

Automated Configuration of MIP solvers

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Parameters in Algorithms

Most algorithms have parameters

- ▶ Decisions that are left open during algorithm design
 - numerical parameters (e.g., real-valued thresholds)
 - categorical parameters (e.g., which heuristic to use)
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Prominent parameters in MIP solvers

- ▶ Preprocessing
- ▶ Which type of cuts to apply
- ▶ MIP strategy parameters
- ▶ Details of underlying linear (or quadratic) programming solver

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- ▶ “Experiment with them”
 - Perform manual optimization in 76-dimensional space
 - Complex, unintuitive interactions between parameters
 - **Humans are not good at that**
- ▶ CPLEX automated tuning tool (since version 11)
 - Saves valuable human time
 - Improves performance

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 - Runnable algorithm A , its parameters and their domains
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This paper: application study for MIP solvers

- ▶ Use existing algorithm configuration tool (PARAMILS)
- ▶ Use different MIP solvers (CPLEX, GUROBI, LPSOLVE)
- ▶ Use six different MIP benchmark sets
- ▶ Optimize different objectives (runtime to optimality/MIP gap)

Outline

1. Related work
2. Details about this study
3. Results
4. Conclusions

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- ▶ COMPOSER [Gratch & Dejong, '92; Gratch and Chien, '96]
 - Spacecraft communication scheduling
- ▶ CALIBRA [Diaz and Laguna, '06]
 - Optimized various metaheuristics
- ▶ F-RACE [Birattari et al., '04-present]
 - Iterated Local Search and Ant Colony Optimization
- ▶ PARAMILS [Hutter et al, '07-present]
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 - Main problem: only optimized performance for single instances
 - Only used small subset of 10 CPLEX parameters

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PARAMILS [Hutter et al., AAAI'07 & '09]:

Iterated local search: biased random walk over local optima

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 - typically outperforms BASICILS
 - used in this study

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- ▶ Results
 - Provably never hurts
 - Sometimes substantial speedups

[Hutter et al., JAIR'09]

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	Categorical	45	3–7	
	Integer	18	discretized: 5–7	
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- ▶ Concise bug reports ↪ helped to fix 2 bugs in GUROBI (!)

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Benchmark sets used

Domain	Type	#instances	Citation
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Combinatorial auctions (WDP)	MILP	2 000	[Leyton-Brown et al., '00]
Mixed integer knapsack (MIK) and 3 more ...	MILP	120	[Atamtürk, '03]

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Split benchmarks 50:50 into training and test sets

- ▶ Optimized parameters on the training set
- ▶ Reported performance on the test set
- ▶ Necessary to check for *over-tuning*

Setup of configuration experiments

Perform 10 independent runs of PARAMILS

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Compare test performance of:

- ▶ PARAMILS's configuration $\hat{\theta}^*$
- ▶ Default algorithm settings
- ▶ CPLEX tuning tool
 - GUROBI and LPSOLVE: no tuning tool available

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Minimization of Runtime to Optimal Solution

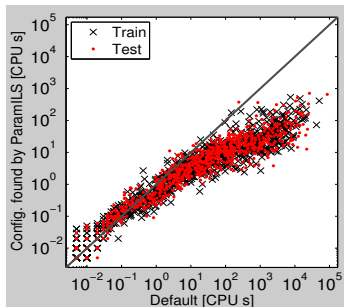
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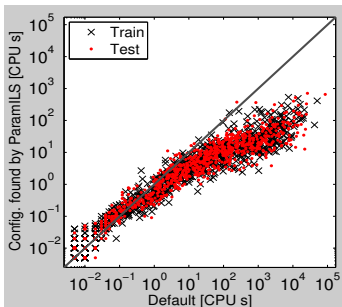
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- ▶ Mean speedup (on test instances)
 - CPLEX 2x to 50x



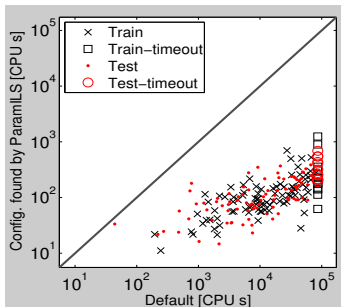
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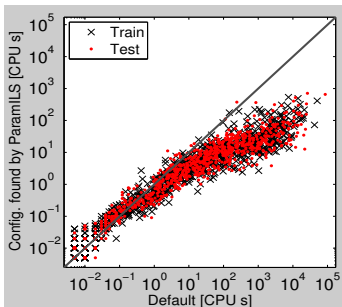
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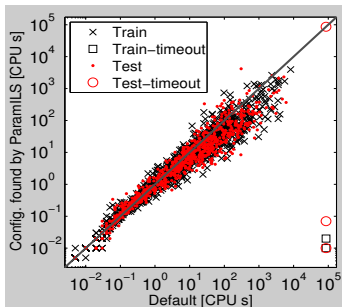
LPSOLVE on WDP instances (150x)

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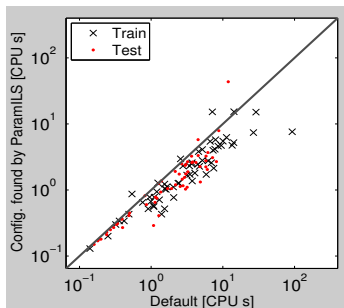
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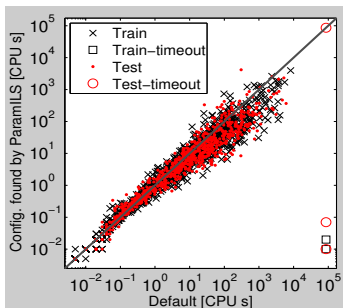
GUROBI on SUST instances (2.3x)

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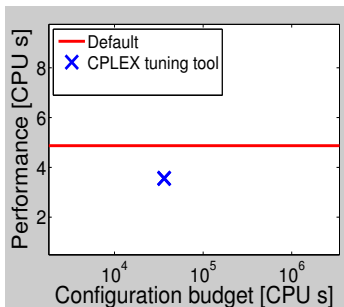
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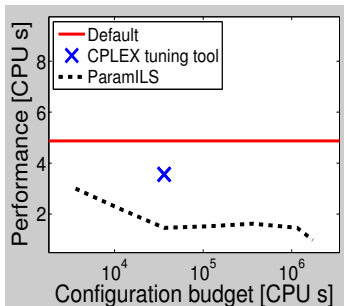
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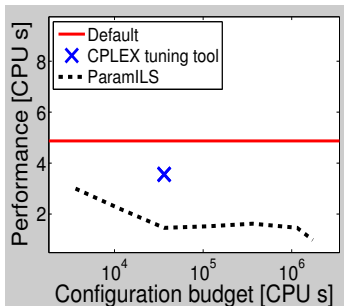
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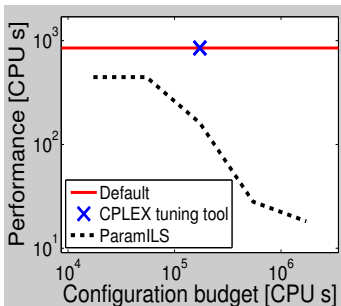
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- ▶ Reduction factors of average optimality gap (on test set)
 - CPLEX 1.3x to 8.6x
 - LPSOLVE 1x (no reduction) to 46x
 - GUROBI 1.1x to 2.2x

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- ▶ CPU time (here: 10×2 days per domain)

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- ▶ Per-instance approaches for heterogeneous benchmarks
 - Given a new unseen instance:
 - + Compute instance characteristics (fast)
 - + Use parameter config. predicted to be best for the instance

Thanks to:

- ▶ Providers of instance benchmark sets
 - Louis-Martin Rousseau
 - Bistra Dilkina
 - Berkeley Computational Optimization Lab
- ▶ Commercial MIP solvers for free full academic license
 - IBM (CPLEX)
 - GUROBI
- ▶ LPSOLVE developers for their solver
- ▶ Compute clusters
 - Westgrid
 - CFI-funded arrow cluster
- ▶ Funding agencies
 - Postdoc fellowship from CBIE
 - MITACS
 - NSERC

Backup slides

Differences to STOP [Baz et al, '09]

Baz et al optimized for single instances

“In practice, users would typically be tuning for a family of related instances rather than for an individual instance”

- ▶ Generalization to *sets* of instances is nontrivial
 - Cannot afford to run all instances for each configuration
 - ↪ FOCUSEDILS adapts # runs per configuration

Further differences

- ▶ Baz et al used older CPLEX version (9.0)
 - defaults improved in newer CPLEX versions
- ▶ Baz et al considered (only) 10 CPLEX parameters
 - and also not all possible values for each parameter
 - in order to improve STOP's performance
 - ↪ requires domain knowledge

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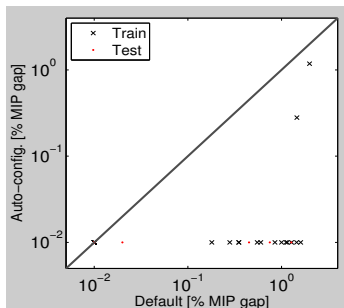
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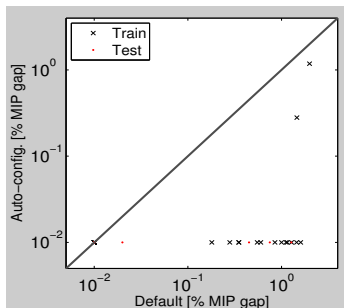
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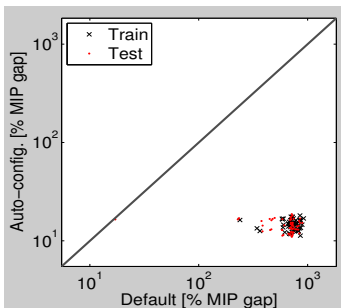
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