Investigating the Viability of Exact Feasibility Testing

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Exact Feasibility Testing

• **Given:** a subset of American TV stations

• **Ask:** can they be packed into a reduced set of channels (e.g., UHF 14-30)?
  – Must respect all interference constraints
  – Must introduce no additional simplifying assumptions (“exact”)

• **Goal:** obtain a correct yes/no answer to this question within a reasonable amount of time
Interference Constraints

• **Pairwise interference:** prohibit channel assignments in which interference between any pair of stations exceeds 0.5% of served population (NPRM “Option 2”)
  
  – **Short spacing:** pairs of stations now interfering above 0.5% can continue to cause the same pairwise interference

• **Land mobile operations:** restricted joint channel assignments for stations broadcasting from given tower pairs

• **Border constraints:** protected channels near Canadian, Mexican borders

We’re developing software to output “problem instances” (sets of stations + constraints) in flat, human-readable form.
Satisfiability Testing

• Given a propositional logic formula, does there exist an assignment of (true/false) values to its variables that makes the formula true?

• E.g., a formula with 4 variables and 2 “clauses”:

\[(v_1 \lor \neg v_2 \lor v_4) \land (\neg v_1 \lor \neg v_3 \lor v_4)\]

\[ [v_1, v_2, v_3, v_4] = [true, true, false, false] \]
Encoding Station Packing as SAT

One variable $v_{i,j}$ for each station $i$ and channel $j$

Each station $i$ is assigned some channel:

$$\left(v_{i,14} \lor \cdots \lor v_{i,30}\right) \ \forall i$$

No station $i$ is assigned two channels $k \neq l$:

$$\left(\neg v_{i,k} \lor \neg v_{i,l}\right) \ \forall k, l$$

A pair of stations $i, j$ are not given a forbidden joint channel assignment $k, l$:

$$\left(\neg v_{i,k} \lor \neg v_{j,l}\right) \ \forall i, j, \text{constrained } k, l$$
Generating Problem Instances

• We need data to study
  – An academic research project: must rely only on publicly available (non-confidential) information

• Our approach:
  – probability distribution $P$ over stations, probability proportional to population served (a proxy for value)
  – Start with $S = \emptyset$. Then repeatedly:
    • sample a station $i$ from $P$ without replacement
    • check feasibility of packing $S \cup \{i\}$ into UHF 14-30
      – 30 minute cutoff
    • if proven feasible, $S \leftarrow S \cup \{i\}$
  – Result: a dataset of problem instances
Is Exact Feasibility Checking Feasible?

• Enormous SAT instances
  – 10,000s of variables; 100,000s of constraints
  – Are they solvable within a reasonable amount of time?

• I’ll report on a research project investigating this question. I’d like to acknowledge:

Alexandre Fréchette
Comparing SAT Solvers
Picosat in more detail

Instance Size versus Satisfiability and Runtime for picosat

Number of Unfixed Stations (out of 1830)

Log Runtime

1 sec

30 min

1 min

1 sec
Automated Algorithm Configuration

• Many design choices are faced in the implementation of a heuristic algorithm
  – exposed by an algorithm designer as parameters

• A decade-long focus of my research group: automated algorithm configuration
  – replace human design effort with machine time
  – achieve better performance

• We used SMAC [Hutter, Hoos & Leyton-Brown, 2011]
  – a Bayesian optimization method
Automatic Configuration

![Graph showing runtime distribution]

- 1 sec
- 1 min
- 30 min
- <30 min
- <100 sec
- <10 sec
- <1 sec
- <0.1 sec
- <0.01 sec

Fraction of instances with At Most Runtime

Log Runtime

Capped
Automatic Configuration
Ongoing Research

• Longer, more exhaustive configuration runs
• Configuring additional solvers
• New datasets
  – same heuristic; stronger solver, more machine time
  – based on more realistic simulations
• Iterative SAT solving
• Algorithm portfolios
  – initial investigation: 2× speedup
  – could be much stronger by leveraging less similar algorithms (e.g., DAC’s feasibility checker)