# My Research:

# Algorithms for Making Good Decisions



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# My Field

- Computer Science
  - Hub and spokes model: CS as an enabling ingredient for a wide variety of interdisciplinary projects
  - My own interdisciplinary connections: Microeconomic Theory, Math, Operations Research, Philosophy, Statistics, Cognitive Science
- Artificial Intelligence
  - Getting computers to do things that previously, only people could do
- Computer Science Theory
  - Mathematical underpinnings of computer science, particularly in the design and analysis of algorithms
- My own work: Algorithms for making good decisions
  - Game theory: decisions depend on what other actors will do
  - Empirical Algorithmics: algorithms that work well in practice

#### 

## **REASONING ABOUT LARGE GAMES**

## **Game Theory**

- Mathematical study of interaction between self-interested, rational agents
- Game
  - players/agents
  - actions
  - payoffs
- Strategies:
  - pure strategy: picking a single action
  - mixed strategy: randomizing over actions



## Game Theory



 Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?

$$C \qquad D$$
  
if  $C \qquad -1, -1 \qquad -4, 0$   
if  $D \qquad 0, -4 \qquad -3, -3$   
if  $C \qquad 0$  ms for defective

- Consider this situatic (
  - both use a correct in
  - one correct, one def
  - both defective: both \_

## **Analyzing Games**

- TCP backoff game is a Prisoner's Dilemma
  - both players have a dominant strategy: defective
    - if player 2 plays C, D is player 1's best response
    - if player 2 plays D, D is player 1's best response
    - likewise for player 2
- Not all games are so simple to analyze
  - the best thing for one player to do can depend on what the other player does
    - rock-paper-scissors
    - poker
- What can we say about such games?

$$C$$
  $D$ 



## **Game Theory**

- Key insight:
  - don't just think about single players' actions
  - find strategy profiles where all players
     simultaneously play best responses



- Such a strategy profile is called a Nash equilibrium
  - at least one Nash equilibrium exists in every finite game
    - as long as agents are allowed to randomize their strategies
  - best known algorithms for finding Nash equilibrium require exponential time

## The Kind of Games Often Studied

- The analysis of such 2 x 2 games has proven surprisingly interesting, and has had a profound impact both on our understanding of strategic situations and popular culture
  - e.g., google "dark knight game theory" or "strangelove game theory"





## The Kind of Games We'd Like to Study

- When we use game theory to model real systems, we'd like to consider games with more than two agents and two actions
- Some examples of the kinds of questions we would like to be able to answer:
  - How will heterogeneous users route their traffic in a network?
  - How will advertisers bid in a sponsored search auction?
  - Which job skills will students choose to pursue?
  - Where in a city will businesses choose to locate?
- Most GT work is analytic, not computational
- What's holding us back?
  - the size of classical game representations grows exponentially in the number of players
    - this makes all but the simplest games infeasible to write down
  - even when games can be represented, the best algorithms tend to have worst-case performance exponential in the game's size









## **Compact Representations**

#### Research program for advancing the computational analysis of games:

- 1. find representations that can encode games of interest in **exponentially-less space** than the normal form
- 2. find **efficient algorithms** for working with these representations
- Action Graph Games: compactly represent games exhibiting context-specific independence, anonymity or additive structure
- Generalizes all major, existing compact representations of simultaneous-move games
- Fast algorithms for computing quantities of interest
  - Nash equilibrium, correlated equilibrium, pure-strategy Nash equilibrium, others...

Introduction

**Reasoning about Large Games** 

Auction Design and Analysis

**Empirical Algorithmics** 

## **Coffee Shop Game**

Search

Search the map

Find businesses

Get directions



Local

(G)

 $\bigcirc$ 

Local<sup>New!</sup> Web Groups News more » Images category: Coffee Houses

e.g., "hotels in calgary" or "5000 dufferin street, toronto"

Print 🖂 Email 📾 Link to this page

#### Search results for category: Coffee Orith Harbour Park Harbour Map Satellite Hybrid $|\uparrow|$ Houses in this map 99 ∈₩∋ Westin Bayshore Connoisseurs' Coffee ¢ Resort 1075 Georgia Street West, Vancouver, BC + V6E 3C9 ŧ (604) 683-1486 Melriches Coffeehouse 1244 Davie Street, Vancouver, BC V6E 1N3 (604) 689-5282 Canada Hole In The Wall Cappuccino Bar arriott-Vanco Empire Pan Pacific 1030 Georgia Street West, Vancouver, BC andmark Hotel 1A V6E 2Y3 (604) 646-4653 Starbucks Coffee Co English 1055 W Georgia, Vancouver, BC V5K 1A1 Bay Park (604) 685-5882 Five Roses Bakery Cafe 1220 Bute Street, Vancouver, BC V6E 1Z8 (604) 669-8989 Starbucks Coffee Co Sutton Place Nelsor 1095 Howe Street, Vancouver, BC V6Z 1P6 Park (604) 685-7083 incouver Cmt ollege-City Ct Uptown Espresso 808 Nelson Street, Vancouver, BC V6Z 2H2 (604) 689-1920 Orpheum. Theatre Sunset Caffe Artigiano 99A Beach 99 763 Hornby Street, Vancouver, BC V6Z 1S2 (604) 696-9222 99A Skyline Expresso 1/ 900 Howe Street, Vancouver, BC V6Z 2M4 (604) 683-4234 Ave BC Place Stadium Pacific I Ave Farenheit Celsius Coffee Space Centre 1225 Burrard Street, Vancouver, BC V6Z 1Z5 Plaza Of Nations (604) 682-6675 Vanier Park Chicco Dall Oriente \_100@ft ● ©2005 Google Map data ©200999897EQ™ -1504 Robson Street, Vancouver, BC V6G 1C2 öl 200 m

## **Action-Graph Games**

- set of players: want to open coffee shops
- actions: locations where a shop could be opened
- utility: profitability of a location
  - depends only on number of other players who choose same or adjacent location



### **Experimental Results: Representation Size**



Coffee shop game, 5 x 5 grid *NF grows exponentially; AGG grows polynomially* 

### **Experimental Results: Expected Payoff**



Coffee Shop Game, 5 x 5 grid, 1000 random strategy profiles *NF grows exponentially; AGG grows polynomially* 

## **AUCTIONS AND MARKET DESIGN**

## Auctions: why do computer scientists care?

### • Efficient resource allocation

- a core interest of computer science
- auctions solve this problem when agents are self interested
- They're **big** (\$\$\$)
  - and the internet is changing the way they're used

home	pay register s	ign in   services   site map	Start new search Search		
	Sell My eBay	Community Help	Advanced Search		
			Powered By		
Back to home page	Listed in category	: <u>Everything Else</u> > <u>Metaphys</u>	<u>ical</u> > <u>Psychic, Paranormal</u>		
Virgin Mary In Grill	ed Cheese N	IOT A HOAX ! LOOK	& SEE ! Item number: 5535890757		
Bidder or seller of this item? Sign in for your status Email to a friend   Watch this item in My eBay					
Note: This listing is r	estricted to n	re-approved hidders	or huvers only		
Email the seller to be placed	l on the pre-appro	ved hidder/buver list	or buyers only.		
Aug.	Current bid <sup>.</sup>	US \$7.600.00	Seller information		
		Place Bid >	<u>dltdesigns2002</u> ( <u>47</u> 😭)		
	Time left:		Feedback Score: 47		
		3 days 23 hours	Positive Feedback: 96.1%		
		7-day listing Ends Nov-22-04	States		
		17:22:07 PST	Read feedback comments		
Larger Picture	Start time:	Nov-15- 04 17:22:07 PST	Add to Favorite Sellers		
			Ask seller a question		
	History:	<u>4 bids</u> (US \$3,000.00 starting bid)	View seller's other items		
			Safe Buying Tips		
	High bidder:	User ID kept	Financing available NEWI		
		private	No payments until April, and no interest if naid by April		

## Auctions: a key application of game theory

- A broader category than often perceived
- Generally, auctions are markets in which:
  - agents make binding declarations of interest in one or more resources
  - these resources are allocated according to known rules
  - payments to/from agents may be imposed
- Modeled using game theory. Some new wrinkles:
  - infinite action space
  - imperfect information about payoffs (other agents' valuations)
- How do sellers choose the **particular auctions** they do?
  - mechanism design (Nobel prize 2007): "inverse game theory"

## **Second-Price Auctions**

- An auction that might initially seem strange: second-price
  - 1. all bidders submit sealed bids
  - 2. the high bid wins
  - 3. the winner pays the **second-highest** bid amount
- **Theorem:** it is a dominant strategy in a second-price auction to bid your true value for the good.
- Proof:
  - **Case 1:** bidding truthfully **would** make you the high bidder
    - you can't gain by changing your bid
  - Case 2: bidding truthfully would not make you the high bidder
    - you can't gain by changing your bid

## **Second-Price Auctions**

- **Theorem**: it is a dominant strategy in a second-price auction to bid your true value for the good.
- Case 1: bidding truthfully, you're the high bidder



- bid more:
  - no difference (still win, pay same)
- bid less:
  - 1. no difference
  - 2. you lose

## **Second-Price Auctions**

- **Theorem**: it is a dominant strategy in a second-price auction to bid your true value for the good.
- Case 2: bidding truthfully, you're not the high bidder



- bid less:
  - no difference (still lose, pay nothing)
- bid more:
  - 1. no difference
  - 2. you win, pay too much



Help 🕐



Home > Help > Buying > Different Ways of Buying > Bidding > Bidding on eBay

#### eBay Help

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#### **Bidding on eBay**

eBay uses an automatic bidding system to make bidding on auctions more convenient and less timeconsuming for buyers. There is nothing you have to set up in order to bid in this way. When you bid on an auction style listing you will be placing bids using this method. Practice bidding on eBay from this <u>test auction</u>!

#### Here's how bidding on eBay works:

- When you place a bid, you enter the maximum amount you'd be willing to pay for the item. Your maximum amount is kept confidential from other bidders and the seller.
- 2. The eBay system compares your bid to those of the other bidders.
- The system places bids on your behalf, using only as much of your bid as is necessary to maintain your high bid position (or to meet the reserve price). The system will bid up to your maximum amount.
- 4. If another bidder has a higher maximum, you'll be outbid. BUT, if no other bidder has a higher maximum, you win the item. And you could pay significantly less than your maximum price! This means you don't have to keep coming back to re-bid every time another bid is placed.



Campaig Campaign S Traf Traffic E	<ul> <li>Ranking: descending by (quality score) x (bid amount)</li> <li>quality score is click-through rate plus other measures of advertisement relevance</li> </ul>	ed Search Search
<u>« Revise se</u> All estimate vary. To vie Group. <u>Lea</u>	"The AdWords Discounter will charge you the lowest CPC you can be charged while still maintaining your position"	ositions may opriate Ad

Estimated clicks per day: 309 - 445 (at a daily budget of \$100.00)

Maximum CPC: .25	Daily	budget: 100	Get New Estim	ates	
Keywords <del>v</del>	<u>Search</u> Volume	Estimated Avg. <u>CPC</u>	Estimated Ad Positions	Estimated Clicks / Day	Estimated Cost / Day
Search Total		\$0.10 - \$0.23	4 - 6	309 - 445	\$30 - \$100
computer science		\$0.09 - \$0.20	1 - 3	8 - 11	\$1 - \$3
game theory		\$0.08 - \$0.12	1 - 3	2	\$1
insurance		\$0.11 - \$0.25	4 - 6	156 - 235	\$20 - \$60
kevin		\$0.08 - \$0.16	1 - 3	84 - 108	\$7 - \$20
mortgage		\$0.11 - \$0.25	4 - 6	59 - <mark>8</mark> 8	\$7 - \$30
university of british columbia		\$0.08 - \$0.15	1 - 3	1	\$1

Estimates for these keywords are based on clickthrough rates for current advertisers. Some of the keywords above are subject to review by Google and may not trigger your ads until they are approved. Please note that your traffic estimates assume your keywords are approved.

# **Analyzing Ad Auctions**

- Search engines used **different auctions** over the years
  - GFP: Yahoo! and Overture 1997-2002
  - uGSP: Yahoo! 2002-2007
  - wGSP: Google, Microsoft, Yahoo! 2007-present
- Question
  - Is wGSP better than GFP and uGSP?
- Better by what metric:
  - revenue?
  - efficiency?
- Answer this question by representing the ad auction as an AGG, and computing Nash equilibria

## **Analyzing Ad Auctions: Efficiency**



## **Analyzing Ad Auctions: Revenue**





# DESIGNING ALGORITHMS TO WORK WELL IN PRACTICE

- Many **important problems** are computationally hard
  - circuit verification, planning, protein folding, probabilistic inference, vehicle routing, ...
- We need to be able to solve hard problems in practice, even if their worst-case complexity is exponential
  - Luckily, many instances are easy in practice
- Overall research agenda:
  - bypass the theoretical question of worst-case hardness
  - design algorithms that do well on "typical" inputs

## **One Motivating Question**

"How hard is it to solve a given problem in practice, using the best available methods?"

### The best available methods tend

- to offer no interesting theoretical guarantees
- work astoundingly well in practice
- often exhibit exponentially varying performance
   (e.g., milliseconds to days) even on fixed-size problems

## **Our Key Finding**

Even in settings where **formal analysis seems hopeless**:

- algorithms are complex black boxes
- instance distributions are heterogeneous or richly structured
   ...it is possible to apply rigorous statistical methods to
   answer such questions with high levels of confidence.

## **Empirical Hardness Models**

- Predict how long an algorithm will take to run, given:
  - A set of instances D
  - For each instance  $i \in D$ , a vector  $\mathbf{x}_i$  of feature values
  - For each instance  $i \in D$ , a runtime observation  $y_i$
- We want a mapping  $f(x) \mapsto y$  that accurately predicts  $y_i$  given  $x_i$ 
  - This is a **regression** problem
- The amazing thing: this works at all!

## **Examples**



Red Cockaded Woodpecker Habitat, CPLEX



Travelling Salesperson, Concorde

## **Design Patterns**

- It's a lot of work to design new heuristic algorithms
  - Algorithms that do well on instances arising from a given application often perform poorly elsewhere
- Solution: automatic analysis and design patterns
  - general methods for predicting algorithm performance and constructing new algorithms, based on representative sets of "typical" problem instances
  - exchange expensive human expertise for cheap computer time

# Satisfiability (SAT) Solvers

- There are many high performance SAT solvers
  - indeed, for years a biannual international competition has received >20 submissions in each of 9 categories
- However, no solver is dominant
  - different solvers work well on different problems
    - hence the different categories
  - even within a category, the best solver varies by instance



- The idea: an algorithm portfolio, leveraging the power of all available algorithms
- SATzilla:
  - an algorithm portfolio constructed from all available state-of-the-art complete and incomplete SAT solvers
  - it won 5 medals in each of the
     2007 and 2009 SAT competitions



## SATzilla



- Given:
  - training set of instances
  - performance metric
  - candidate solvers
  - portfolio builder (incl. instance features)
- Training:
  - collect performance data
  - portfolio builder learns empirical hardness models
- At Runtime:
  - predict performance
  - select solver

## **Algorithm Design Philosophies**

- Traditional approach
  - Hard-code various design choices
  - Iteratively conduct small experiments to improve the design
- Our approach
  - Make all design options explicit, encoding them as parameters
    - Results in a generalized, highly parameterized algorithm
    - Instantiation produces many different solvers
  - Given a distribution, set the parameters using an automatic algorithm configuration procedure

## **SATenstein?**

- Frankenstein's goal:
  - Create "perfect" human being from scavenged body parts
- **SATenstein's** goal:
  - Create high-performance SAT solvers using components scavenged from existing solvers



## **How does SATenstein work?**



#### Existing Algorithm Components



- Designer creates highlyparameterized algorithm from existing components
- Given:
  - training set of instances
  - performance metric
  - parameterized algorithm
  - algorithm configurator
- Configure algorithm:
  - run configurator on training instances
  - output is a configuration that optimizes metric

## **How does SATenstein work?**



- Designer creates highlyparameterized algorithm from existing components
- Given:
  - training set of instances
  - performance metric
  - parameterized algorithm
  - algorithm configurator
- Configure algorithm:
  - run configurator on training instances
  - output is a configuration that optimizes metric

## **Summary of Results**

- Performance summary:
  - Factor of 70 1300 performance improvement over best challenger on QCP, HGEN, CBMC-SE
  - Factor of 1.4 2 performance improvement over best challenger on SW-GCP, R3SAT and FAC
- Impact on state of the art:
  - in all cases, generated the best SLS algorithm we're aware of
  - for some distributions, our new algorithm is the very best of which we're aware

## **Advantages and Disadvantages**

SATzilla portfolio-based algorithm selection

#### **SATenstein**

algorithm design via automatic configuration





## **Advantages and Disadvantages**

#### SATzilla portfolio-based algorithm selection



# Exploit per-instance variation between solvers using learned runtime models

- **practical:** e.g., won competition medals
- fully automated: requires only cluster time rather than human design effort

#### Key drawback:

- requires a set of strong, relatively uncorrelated candidate solvers
- can't be applied in domains for which such solvers do not exist

## **Advantages and Disadvantages**

#### SATenstein

[KhudaBukhsh, Xu, Hoos, Leyton-Brown, 2009] algorithm design via automatic configuration

- Instead of manually exploring a design space, build a highly-parameterized algorithm and then configure it automatically
- Can find **powerful**, **novel designs**
- But: only produces single algorithms designed to perform well on the entire training set









Starting from a single parameterized algorithm, automatically find a set of uncorrelated configurations that can be used to build a strong portfolio.

## Hydra Procedure: Iteration 1



## Hydra Procedure: Iteration 2



## Hydra Procedure: Iteration 3



## Hydra Procedure: After Termination



## **Performance Summary**

Solver	RAND	HAND	BM	INDU
Best Challenger (of 17)	1128.63	2960.39	224.53	11.89

\* Statistically insignificant performance difference (sign rank test). Hydra's performance was significantly better in all other pairings.

## **Algorithms for Making Good Decisions**

- Reasoning about Large Games: can compute equilibria (etc.) of large game-theoretic interactions by representing them as action-graph games.
- Auction Design and Analysis: game theory can be leveraged to construct protocols that work even if agents aren't cooperative. Computational techniques can help us understand what will happen under a new design.
- Empirical Algorithmics: algorithms that work well in practice
  - Empirical hardness models: predict algorithm behavior
  - SATzilla: use these models to build algorithm portfolios
  - **SATenstein:** solve the design problem using automatic configuration
  - Hydra: design portfolios from a single parameterized algorithm