Level-0 Meta-Models for Predicting Human Behavior in Games

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Behavioral Game Theory

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- Do people actually follow them?
- Not reliably, as demonstrated by a large body of experiments
- Behavioral game theory: Aims to model actual human behavior in games

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- Nash equilibrium often makes counterintuitive predictions
  - In Traveler's Dilemma: The vast majority of human players choose 97–100. The Nash equilibrium is 2
- Modifications to a game that don't change Nash equilibrium predictions at all can cause large changes in how human subjects play the game [Goeree & Holt 2001]
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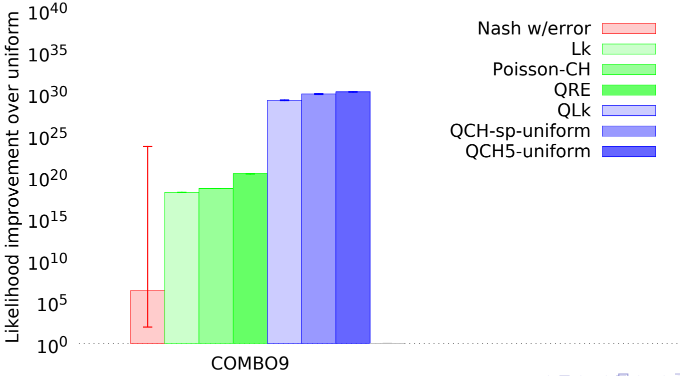
  - In Traveler's Dilemma: When the penalty is large, people play much closer to Nash equilibrium
  - But the size of the penalty does not affect equilibrium
- Clearly Nash equilibrium is not the whole story
- Behavioral game theory proposes a number of models to better explain human behavior

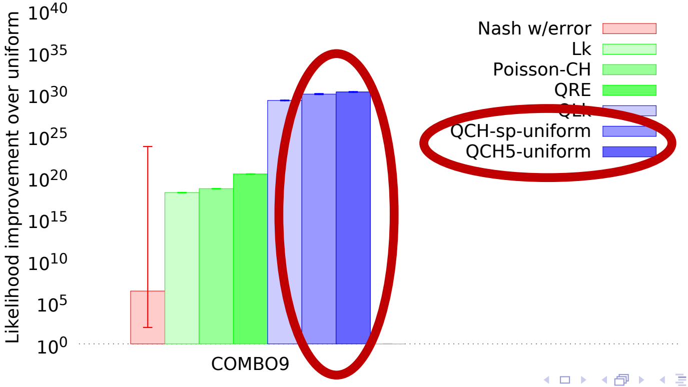
BGT State of the art

- In previous work [Wright & Leyton-Brown, 2010; 2014a], we compared several behavioral models' predictive performance.



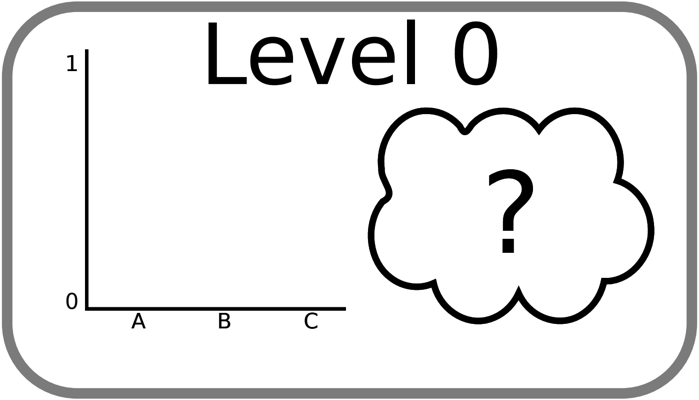
BGT State of the art

- In previous work [Wright & Leyton-Brown, 2010; 2014a], we compared several behavioral models' predictive performance.
- Quantal cognitive hierarchy is the current state of the art model.



Iterative reasoning

Quantal cognitive hierarchy is an iterative model:



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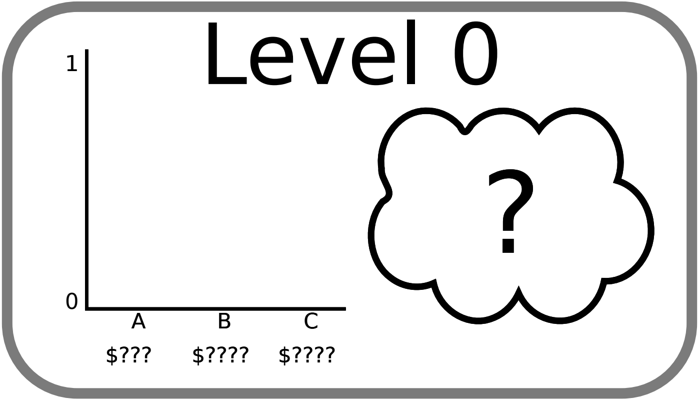

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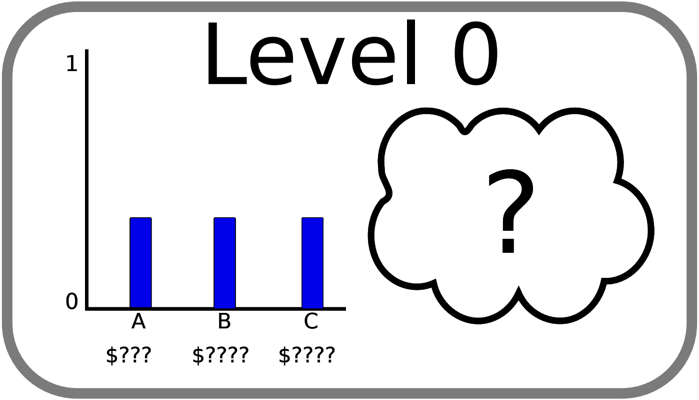

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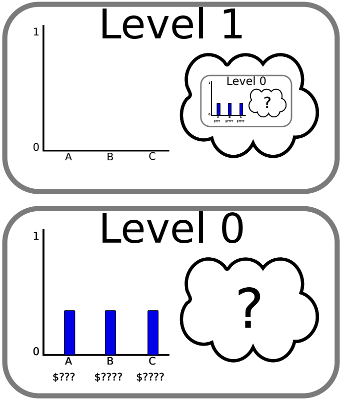

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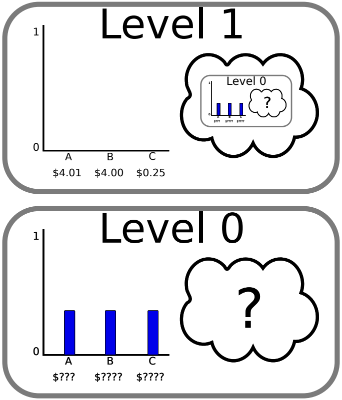

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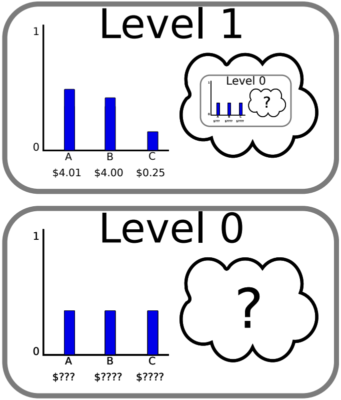

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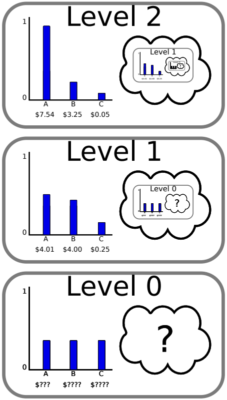
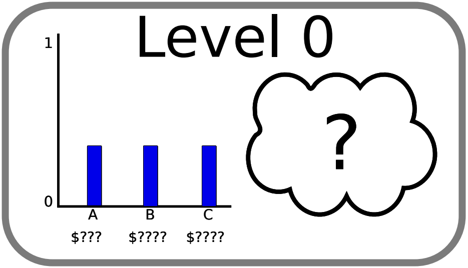


Quantal cognitive hierarchy (QCH)

- Agents' levels drawn from a distribution g
- An agent of level m responds to the truncated, true distribution of levels from 0 to $m-1$
- Agents quantally respond to their beliefs

$$\pi_{i,0}(a_i) = |A_i|^{-1},$$
$$\pi_{i,m}(a_i) = QBR_i(\pi_{-i,0:m-1}; \lambda)$$
$$\pi_{i,0:m-1} = \frac{\sum_{\ell=0}^{m-1} \pi_{i,\ell}\, g(\ell)}{\sum_{\ell=0}^{m-1} g(\ell)}$$

Level-0



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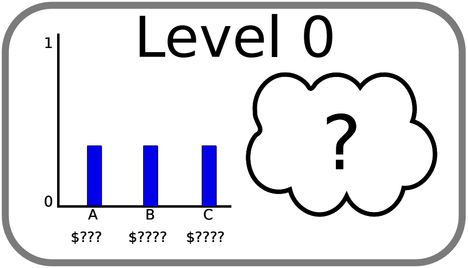

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- And yet best performing parameters for QCH suggest large numbers of level-0 agents
- Level-0 agents' actions influence every other level
- Take modeling level-0 behavior more seriously?

Level-0 meta-model

- Define a level-0 meta-model:
  - A mapping from an (arbitrary) game to a (potentially nonuniform) level-0 distribution over that game's actions
  - Leverage some of what we know about how people reason nonstrategically about games
  - The meta-model can have its own parameters
- Use an existing iterative model (quantal cognitive hierarchy) on top of the improved level-0 model to make predictions
- What distinguishes level-0 from level-1?
  - Our line in the sand: no explicit beliefs about how other agents will play

Features

Five binary features of each action:

1. Minmin Unfairness
   - Does this action contribute to the least unfair outcome?
2. Maxmax payoff ("Optimistic")
   - Does this action contribute to my own highest-payoff outcome?
3. Maxmin payoff ("Pessimistic")
   - Is this action best in the (deterministic) worst case?
4. Minimax regret
   - Does this action have the lowest maximum regret?
5. Efficiency (Total payoffs)
   - Does this action contribute to the social-welfare-maximizing outcome?

Linear meta-model

We say that a feature is informative if it can distinguish at least one pair of actions.

For each action, compute a sum of weights for features that are both informative and that “fire”, plus a noise weight.

$$\text{prediction for } a_i \propto w_0 + \sum_{f \in F} \mathbb{I}[f \text{ is informative}] \cdot \mathbb{I}[f(a_i) = 1] \cdot w_f$$

Example: Consider Player 1

	A	B	C
X	$100, 20$	$10, 67$	$30, 40$
Y	$40, 35$	$50, 49$	$90, 70$
Z	$41, 21$	$42, 22$	$40, 23$

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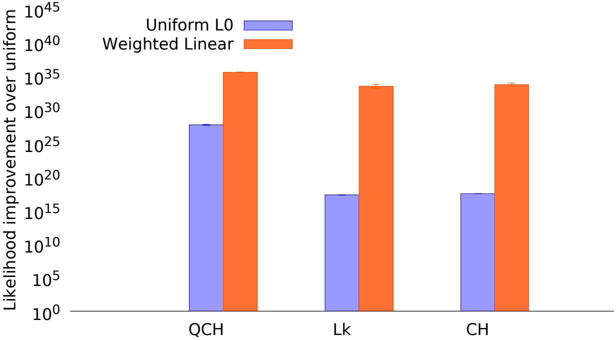
Action X's weight: $w_0 + w_{\text{maxmax}}$
Action Y's weight: $w_0 + w_{\text{minmin}} + w_{\text{total}} + w_{\text{fairness}}$
Action Z's weight: $w_0 + w_{\text{minmin}}$

Performance results



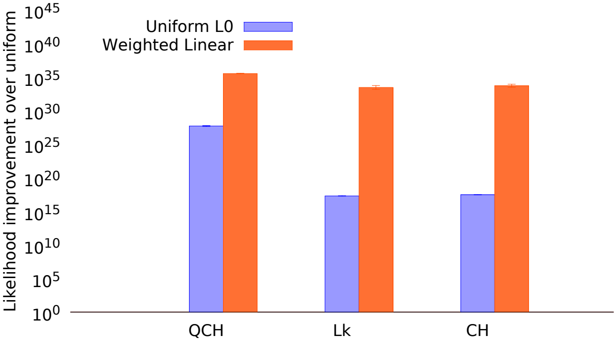
Three iterative models:

1. Quantal Cognitive Hierarchy
2. Level-k
3. Cognitive Hierarchy

Two level-0 meta-models:

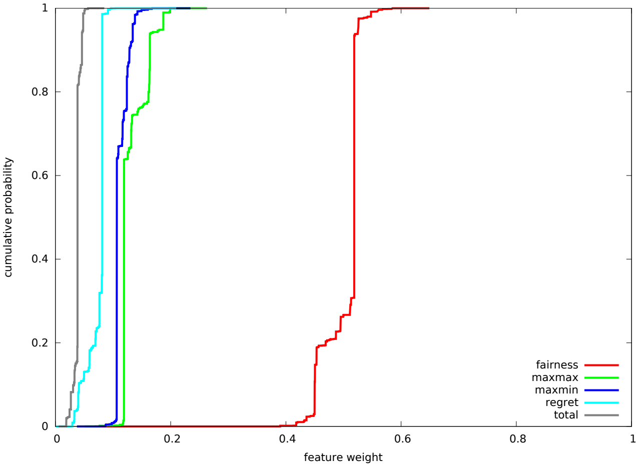
1. Uniform L0
2. Weighted Linear

Performance results



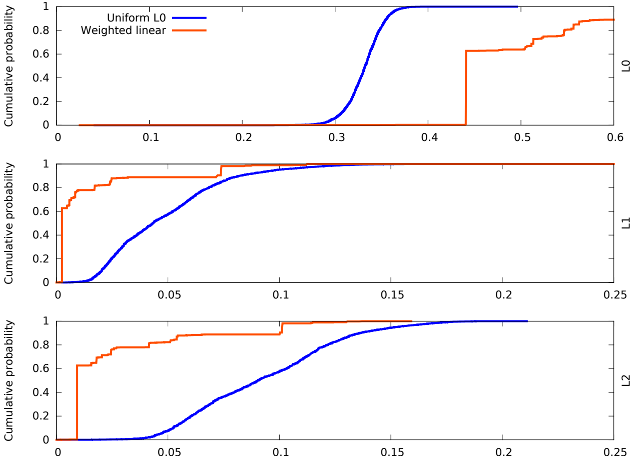
- Weighted linear meta-model for level-0 agents dramatically improved the performance of all three iterative models.
    - Almost erases the difference between the models themselves.

Bayesian parameter analysis



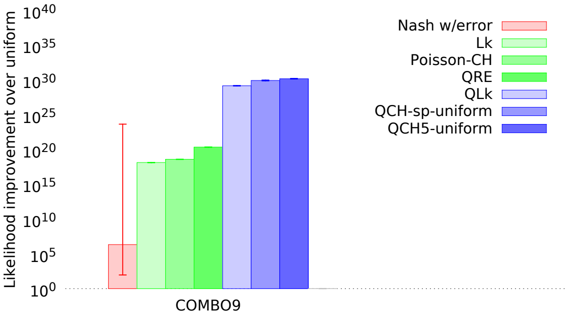
- Fairness is by far the highest-weighted feature
- All the features quite well identified

Parameter analysis: Levels



- Weighted linear $\implies$ much lower variance estimates
- Predicts that about half the population is level-0!

Conclusions



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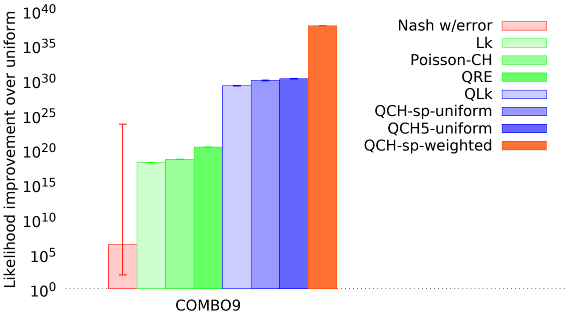

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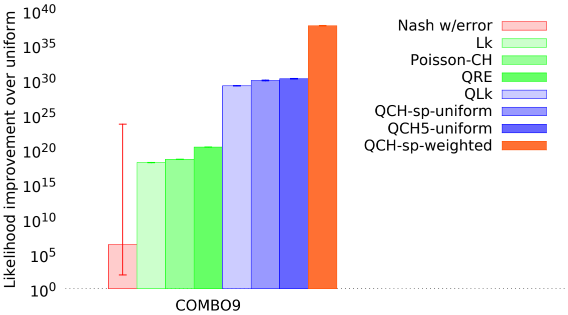

- Weighted linear meta-model for level-0 agents dramatically improved the performance of iterative models.
- Strong evidence for the existence of level-0 agents.
  - For any meta-model, including uniform!
  - Contrary to conventional wisdom.

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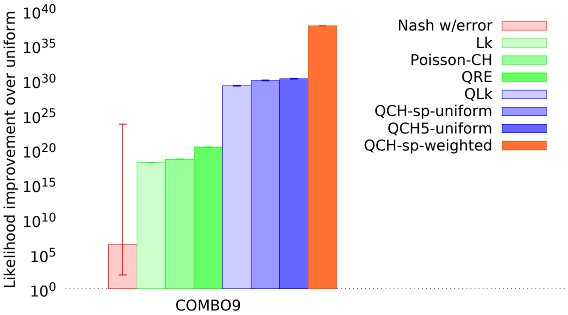

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