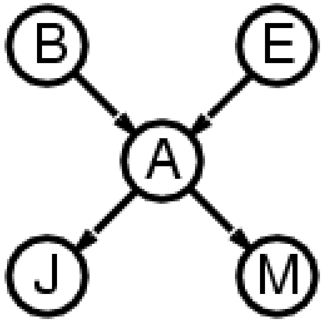
Reasoning Under Uncertainty: More on BNets structure and construction

CPSC 322 Lecture 27

Belief networks Recap

- By considering **causal dependencies**, we order variables in the joint.
- **Apply** chain rule and **simplify**



- **Build a directed acyclic graph** (DAG) in which the parents of each variable *X* are those variables on which *X* directly depends.
- **By construction**, a variable is independent from its non-descendants given its parents.

Belief Networks: open issues

- **Independencies:** Does a BNet encode more independencies than the ones specified by construction? Yes

- **Compactness**: We reduce the number of probabilities from $O(2^n)$ to $O(n 2^k)$

In some domains we need to do better than that!

- Still too many and often there are no data/experts for accurate assessment

Solution: Make stronger (approximate) independence assumptions

Learning Goals for this class

You can:

- Given a Belief Net, determine whether one variable is conditionally independent of another variable, given a set of observations.

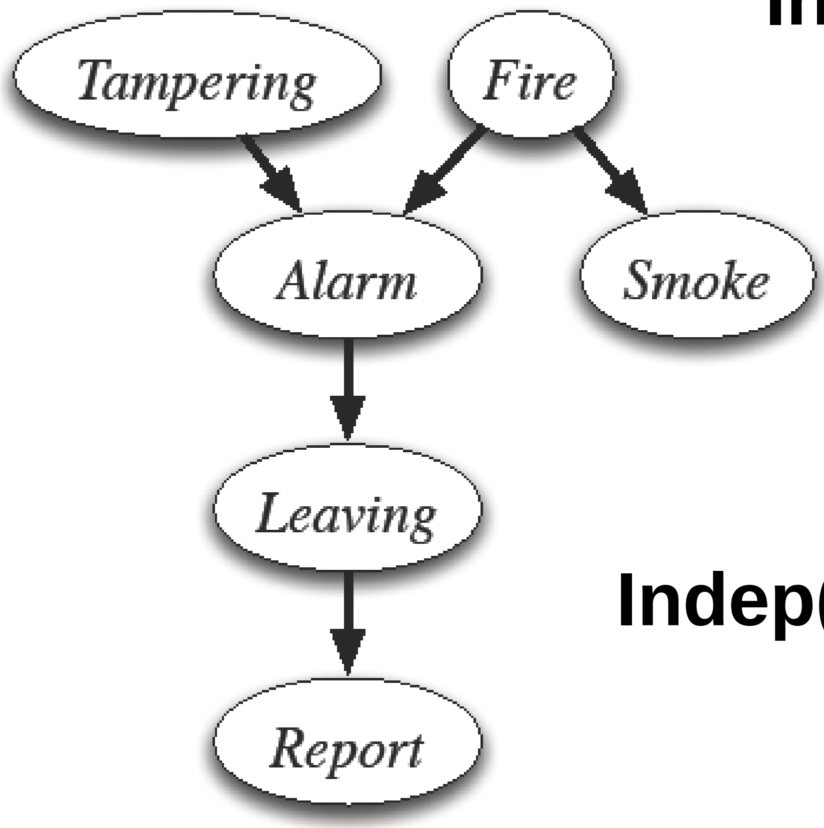
- Define and use **Noisy-OR** distributions. Explain assumptions and benefit.

- Implement and use a **naïve Bayesian classifier**. Explain assumptions and benefit.

Lecture Overview

- Implied Conditional Independence relations in a Bnet
- Compactness: Making stronger Independence assumptions
  - Representation of Compact Conditional Distributions
  - Network structure(Naïve Bayesian Classifier)

Bnets: Entailed (in)dependencies

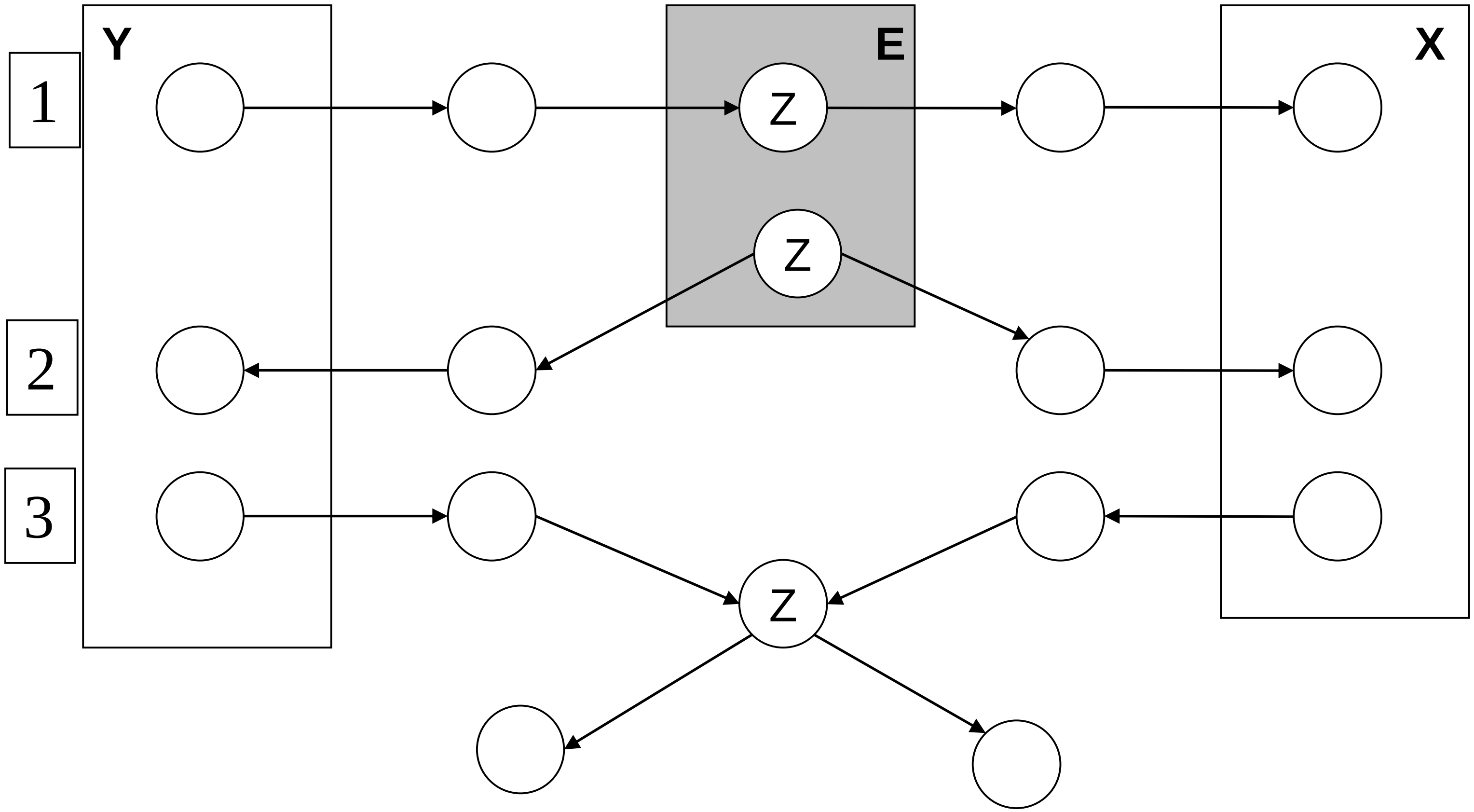


Indep(Report, Fire,{Alarm})?

Indep(Leaving, Smoke,{Fire})?

Conditional Independencies

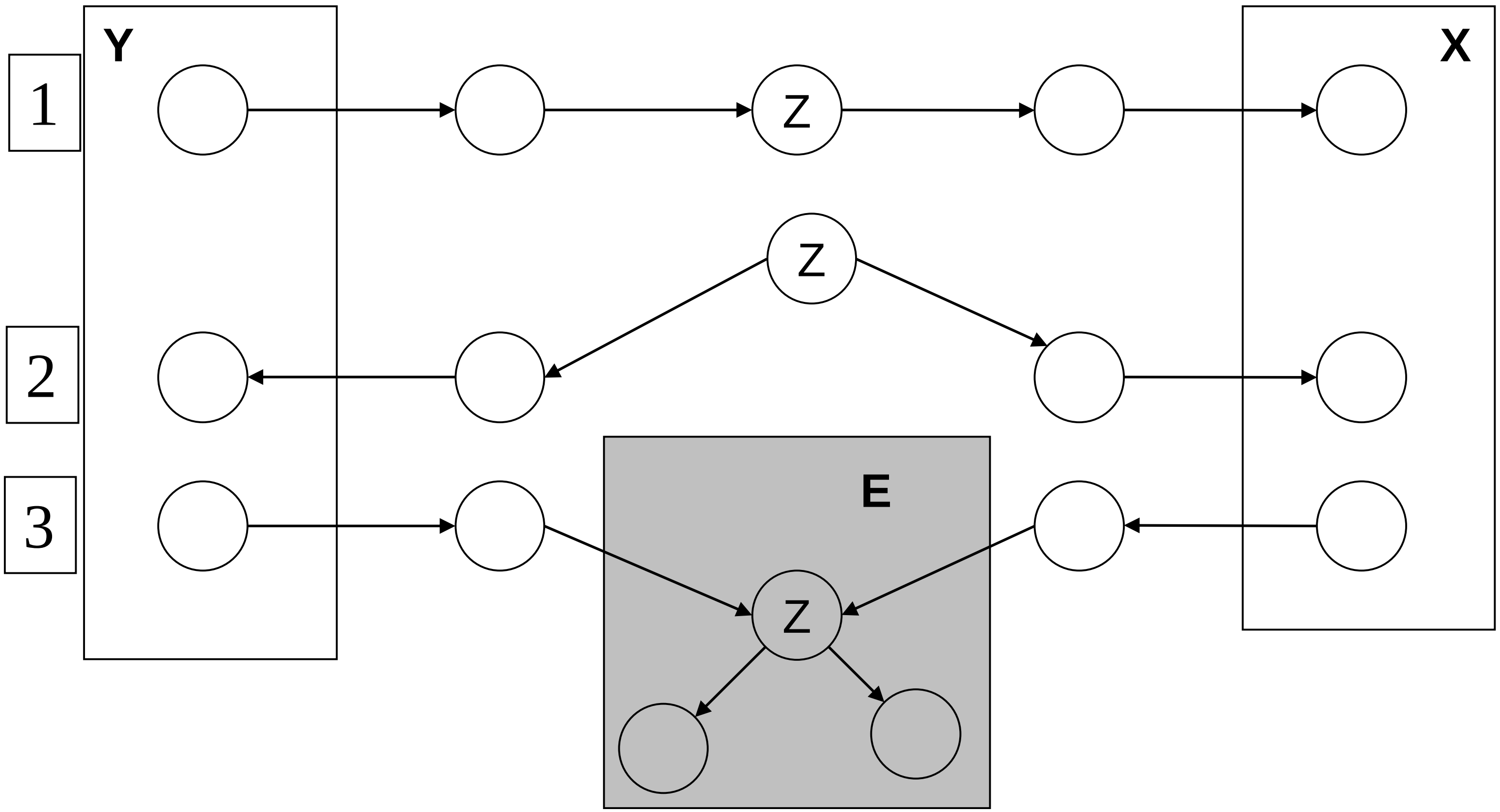
Or, blocking paths for probability propagation. Three ways in which a path between X to Y can be blocked, (1 and 2 given evidence E)



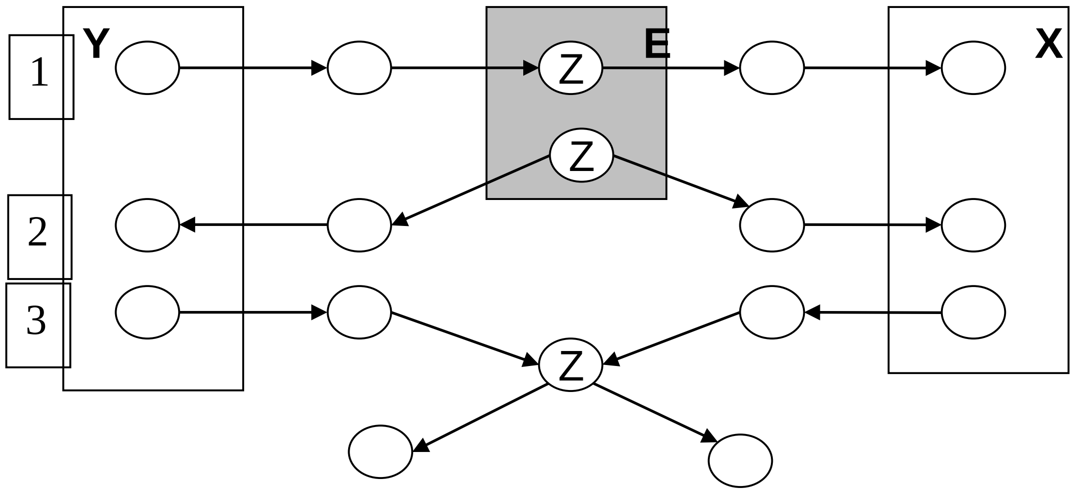
Note that, in 3, X and Y become dependent as soon as I get evidence on Z or on *any of its descendants*

Or ….Conditional Dependencies

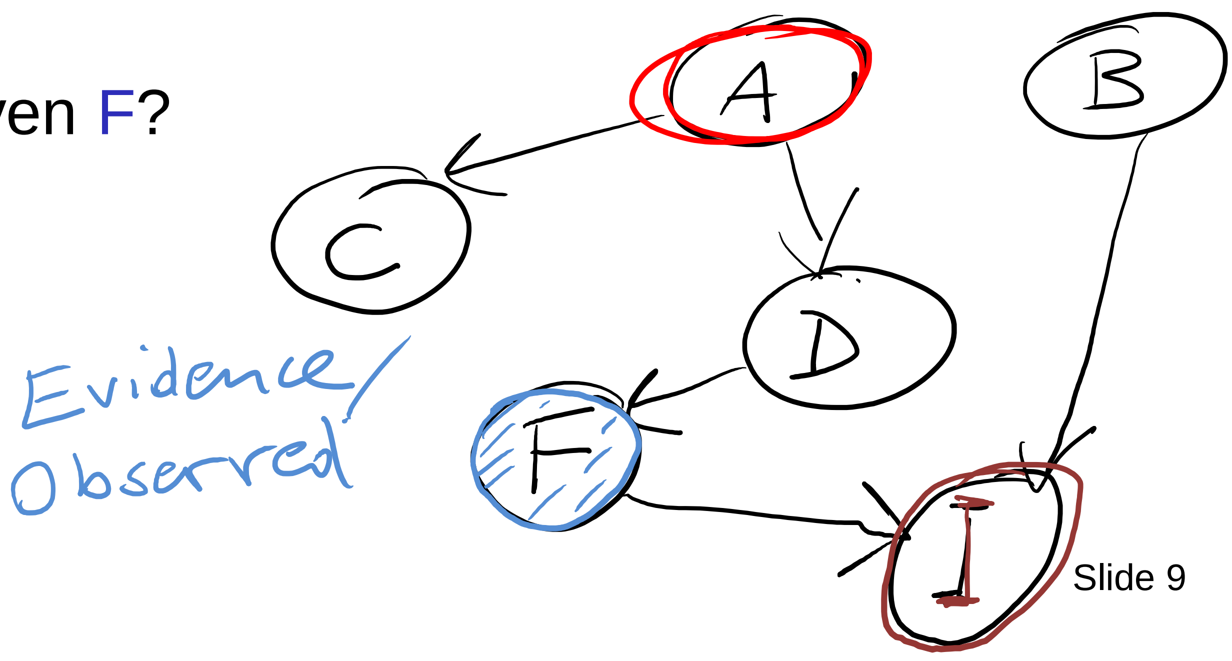
These are cases where X and Y are conditionally **dependent** on each other



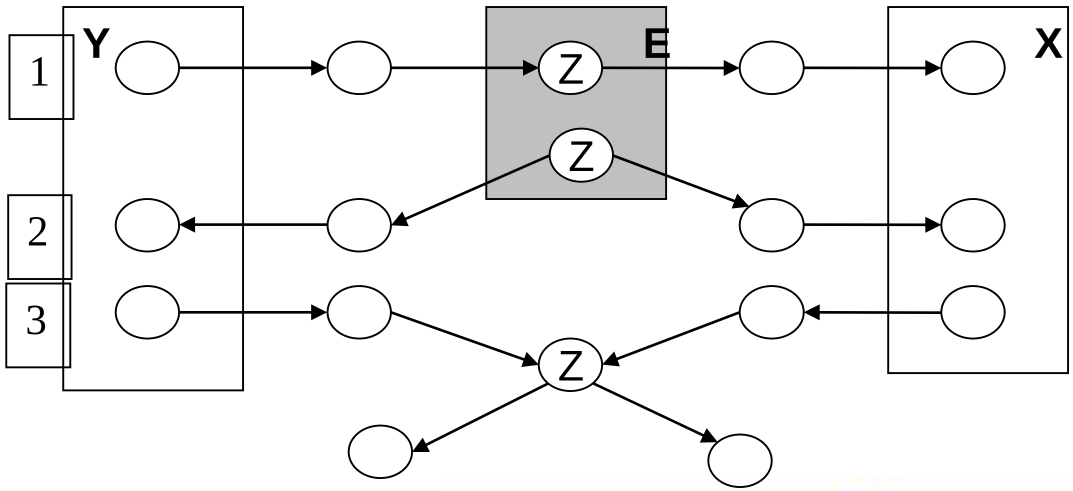
In/Dependencies in a Bnet : Example 1



Is A conditionally independent of I given F?



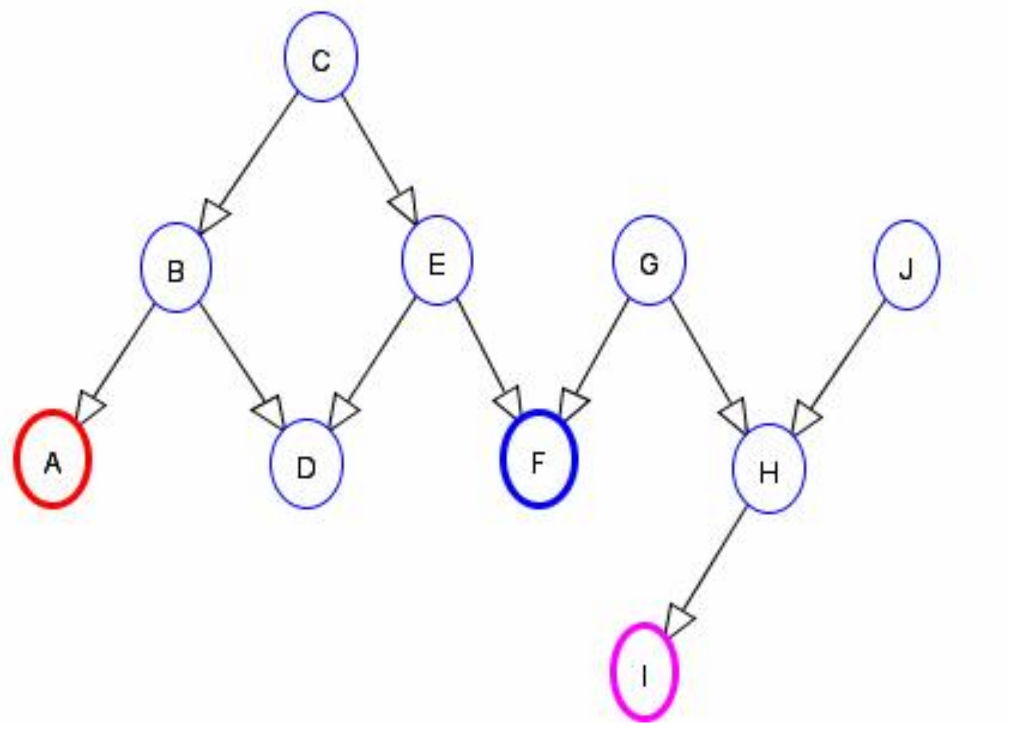
In/Dependencies in a Bnet : Example 2



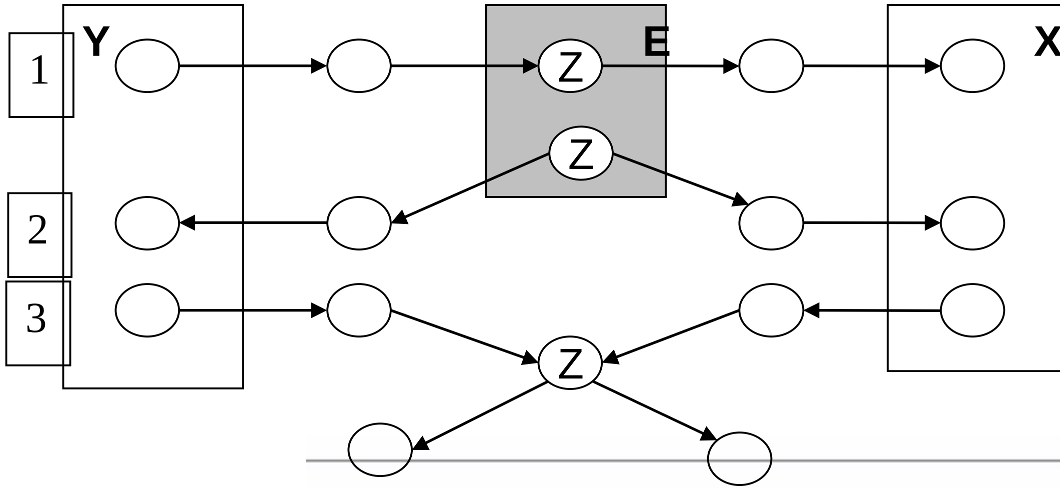
Is A conditionally independent of I given F?

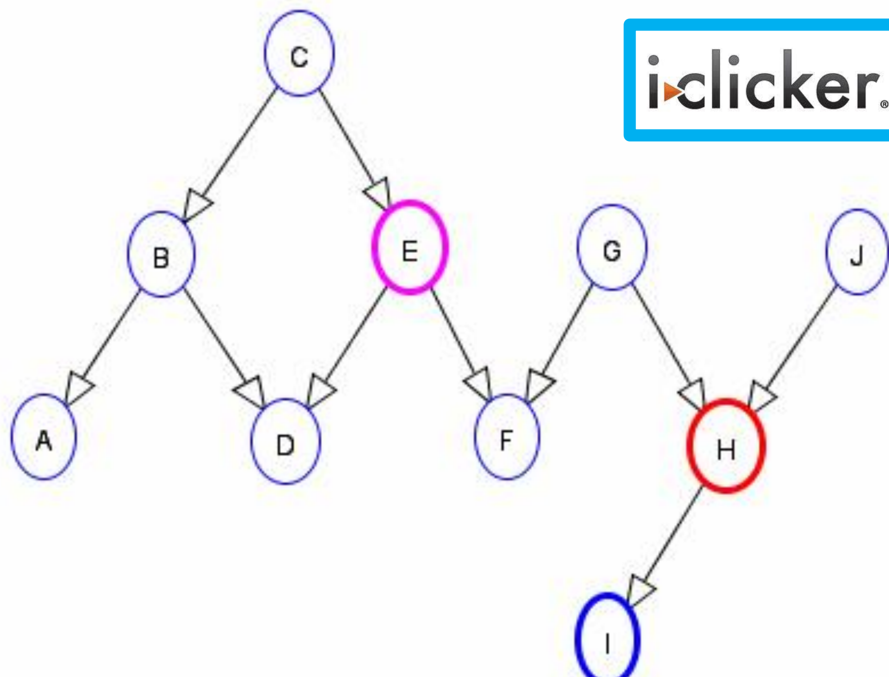
iclicker

A. Yes
B. No
C. My brain hurts



In/Dependencies in a Bnet : Example 3





Is H conditionally independent of E given I?

A. Yes
B. No
C. My brain hurts

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More on Construction and Compactness: Compact Conditional Distributions

Once we have established the topology (structure) of a Bnet, we still need to specify the conditional probabilities

How?

- **From Data**
- **From Experts**

To facilitate acquisition, we aim for compact representations for which data/experts can provide accurate assessments

More on Construction and Compactness: Compact Conditional Distributions

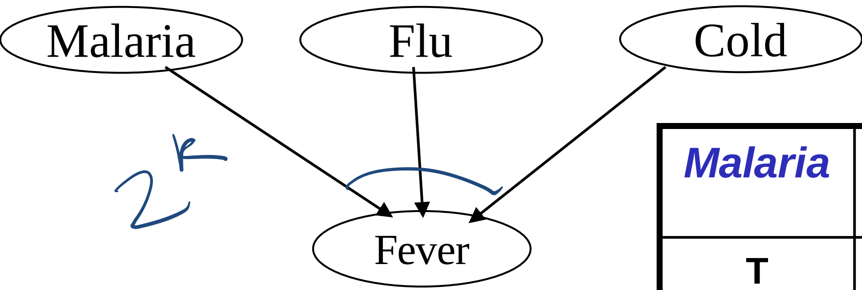
From JointPD 2^n to $n2^k$

But still, CPT grows exponentially with number of parents

In a **semi-realistic model of internal medicine** with **448** nodes and **906** links **133,931,430** values are required!

And often there are no data/experts for accurate assessment

Effect with multiple non-interacting causes



What do we need to specify?

Malaria	*Flu*	*Cold*	*P(Fever=T | ..)*	*P(Fever=F|..)*
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

What do you think data/ experts could easily tell you?

More difficult to get info to assess more complex conditioning….

Solution: Noisy-OR Distributions

- Models multiple non interacting causes
- Logic OR with a probabilistic twist.
  - Logic OR Conditional Prob. Table.

Malaria	*Flu*	*Cold*	*P(Fever=T | ..)*	*P(Fever=F|..)*
T	T	T	1	0
T	T	F	1	0
T	F	T	1	0
T	F	F	1	0
F	T	T	1	0
F	T	F	1	0
F	F	T	1	0
F	F	F	0	1

Solution: Noisy-OR Distributions

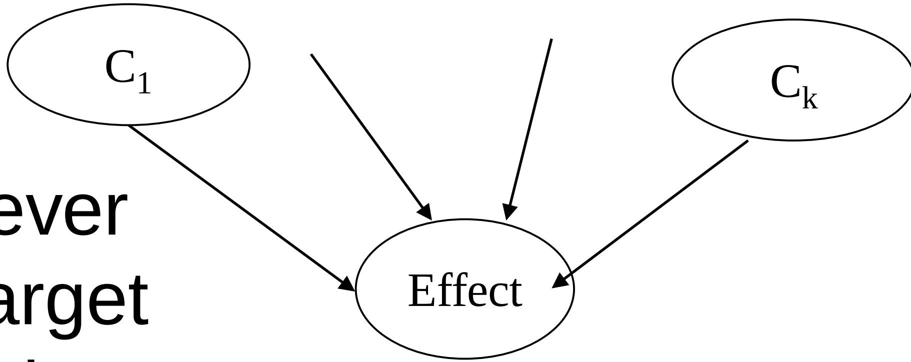
The **Noisy-OR model** allows for **uncertainty in the ability of each cause to generate the effect** (e.g.. one may have a cold without a fever)

Malaria	*Flu*	*Cold*	***P(Fever=T | ..)***	***P(Fever=F|..)***
T	**T**	**T**		
T	**T**	**F**		
T	**F**	**T**		
T	**F**	**F**		
F	**T**	**T**		
F	**T**	**F**		
F	**F**	**T**		
F	**F**	**F**		

Two assumptions:

1. All possible causes are listed
2. For each of the causes, whatever **inhibits** it to generate the target effect is independent from the **inhibitors** of the other causes

Noisy-OR: Derivations



For each of the causes, whatever inhibits it to generate the target effect is independent from the inhibitors of the other causes

Independent Probability of failure q_i for each cause alone:

- P(Effect=F | C_i = T, *and no other causes*) = q_i
- P(Effect=F | C_1 = T,.. C_j = T, C_{j+1} = F,., C_k = F)=

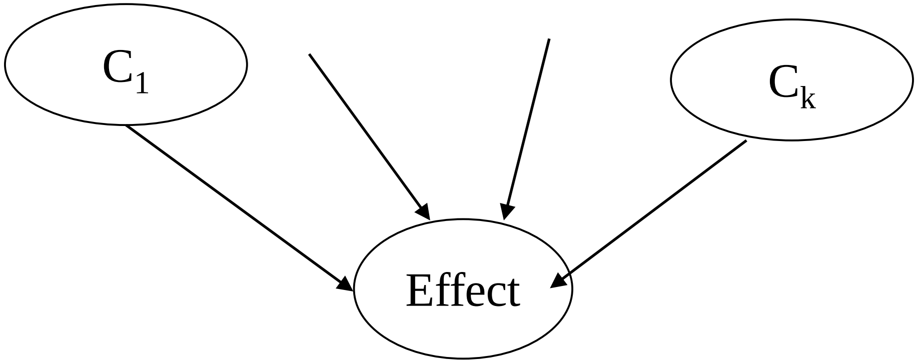
A. $\sum_{i=1}^{J} q_i$

B. $\prod_{i=1}^{J} q_i$

C. $1 - q_i$

D. None of those

Noisy-OR: Derivations



For each of the causes, whatever inhibits it to generate the target effect is independent from the inhibitors of the other causes

Independent Probability of failure q_i for each cause alone:

- P(Effect=F | C_i = T, *and no other causes*) = q_i
- P(Effect=F | C_1 = T,.. C_j = T, C_{j+1} = F,., C_k = F)= $\Pi_i\, q_i$
- P(Effect=T | C_1 = T,.. C_j = T, C_{j+1} = F,., C_k = F) = $1 - \Pi_i\, q_i$

Noisy-OR: Example

P(Fever=F| Cold=T, Flu=F, Malaria=F) = 0.6

P(Fever=F| Cold=F, Flu=T, Malaria=F) = 0.2

P(Fever=F| Cold=F, Flu=F, Malaria=T) = 0.1

Model of internal medicine

133,931,430 → 8,254

- P(Effect=F | C_1 = T,.. C_j = T, C_{j+1} = F,., C_k = F)= $\prod^{j}_{i=1} q_i$

Malaria	*Flu*	*Cold*	***P(Fever=T | ..)***	***P(Fever=F|..)***
T	**T**	**T**		0.1 x 0.2 x 0.6 **= 0.012**
T	**T**	**F**		0.2 x 0.1 **= 0.02**
T	**F**	**T**		0.6 x 0.1**=0.06**
T	**F**	**F**	**0.9**	**0.1**
F	**T**	**T**		0.2 x 0.6 **= 0.12**
F	**T**	**F**	**0.8**	**0.2**
F	**F**	**T**	**0.4**	**0.6**
F	**F**	**F**		**1.0**

- Number of required probabilities linear in …..

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  - **Network structure (Naïve Bayesian Classifier)**

Naïve Bayesian Classifier

A very simple and successful BNet that allows us to classify entities in a set of classes C, given a set of attributes

Example:

- Determine whether an email is spam (only two classes spam=T and spam=F)
- Useful attributes of an email ? *the words in it*

Assumptions

- The value of each attribute depends on the classification
- **(Naïve)** The attributes are independent of each other given the classification

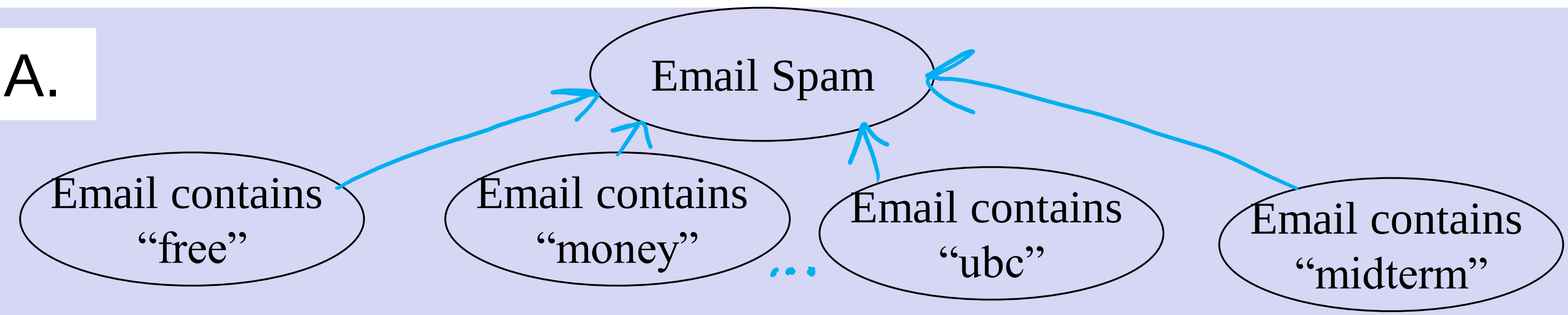
P("bank" | "account" , spam=T) != P("bank" | spam=T)

What is the structure?

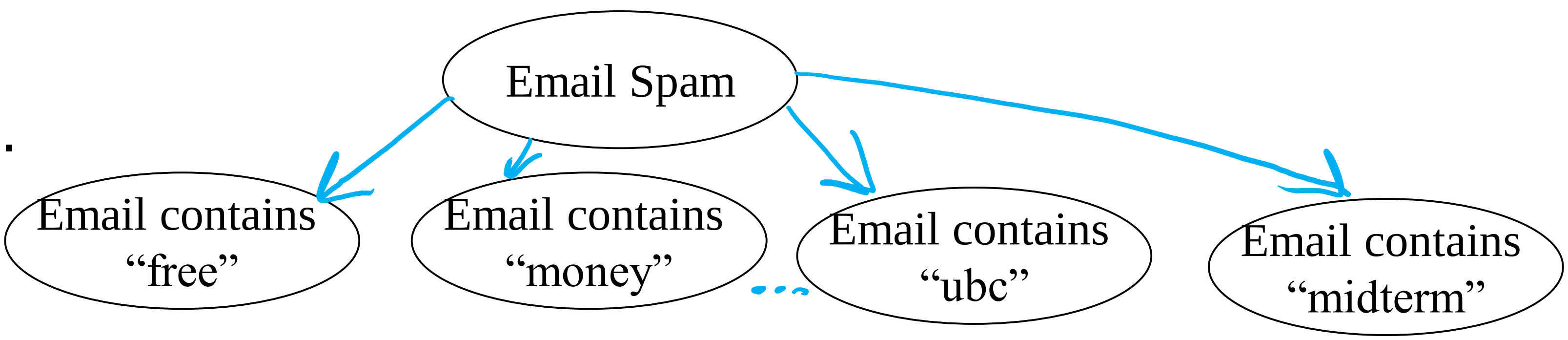
Assumptions

- The value of each attribute depends on the classification
- **(Naïve)** The attributes are independent of each other given the classification

A.



B.

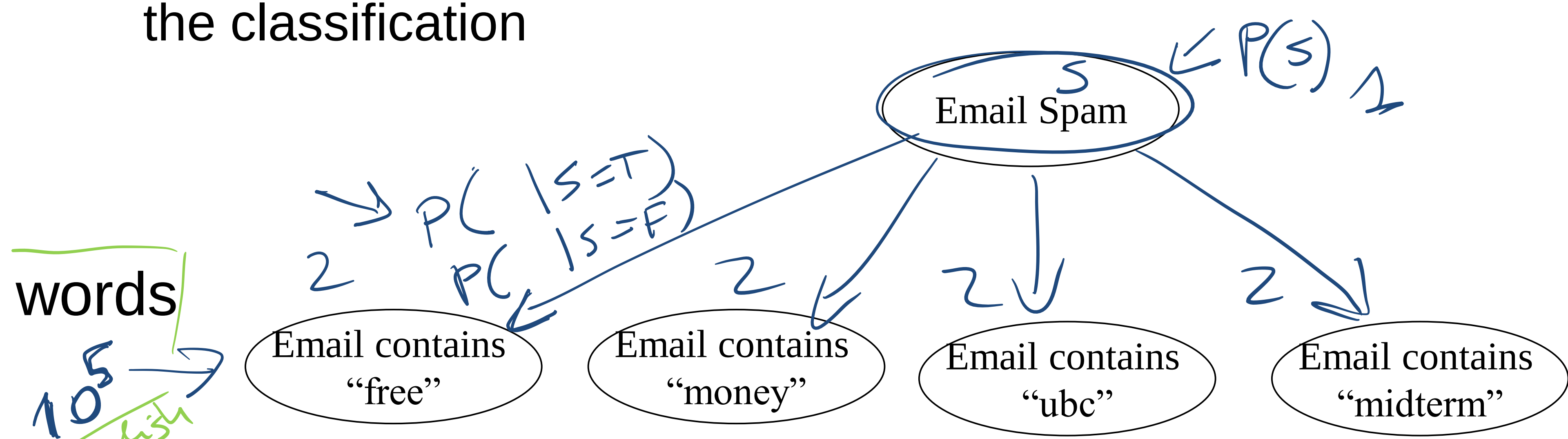


C.

Naïve Bayesian Classifier for Email Spam

Assumptions

- The value of each attribute depends on the classification
- **(Naïve)** The attributes are independent of each other given the classification



Number of parameters?

$2 \cdot 10^5$ vs 2^{10^5} Joint

Easy to acquire?

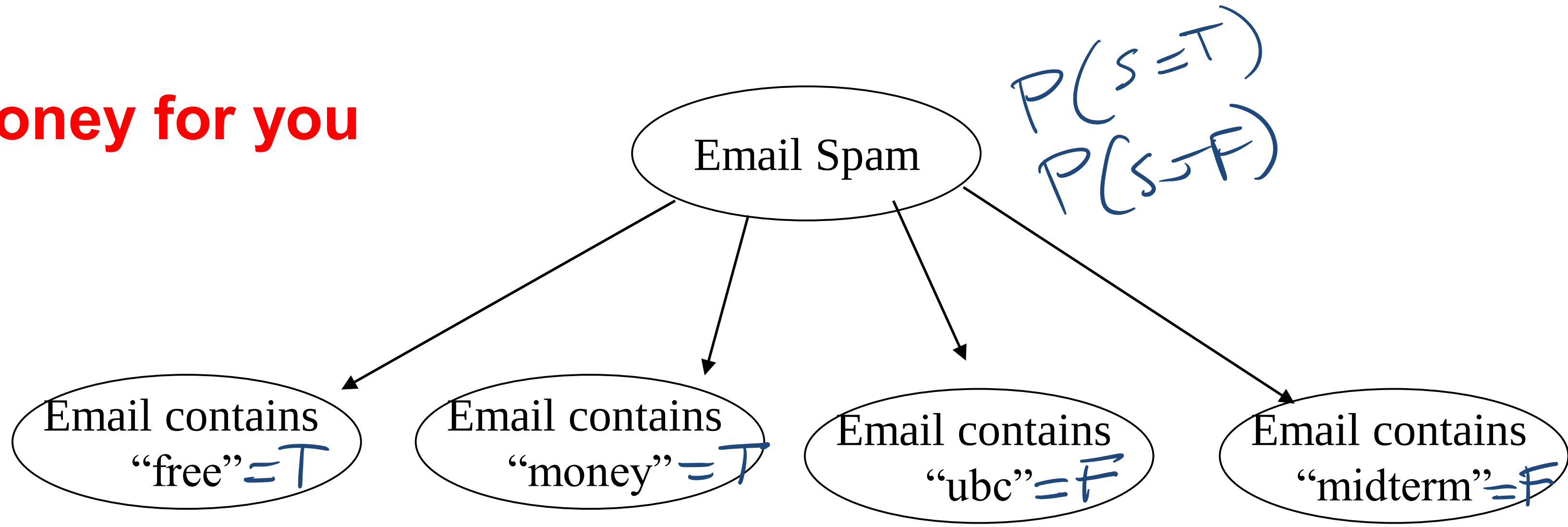
If you have a large collection of emails for which you know if they are spam or not……

NB Classifier for Email Spam: Usage

Most likely class given set of observations

Is a given Email *E* spam?

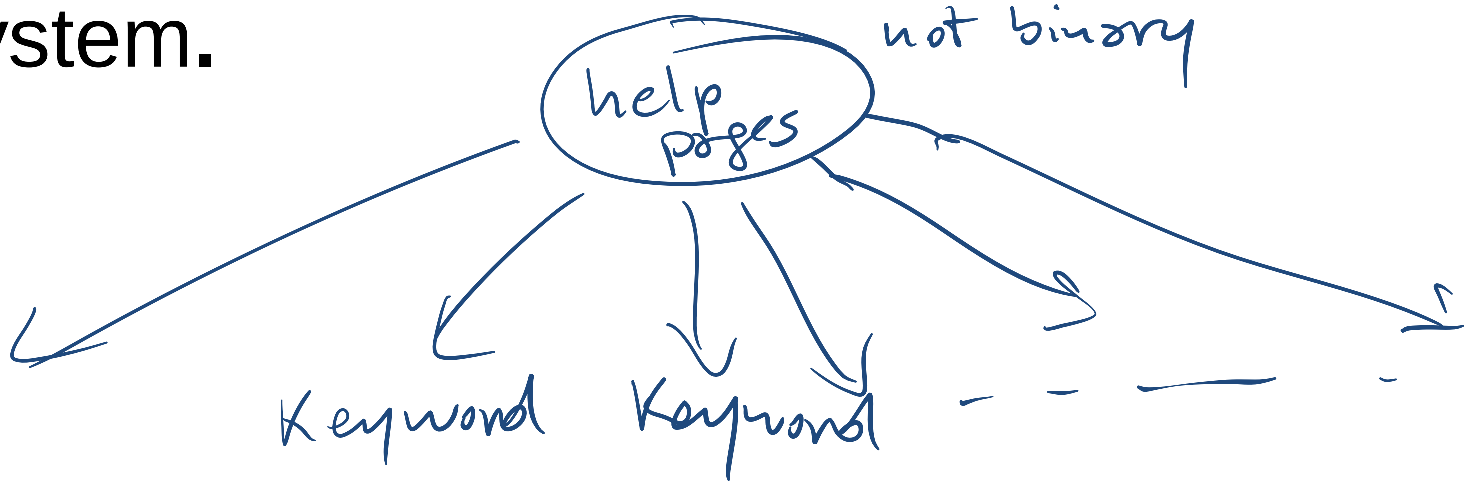
"free money for you now"



Email is spam if……. P(S=T | …) > P(S=F | …) after the words in the email have been taken into account (i.e. set to T, with all others set to F)

For another example of naïve Bayesian Classifier

help system to determine what **help page a user is interested in** based on **the keywords they give in a query** to a help system**.**



Next Class

Bayesian Networks Inference: **Variable Elimination**