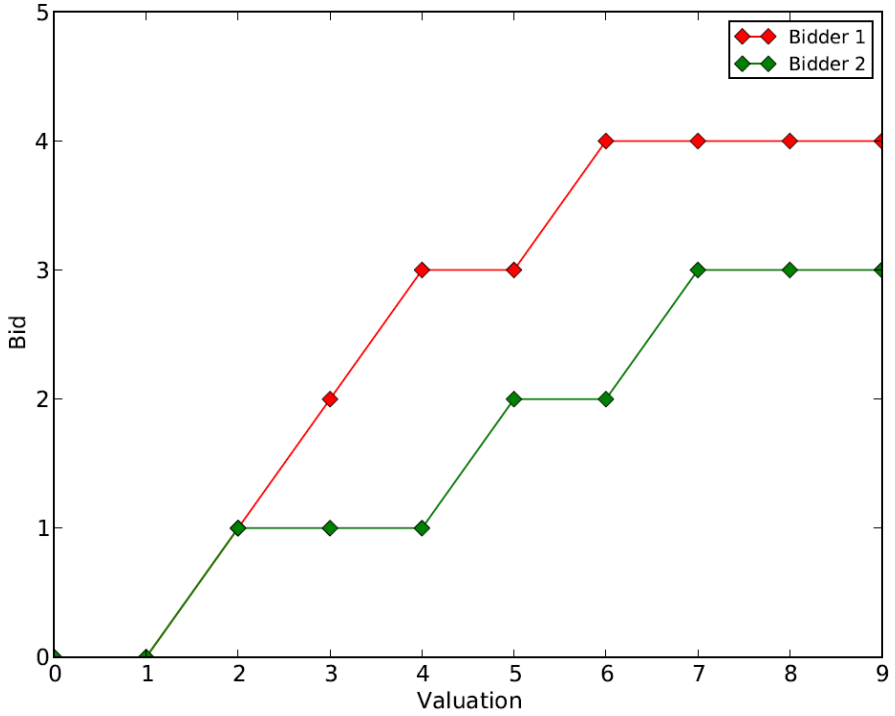
- **Asymmetric, single-unit, first-price auction**
- n = 2 players, t = 10 types each, a = 10 actions each



- Induced normal form: $(a^t)^n \geq$ **10^8 TB**
- BAGG found by posec: **27kb file, produced in ~6s**

posec Performance





- GNM finds a (pure strategy) **BNE in ~53s**
- Gives us a **price of anarchy lower bound: 1.068**
- Try it out: https://github.com/davidrmthompson/positronic-economist

KUDU: A MOBILE MARKET FOR AGRICULTURAL COMMODITIES IN DEVELOPING COUNTRIES

[Ssekibuule, Quinn & LB, 2013]

African produce market circa 1900

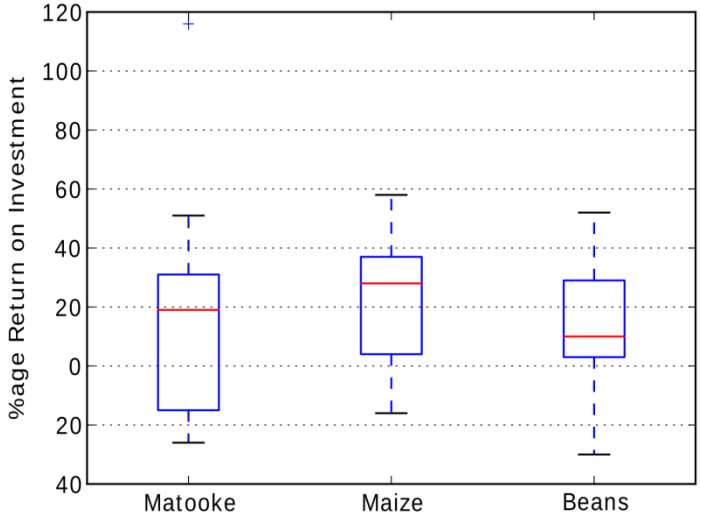
Ugandan produce market circa 2011

Sometimes the scale is a bit bigger…

Sometimes the scale is a *lot* bigger…

Problem: Market Inefficiency

- **Subsistence agriculture** is the main occupation in Uganda
- Farmers **waste a lot of time** transporting produce; waiting by the road
- Buyers and sellers have trouble **finding each other**
- Sporadic **food shortages** in urban centers
- Robust **arbitrage** opportunities



The wave of the future?

Kudu: an SMS-based market for agricultural commodities
[Ssekibuule, Quinn & LB, 2013]

- **bids** consider price, reputation, quality, geographic location
- **market** clears daily
  - posted prices for farmers
  - second-pricing for buyers
- can **ban** specific traders

Really works!

- Field trial Jan – July 2013
- http://www.kudu.ug

Kudu | Home | Buy | Sell | Offers | Purchased | Market Information | Bids | Asks | Create Account

Kudu trade

Welcome to Kudu! Buy and sell agricultural goods in Uganda with our easy-to-use auction system.

How to buy » | How to sell »

Create Account

Existing users please log in

Phone number: phone number

Passcode: passcode

Login

Kudu stats

(as of March 18, 2013)

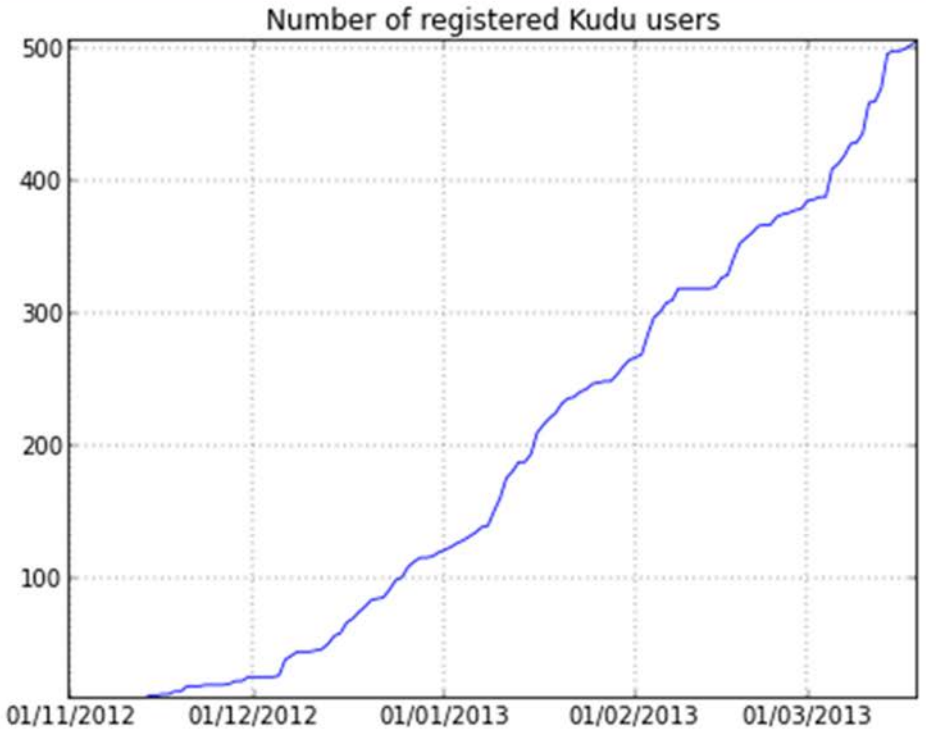
Current number of traders: 506

Current top 5 products among buyers, and total quantities offered:

Maize (Kasooli)	157705 Kgs
Beans (Nambale)	28825 Kgs
Coffee (Kibooko)	14280 Kgs
Coffee (Robusta)	10900 Kgs
Cassava (Muwogo)	5000 Kgs

Current top 5 products among sellers, and total quantities bidded for:

Maize (Kasooli)	480000 Kgs
Simsim (Ntungo)	105000 Kgs
Matooke	81100 Bunches
Beans (Nambale)	71250 Kgs
Coffee (Coronal-Endokwa)	20000 Kgs



Statistics from our Field Trial

- 1024 traders and farmers **registered**
- 520 **asks** (USD $1,700,000); 285 **bids** (USD $960,000)
- 219 users used Kudu only to **learn commodity prices**
- Market activity highly dependent on **radio adverts**
- **Largest bid**: 120,000 Kg of maize (verified genuine)
  - 53 bids, 94 asks exceeding 10,000 Kg

Produce type	Total ask quantity
Peanuts	512,375 Kg
Maize	1,711,935 Kg
Beans mixed	114,900 Kg
Coffee (Robusta)	36,800 Kg
Sweet Potatoes	2,221 Sacks

Table 3: Quantities of the five categories of produce with the highest aggregate ask value.

Produce type	Total bid quantity
Maize	917,300 Kg
Sesame	110,000 Kg
Beans mixed	179,050 Kg
Soya	40,000 Kg
Peanuts	35,050 Kg

Table 4: Quantities of the five categories of produce with the highest aggregate bid value.

Next Steps for Kudu

- National **scale**
- More work on **quality, reputation**
- More sophisticated **matching**
- Dealing with **malformed bids**
- Mostly, advertising and labor

- Looking for **funding**—ideas?

Pragmatic AGT

I argued for the benefits of pragmatic AGT, which

1. measures performance on specific **problems of interest**;
2. adopts **statistical rather than analytical methods**.

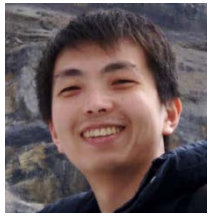
Today I told you about:

- **Spectrum repacking:** computational issues are at the heart of the FCC's upcoming radio spectrum redistribution. We're helping to build high-speed feasibility checkers and investigating novel auction designs.
- **Computational Mechanism Analysis:** can leverage compact game representations to enable quantitative, statistical analysis of existing mechanisms like ad auctions.
  - **Positronic Economist** will make it easier for you to use these tools.
- **Kudu,** an SMS-based market for agricultural commodities, leverages practical ideas from AGT to help Ugandan farmers.

Thanks to my students and collaborators!

- **Alexandre Fréchette**
  - *Spectrum repacking*

- **David R.M. Thompson**
  - *Ad auctions; Positronic Economist*



- **Albert Xin Jiang**
  - *AGGs*

- **John Quinn, Richard Ssekibuule**
  - *Kudu*