Revenue Optimization in the Generalized Second-Price Auction

Kevin Leyton-Brown

joint work with **David R. M. Thompson**
To appear at EC'13
{daveth, kevinlb}@cs.ubc.ca, University of British Columbia

April 18, 2013; Dagstuhl

Introduction

Despite years of research into novel designs, search engines have held on to (quality-weighted) GSP.

Question

How can revenue be maximized within the GSP framework?

Various (reserve price; squashing) schemes have been proposed.

We do three kinds of analysis:

- theoretical: single slot, Bayesian
- computational, perfect information: enumerate all pure equilibria; consider best and worst
- computational: consider the equilibrium corresponding to a DS truthful mechanism with the appropriate allocation rule

Outline

Modeling advertisers

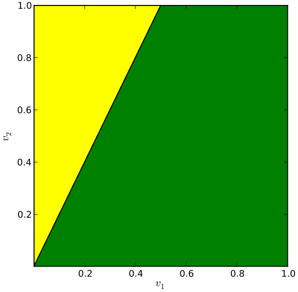
Definition (Varian's model [Varian 07])

Each advertiser i has a valuation v_i per click, and quality score q_i. In position k, i's ad will be clicked with probability $\alpha_k q_i$, where α_k is a position-specific click factor.

"Vanilla" GSP

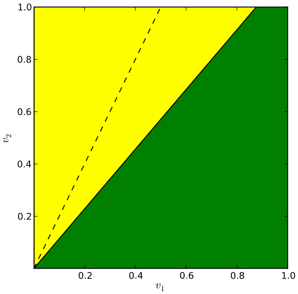
- rank by $b_i q_i$, charge lowest amount that would preserve position in the ranking.

1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$:



GSP with Squashing

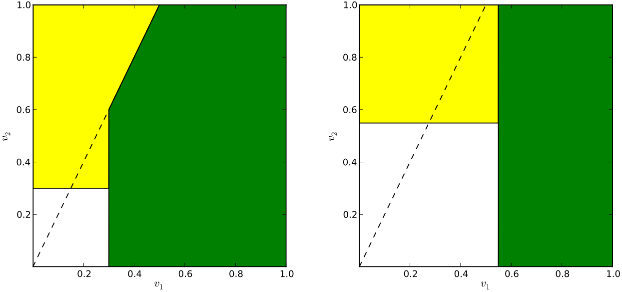
- rank by $b_i(q_i)^s$, $s \in [0, 1]$ [Lahaie, Pennock 07].
  - $s = 1$: vanilla GSP
  - $s = 0$: no quality weighting
- used in practice by Yahoo!, according to media reports



1 slot, 2 bidders, quality scores $q_1 = 1$ *and* $q_2 = 0.5, s = 0.19$.

GSP with unweighted reserves (UWR)

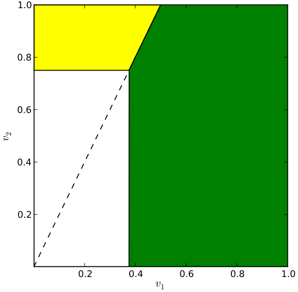
- Vanilla GSP with global minimum bid and payment of r
  - UWR was common industry practice; now replaced by QWR.



1 slot, 2 bidders, quality scores $q_1 = 1$ *and* $q_2 = 0.5, r = .549.$

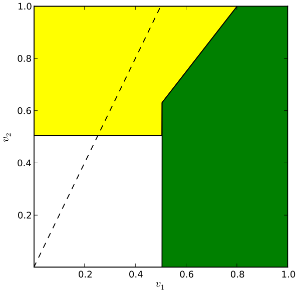
GSP with quality-weighted reserves (QWR)

- Vanilla GSP with per-bidder minimum bid and payment r/q_i
  - UWR was common industry practice; now replaced by QWR.



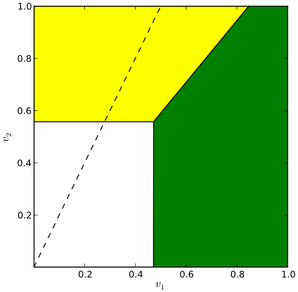
1 slot, 2 bidders, quality scores $q_1 = 1$ *and* $q_2 = 0.5, r = .375.$

GSP with unweighted reserves and squashing (UWR+sq)



1 slot, 2 bidders, quality scores $q_1 = 1$ *and* $q_2 = 0.5, r = .505, s = 0.32$.

GSP: quality-weighted reserves and squashing (UWR+sq)



1 slot, 2 bidders, quality scores $q_1 = 1$ *and* $q_2 = 0.5, r = .472, s = 0.24.$

Our main findings

Considering Varian's valuation model, our main findings:

- QWR is consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring: a new GSP variant that is provably optimal in some settings, and does well in others
- first systematic investigation of the interaction between reserve prices and squashing
- first systematic investigation of the effect of equilibrium selection on the effectiveness of revenue optimization

Outline

1. Model and auctions
2. Theoretical analysis, single-slot auctions
3. What happens in the multi-slot case?
4. Equilibria corresponding to DS truthful mechanisms

Revenue-optimal position auctions

- The auctioneer is selling impressions. A bidder's per-impression valuation is $q_i v_i$, where:
  - the auctioneer knows q_i
  - the auctioneer knows the distribution from which v_i comes
- Thus, even if per-click valuations are i.i.d., each bidder has a different per-impression valuation distribution, and the seller knows about those differences.
  - Strategically, it doesn't matter how q's are distributed, because it is impossible for a bidder to participate in the auction without revealing this information.

Optimality of unweighted reserves

Proposition

Consider any one-position setting where each agent i's per-click valuation v_i is independently drawn from a common distribution g. If g is regular, then the optimal auction uses the same per-click reserve price r for all bidders.

Proof.

- Because g is regular, we must maximize virtual surplus.
- i's value per-impression is $q_i v_i$.
- Transforming g into a per-impression valuation distribution f gives: $f(q_i v_i) = g(v_i)/q_i$ and $F(g_i v_i) = G(v_i)$.
- Substituting into the virtual value function gives:

$$\psi_i(q_i v_i) = q_i \left(v_i - \frac{1 - G_i(v_i)}{g_i(v_i)} \right)$$

- Optimal per-click reserve r_i is solution to $\psi_i(q_i r_i) = 0$, which is independent of q_i.

□

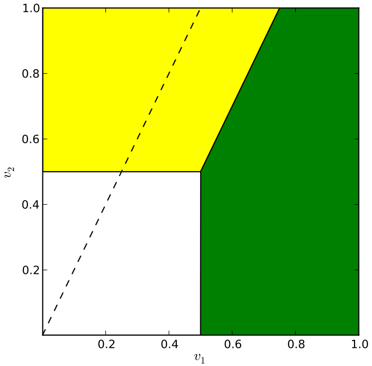
Uniform distribution, single slot

Definition (Anchoring GSP)

Bidders face an unweighted reserve r, and those who exceed it are ranked by $(b_i - r)q_i$.

Proposition

When per-click valuations are drawn from the uniform distribution, anchoring GSP is optimal.



Optimizing GSP variants by grid search: uniform, 2 bidders

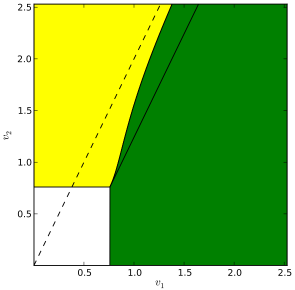
Auction	Revenue ($\pm 1e-5$)	Parameters
VCG/GSP	0.208	—
Squashing	0.255	$s = 0.19$
QWR	0.279	$r = 0.375$
UWR	0.316	$r = 0.549$
QWR+Sq	0.321	$r = 0.472, s = 0.24$
UWR+Sq	0.322	$r = 0.505, s = 0.32$
Anchoring	0.323	$r = 0.5$

- Anchoring's r agrees with [Myerson 81] and QWR's with [Sun, Zhou, Deng 11].
- Optimal parameters for other variants don't correspond to recommendations from the literature.

Optimal auction for the log-normal distribution

Anchoring is not always optimal

(but perhaps it is always a good approximation?)



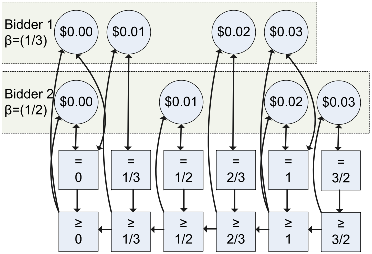
Optimal auction for log normal, 1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$. Anchoring shown for comparison.

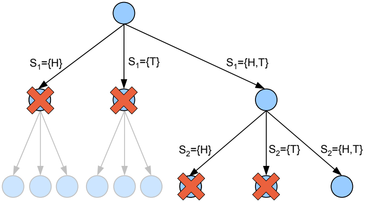
Outline

1. Model and auctions
2. Theoretical analysis, single-slot auctions
3. What happens in the multi-slot case?
4. Equilibria corresponding to DS truthful mechanisms

Computing equilibria

- Action-graph games (AGGs) exploit structure to represent games in exponentially less space than than the normal form [Bhat, LB 04; Jiang, LB 06; Jiang, LB, Bhat 11].
- Games involving GSP and Varian's preference model have such structure [Thompson, LB 09].
- Heuristic tree search can enumerate all pure-strategy Nash equilibria of an AGG [Thompson, Leung, LB 11].

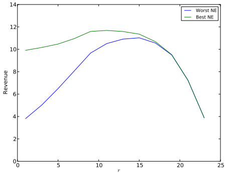




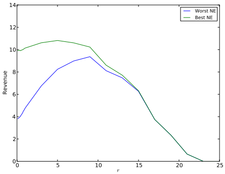
Investigating multiple slots with grid search

- Leverage AGGs to consider more than a single slot, and to examine different equilibria of GSP variants to determine impact of equilibrium selection
  - Sample perfect-information games from the distribution over values and quality scores
    - 5 bidders; 26 bid increments each; 3 slots; uniform valuations
  - enumerate pure-strategy equilibria
  - consider statistics over their best and worst (conservative) NE.
- Identify optimal parameter settings by performing fine-grained grid search.

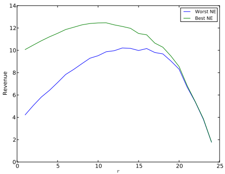
Equilibrium Selection and Reserve Prices



UWR



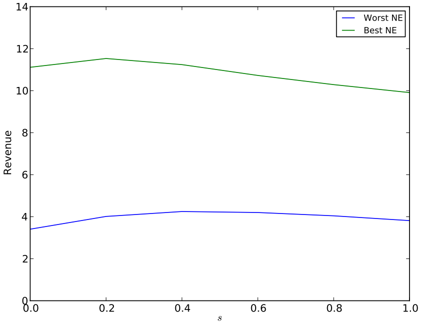
QWR



Anchoring

- Any reserve scheme dramatically improves vanilla GSP's worst-case revenue (look at reserves of $0).
- Optimal unweighted reserves are higher than quality-weighted.
- High bidding can do the work of high reserve prices. Thus, worst-case reserve prices tend to be higher than best case.

Equilibrium Selection and Squashing



- Squashing can improve revenue in best- and worst-case equilibrium. (Recall: $s = 1$ is vanilla GSP.)
- Smaller impact, lower sensitivity than reserve prices.
- Gap between best and worst is consistently large ($\sim 2.5\times$).

Comparing variants optimized for best/worst case

Auction	Revenue
Vanilla GSP	3.814
Squashing	4.247
QWR	9.369
Anchoring	10.212
QWR+Sq	10.217
UWR	11.024
UWR+Sq	11.032

Worst-case equilibrium

Auction	Revenue
Vanilla GSP	9.911
QWR	10.820
Squashing	11.534
UWR	11.686
Anchoring	12.464
QWR+Sq	12.627
UWR+Sq	12.745

Best-case equilibrium

- Worst case: 2-way tie (UWR+Sq, UWR)
- Best case: 3-way tie (UWR+Sq, QWR+Sq, Anchoring)
- UWR's worst case is better than QWR's best case.

Outline

1. Model and auctions
2. Theoretical analysis, single-slot auctions
3. What happens in the multi-slot case?
4. Equilibria corresponding to DS truthful mechanisms

Equilibrium Selection

With vanilla GSP, it's common to study the equilibrium that leads to the efficient (thus, VCG) outcome. Many reasons why this is an interesting equilibrium:

- Existence, uniqueness, polytime computability [Aggarwal et al 06]
- Envy-free, symmetric, competitive eq [Varian 07; EOS 07]
- Impersonation-proof [Kash, Parkes 12]
- Doesn't predict that GSP gets more revenue than Myerson ("Non-contradiction criterion") [ES 10]

Analogously, can compute the equilibrium corresponding to a DS truthful mechanism with the appropriate allocation rule.

- see previous analyses of squashing [LP 07] and reserves [ES 10].

Distributions

For these experiments, we used two distributions:

- **Uniform** v_i's drawn from uniform $(0, 25)$; q_i's drawn from uniform $(0, 1)$.
- **Log-Normal** q_i's and v_i's drawn from log-normal distributions; q_i positively correlated with v_i by Gaussian copula. (Similar to [LP07]; new parameters based on personal communication.)

We compute equilibrium following recursion of [Aggarwal et al 06]. We optimize parameters by grid search.

Revenue across GSP variants, optimal parameters

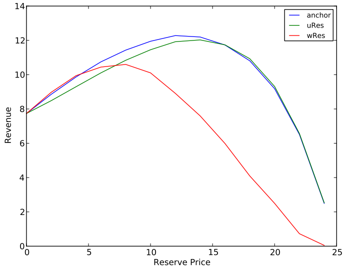
Auction	Revenue
VCG	7.737
Squashing	9.123
QWR	10.598
UWR	12.026
QWR+Sq	12.046
Anchoring	12.2
UWR+Sq	12.220

Uniform distribution

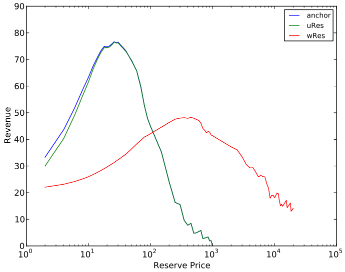
Auction	Revenue
VCG	20.454
QWR	48.071
Squashing	53.349
QWR+Sq	79.208
UWR	80.050
Anchoring	80.156
UWR+Sq	81.098

Log-Normal Distribution

Reserve Prices



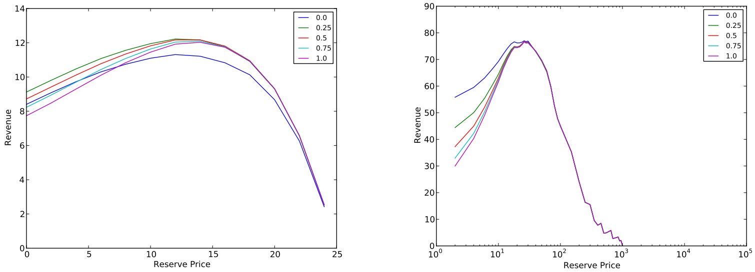
Uniform Distribution



Log-Normal Distribution

- All three reserve-based variants (anchoring, QRW and UWR) provide substantial revenue gains (compare to reserve 0).
- Anchoring very slightly better than UWR; both substantially better than QWR.

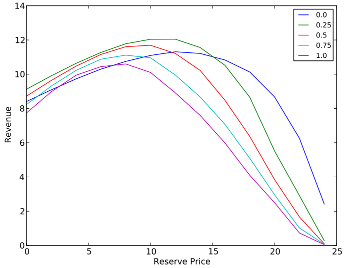
Squashing + UWR



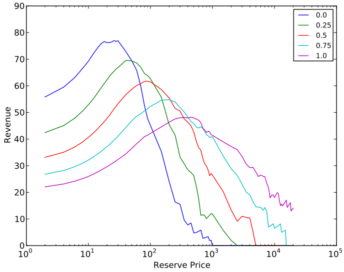
Uniform Distribution   Log-Normal Distribution

- Adding squashing to UWR provides small marginal improvements (compare to $s = 1$) and does not substantially affect the optimal reserve price.

Squashing + QWR



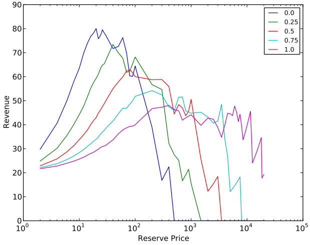
Uniform Distribution



Log-Normal Distribution

- Adding squashing to QWR yields big improvements (compare to $s = 1$); high sensitivity.
- But, the higher the squashing power ($s \to 0$), the less reserve prices are actually weighted by quality.
- Log-normal: optimal parameter setting ($s = 0.0$) removes quality scores entirely and is thus equivalent to UWR.

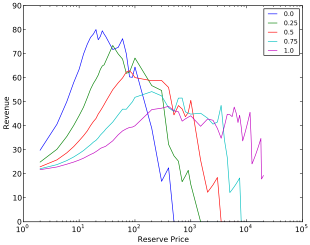
Does squashing help QWR via reserve or ranking?



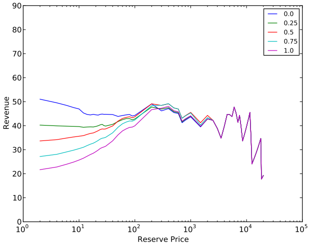
Squashing applied to reserve only
(log normal)

- Applying squashing only to reserve prices can dramatically increase QWR's revenue (compare to $s = 1$).
  - However, there has to be a lot of squashing (i.e., s close to 0)
  - optimal reserve is very dependent on squashing power
  - optimal parameter setting is $s = 0$: identical to UWR

Does squashing help QWR via reserve or ranking?



Squashing applied to reserve only (log normal)

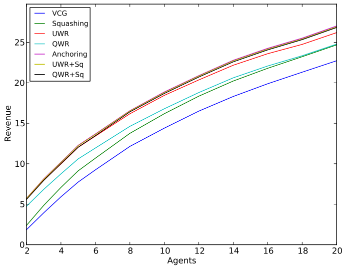


Squashing applied to ranking only (log normal)

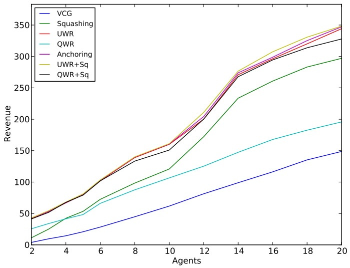
- Applying squashing only to reserve prices can dramatically increase QWR's revenue (compare to $s = 1$).
  - However, there has to be a lot of squashing (i.e., s close to 0)
  - optimal reserve is very dependent on squashing power
  - optimal parameter setting is $s = 0$: identical to UWR
- Applying squashing only to ranking, the marginal gains from squashing over QWR (with optimal reserve) are very small.

Scaling

Because equilibrium computation is cheap, we can scale up.



Uniform Distribution



Log-Normal Distribution

- Top 4 mechanisms are still nearly tied. Squashing and QWR are consistently below.
- As n increases, squashing gains on QWR.
- For log normal, squashing substantially outperforms QWR.

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Why do search engines prefer QWR to UWR? Possible explanations:

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Why do search engines prefer QWR to UWR? Possible explanations:

- Whoops—they should use UWR.

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Why do search engines prefer QWR to UWR? Possible explanations:

- Whoops—they should use UWR.
- Analysis should consider long-run revenue

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Why do search engines prefer QWR to UWR? Possible explanations:

- Whoops—they should use UWR.
- Analysis should consider long-run revenue
- Analysis should consider cost of showing bad ads

Conclusions

We optimized revenue in GSP-like auctions under Varian's valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
- Squashing helps both UWR and QWR.

Why do search engines prefer QWR to UWR? Possible explanations:

- Whoops—they should use UWR.
- Analysis should consider long-run revenue
- Analysis should consider cost of showing bad ads
- Actually, they do some other, secret thing, not QWR.