# Valuation Uncertainty and Imperfect Introspection in Sealed-bid Auctions

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   - Model
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   - Limited
3. General Results
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# Motivation

- Bidding in sealed-bid, IPV settings

**Example**

Purchasing a used car:

- Evaluating a price is difficult.
- Residual uncertainty

**Example**

Pay-per-click advertising:

- i.i.d. value per click: needs data to learn distribution
- Values can change abruptly.

- How to model this for equilibrium analysis?

# Terms

- Type divides into[1]:
  - payoff type (valuation)
  - belief type (private information)
- Deliberation: action that secretly updates belief type
  - introspection: updates agent's beliefs about their own valuation.
  - strategic deliberation[2]: updates agent's beliefs about other agents valuation.
- Beliefs over time with residual uncertainty:
  - *ex ante*: no private information
  - *ex interim*: one agent's private information
  - *ex post*: all agents' private information
  - *ex interim* perfect: one agent's valuation
  - *ex post* perfect: all agents' valuations

[1] Bergemann and Morris, 2006
[2] Larson and Sandholm, 2001

# Taxonomy

We classify settings with deliberative agents along five major axes:

- Valuation distribution: Are agents' valuations independent?
- Privacy: Can agents discover information about each other's valuations?
- Volatility: Can an agent's valuation change?
  - Perfection: Can an agent buy/consume the good without knowing his valuation?
- Costliness: Do deliberations carry a utility penalty?
- Limitations: When can agents perform deliberations?
  - Separability: Can agents condition their deliberations on information from the mechanism?

# Classifications

| Paper | Values | Private | Perfect | Volatile | Costly | Limits | Separable |
|---|---|---|---|---|---|---|---|
| Cremer *et al* (2003) | **ind** | **yes** | yes | **no** | **yes** | no | no |
| Parkes (2005) | all | **yes** | both | **no** | **yes** | no | no |
| Larson & Sandholm (2005) | **ind** | no | yes | yes | **yes** | no | **yes** |
| Larson (2006) | **ind** | no | yes | **no** | **yes** | no | no |
| Larson & Sandholm (2001) | all | no | **no** | yes | **yes** | no | both |
| Blumrosen & Nisan (2002) | **ind** | **yes** | **no** | **no** | no | **yes** | **yes** |
| Bergemann & Valimaki (2002) | all | * | **no** | **no** | **yes** | no | both |
| Persico (2000) | inter | * | **no** | **no** | **yes** | **yes** | **yes** |
| Compte & Jehiel (2001) | **ind** | **yes** | **no** | **no** | **yes** | no | both |
| Sandholm (2000) | **ind** | **yes** | **no** | **no** | **yes** | no | **yes** |
| Rasmusen (2006) | **ind** | **yes** | **no** | **no** | **yes** | **yes** | no |
| **Our Paper** | **ind** | **yes** | **no** | **no** | **yes** | **yes** | **yes** |

- Our interest: imperfection (due to costs or limits) in separable auctions
- Other assumptions: independent, private, non-volatile, symmetric

# Model

## Definition

Our model is a six-tuple $(N, f, Q, A, p, c)$ where:

- $N$ is the set of all agents.
- Each agent has a valuation $v_i$ drawn from distribution $f$ (which has support on the interval $[\underline{v}, \bar{v}]$).
- $Q$ is the set of possible introspections (from which each agent chooses one, $q_i$). $q_\varnothing$ is no introspection.
- $A$ is the set of possible signals the agent can receive, according to conditional probability distribution $p(a_i|q_i, v_i)$.
- $c(q_i, a_i)$ is the cost of the signal (quasi-linear).

# Properties of model

## Proposition

*If the setting is separable, independent valued and private then risk-neutral agents will bid as though their expected valuation was their exact valuation.*

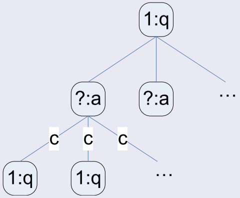
## Proposition

*Our model is without loss of generality regarding some important features:*

- *deliberation costs affected by valuation*
- *multiple deliberations (with limitations)*
- *initial information*

# Properties of model

## Proof Sketch




- *Introspection: policy at every choice node*
- *Signal: leaf node*
- *Signal cost: sum of arc costs*
- *Initial information: Chance moves first*

Larson & Sandholm, 2001

# Outline

# Costly deliberations

**Theorem**

*If deliberations are costly then symmetric, pure-strategy Nash equilibria do not always exist.*

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## Theorem

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## Proof.

- Valuations: $f(v_i = 0) = f(v_i = 1) = 0.5$
- Costly introspection $q^*$: $a_i = v_i$, costs $c$ $(0 < c < 0.25)$

| | $q^*$ | $q_\varnothing$ |
|---|---|---|
| $q^*$ | $.25 - c, .25 - c, .25$ | $.25 - c, .25, .25$ |
| $q_\varnothing$ | $.25, .25 - c, .25$ | $0, 0, .5$ |

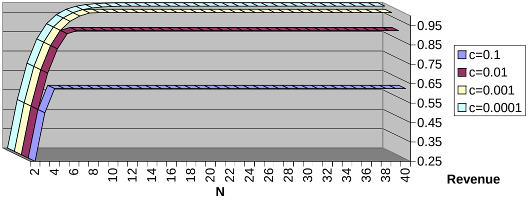
# $\{0,1\}$ valuations, costly, two bidders

| | $q^*$ | $q_\varnothing$ |
|---|---|---|
| $q^*$ | $.25 - c, .25 - c, .25$ | $.25 - c, .25, .25$ |
| $q_\varnothing$ | $.25, .25 - c, .25$ | $0, 0, .5$ |

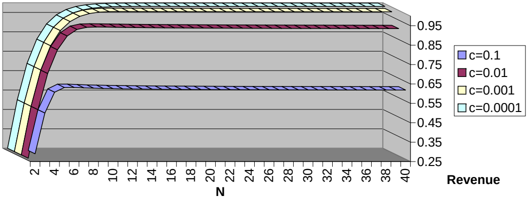
- Revenue differs across Nash equilibria:
  - Constant over a range of $c$ for asymmetric, pure NEs
  - Varying continuously with $c$ for symmetric, mixed NE

# $n$ bidders: revenue

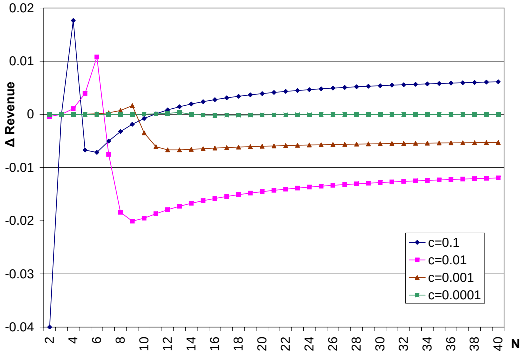
Pure-strategy Nash equilibria:




Symmetric Nash equilibria:



# $n$ bidders: relative revenue

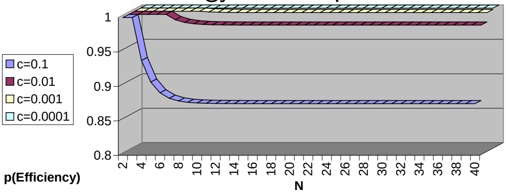



- Pure strategy revenue plateaus.
- Mixed strategy revenue peaks and then declines.

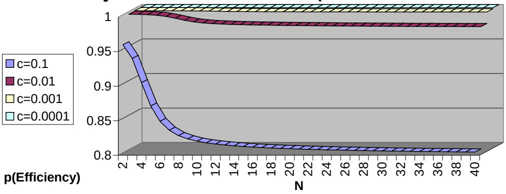
# $n$ bidders: efficiency

- The probability of *ex post* perfect efficiency:

Pure-strategy Nash equilibria:




Symmetric Nash equilibria:



# Simple, limited case, 2-bidders

**Theorem**

*If deliberations are limited then symmetric, pure-strategy Nash equilibria do not always exist.*

# Simple, limited case, 2-bidders

## Theorem

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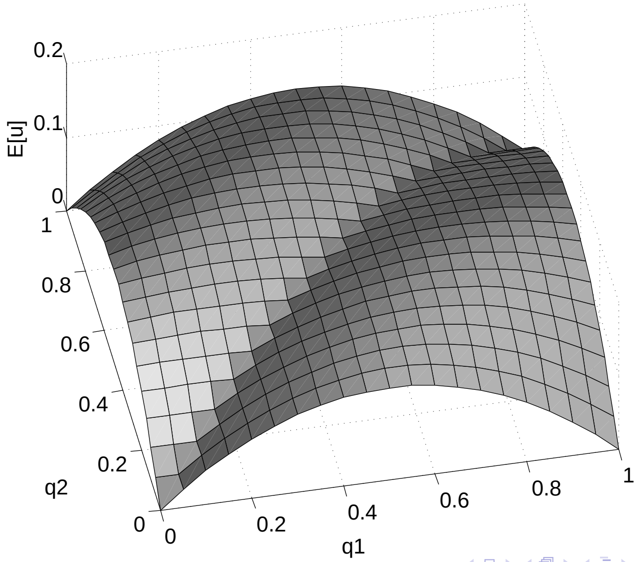
## Proof.

- Valuations: $v_i \sim \text{uniform}[0,1]$
- Limited, free introspection: Is $v_i < q_i$?, $a_i \in \{\text{yes}, \text{no}\}$
- $\mathbb{E}[u_1|q_1, q_2] = \begin{cases} (1-q_1)q_2(1+q_1-q_2)/2 & q_1 < q_2 \\ (1-q_2)q_1(1+q_2-q_1)/2 & o.w. \end{cases}$
- Pure NEs: $[q_1 = 1/3, q_2 = 2/3], [q_1 = 2/3, q_2 = 1/3]$

# Simple, limited case, 2-bidders

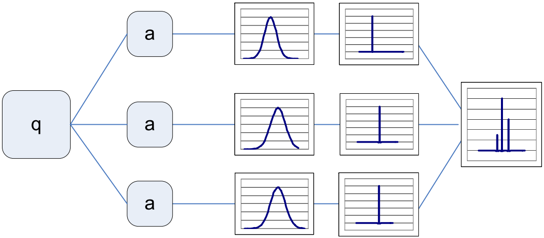


# Outline

# Induced valuation distribution

## Definition (induced valuation distribution)

The induced valuation distribution (IVD) $f_q$ of an introspection $q$ is the distribution agents act as though their valuations were drawn from, given that they chose introspection $q$.



# Theorem: revenue equivalence

**Theorem**

*Under symmetric equilibria, all ex post efficient, separable auctions have equivalent revenue if:*

- *the usual revenue equivalence assumptions hold: independence, privacy, symmetry, risk-neutrality*
- $\forall_{q \in Q}$ *IVD* $f_q$ *is atomless on* $[\underline{v}, \bar{v}]$

# Theorem: revenue bounds

**Theorem**

*In any ex ante individually-rational, separable auction, the expected revenue is bounded above by* $\bar{v} - \sum_i \mathbb{E}[c_i]$.

- Not assumed: independence, privacy, non-volatility, efficiency

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**Theorem**

*In any ex ante individually-rational, separable auction, the expected revenue is bounded above by* $\bar{v} - \sum_i \mathbb{E}[c_i]$.

- Not assumed: independence, privacy, non-volatility, efficiency

**Corollary**

*No ex ante budget-balanced, ex ante individually-rational, separable mechanism can have a dominant strategy which involves an unbounded number of agents performing costly deliberations.*

# Theorem: value of information

## Theorem

*In efficient, separable auctions with independent private values, the value of information for any deliberation policy $q$ falls off exponentially in the number of agents performing it.*

## Proof Sketch

*If we only allow the $k$ agents that perform $q$ to participate, we have symmetric IVDs and standard $\mathbb{E}[u]$ equations apply (i.e. an agent receives the good with probability $F_q^{k-1}(\mathbb{E}[v_i|q_i = q, a_i])$.) If we re-introduce bidders who do not perform $q$, $\mathbb{E}[u_i|q_i = q]$ weakly decreases.*

# Outline

# Future work

- Price of anarchy in separable auctions
- Continuous trade-off between separable and inseparable mechanisms (eg. optimal search)
- Relax assumptions

# Summary

- Taxonomy
- Model (for separable auctions with independent values and privacy)
- Analytic Nash equilibria of second price auctions:
    - Symmetric, pure-strategy NE don't always exist.
    - Revenue and efficiency differ across classes of equilibria.
- Revenue equivalence
- Revenue bound: In the limit, seller pay the costs.
- Value of information: Small costs have strategic impact.