Level-0 Meta-Models for Predicting Human Behavior in Games

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Behavioral Game Theory

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- Do people actually follow them?
Behavioral Game Theory

- Many of game theory’s recommendations are counterintuitive.
- Do people actually follow them?
- **Not reliably**, as demonstrated by a large body of experiments.
- **Behavioral game theory**: Aims to model actual human behavior in games.
Nash equilibrium and human subjects

- Nash equilibrium often makes **counterintuitive predictions**
  - In Traveler’s Dilemma: The vast majority of human players choose 97–100. The Nash equilibrium is 2
- Modifications to a game that don’t change Nash equilibrium predictions at all can cause large changes in how human subjects play the game [Goeree & Holt 2001]
  - In Traveler’s Dilemma: When the penalty is large, people play much closer to Nash equilibrium
  - But the size of the penalty does not affect equilibrium
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  - In Traveler’s Dilemma: When the penalty is large, people play much closer to Nash equilibrium
  - But the size of the penalty does not affect equilibrium
- Clearly Nash equilibrium is not the whole story
- Behavioral game theory proposes a number of models to better explain human behavior
In previous work [Wright & Leyton-Brown, 2010; 2014a], we compared several behavioral models’ predictive performance.
BGT State of the art

- In previous work [Wright & Leyton-Brown, 2010; 2014a], we compared several behavioral models’ predictive performance.
- Quantal cognitive hierarchy is the current state of the art model.
Iterative reasoning

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Level 0

Level 1
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Level 0
$\ldots$

Level 1
$\ldots$

Level 2
$\ldots$
Quantal cognitive hierarchy (QCH)

- Agents’ levels drawn from a distribution $g$
- An agent of level $m$ responds to the truncated, true distribution of levels from 0 to $m - 1$
- Agents quantally respond to their beliefs

\[
\begin{align*}
\pi_{i,0}(a_i) &= |A_i|^{-1}, \\
\pi_{i,m}(a_i) &= QBR_i(\pi_{-i,0:m-1}; \lambda) \\
\pi_{i,0:m-1} &= \frac{\sum_{\ell=0}^{m-1} \pi_{i,\ell}g(\ell)}{\sum_{\ell=0}^{m-1} g(\ell)}
\end{align*}
\]
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Level-0 agents’ actions influence every other level.
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And yet best performing parameters for QCH suggest large numbers of level-0 agents
Level-0 agents’ actions influence every other level
Take modeling level-0 behavior more seriously?
Level-0 meta-model

- Define a level-0 meta-model:
  - A mapping from an (arbitrary) game to a (potentially nonuniform) level-0 distribution over that game’s actions
  - Leverage some of what we know about how people reason nonstrategically about games
  - The meta-model can have its own parameters

- Use an existing iterative model (quantal cognitive hierarchy) on top of the improved level-0 model to make predictions

- What distinguishes level-0 from level-1?
  - Our line in the sand: no explicit beliefs about how other agents will play
Features

Five binary features of each action:

1. Minmin Unfairness
   - Does this action contribute to the least unfair outcome?

2. Maxmax payoff ("Optimistic")
   - Does this action contribute to my own highest-payoff outcome?

3. Maxmin payoff ("Pessimistic")
   - Is this action best in the (deterministic) worst case?

4. Minimax regret
   - Does this action have the lowest maximum regret?

5. Efficiency (Total payoffs)
   - Does this action contribute to the social-welfare-maximizing outcome?
Linear meta-model

We say that a feature is **informative** if it can distinguish at least one pair of actions.

For each action, compute a **sum of weights** for features that are both informative and that “fire”, plus a noise weight.

\[
prediction\text{ for } a_i \propto w_0 + \sum_{f \in F} \mathbb{I}[f \text{ is informative}] \cdot \mathbb{I}[f(a_i) = 1] \cdot w_f
\]
Example: Consider Player 1

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Minimax regret is not informative: 60 for all actions

E.g., Player 1 plays X; if Player 2 plays C, his regret is 60.

40, 35 is the fairest outcome, so Y is minmin unfair.

Y and Z maximize minimum payoff (40 vs. 10 for X).

Y leads to the highest sum of utilities (90 + 70 = 160).

X has the highest best-case utility (100).

Action X’s weight: \( w_0 + w_{\text{maxmax}} \)

Action Y’s weight: \( w_0 + w_{\text{minmin}} + w_{\text{total}} + w_{\text{fairness}} \)

Action Z’s weight: \( w_0 + w_{\text{minmin}} \)
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Performance results

Three iterative models:
1. Quantal Cognitive Hierarchy
2. Level-κ
3. Cognitive Hierarchy

Two level-0 meta-models:
1. Uniform L0
2. Weighted Linear
Performance results

- Weighted linear meta-model for level-0 agents dramatically improved the performance of all three iterative models.
- Almost erases the difference between the models themselves.
Bayesian parameter analysis

- **Fairness** is by far the highest-weighted feature
- All the features quite **well identified**
Parameter analysis: Levels

- Weighted linear $\implies$ much lower variance estimates
- Predicts that about half the population is level-0!
Conclusions

Weighted linear meta-model for level-0 agents dramatically improved the performance of iterative models. Strong evidence for the existence of level-0 agents. For any meta-model, including uniform!

Contrary to conventional wisdom.

Likelihood improvement over uniform

Nash w/error
Lk
Poisson-CH
QRE
QLk
QCH-sp-uniform
QCH5-uniform
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