Propositional Definite Clause Logic: Syntax, Semantics and Bottom-up Proofs

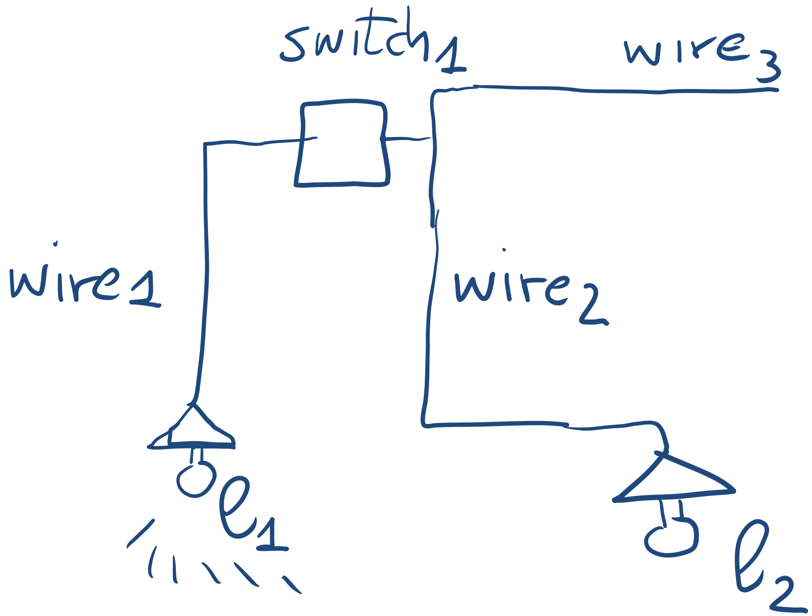
CPSC 322 Lecture 19

Lecture Overview

- **Recap: Logic intro**
- Propositional Definite Clause Logic: Semantics
- PDCL: Bottom-up Proof

Logics as a R&R system

- **represent a domain**



on_l1 ← live_wire1

live_wire1 ← on_switch1 ^ live_wire3

- **reason about it**

if the agent knows on_switch1 and live_wire3, it should be able to infer on_l1

Propositional (Definite Clauses) Logic: Syntax

We start from a restricted form of Prop. Logic

Only two kinds of statements

- that a proposition is true
- that a proposition is true if one or more other propositions are true

Learning Goals for today's class

You can:

- Verify whether an **interpretation** is a **model** of a PDCL KB.
- Verify when a conjunction of atoms is a **logical consequence** of a knowledge base.
- Define/read/write/trace/debug the **bottom-up proof procedure**.

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Propositional Definite Clauses Semantics: Interpretation

Semantics allows you to relate the symbols in the logic to the domain you're trying to model. An **atom** can be **T** or **F**

Definition (interpretation)

An interpretation I assigns a truth value to each atom.

So an interpretation is just a **possible world**

PDC Semantics: Body

We can use the **interpretation** to determine the truth value of **clauses** and **knowledge bases**:

> **Definition (**truth values of statements**):** A body $b_1 \wedge b_2$ is true in I if and only if b_1 is true in I and b_2 is true in I.

	p	q	r	s	p ^ r	p ^ r ^ s
I_1	*true*	*true*	*true*	*true*	T	T
I_2	*false*	*false*	*false*	*false*	F	F
I_3	*true*	*true*	*false*	*false*	F	F
I_4	*true*	*true*	*true*	*false*		
I_5	*true*	*true*	*false*	*true*		

PDC Semantics: definite clause

Definition (truth values of statements cont'**):** A rule $h \leftarrow b$ is false in I if and only if b is true in I and h is false in I.

	p	q	r	s	$p \leftarrow s$	$s \leftarrow q \wedge r$
I_1	*true*	*true*	*true*	*true*	T	T
I_2	*false*	*false*	*false*	*false*	T	T
I_3	*true*	*true*	*false*	*false*	T	T
I_4	*true*	*true*	*true*	*false*	T	F
…..	….	…..	….	….		

In other words: *"if b is true I am claiming that h must be true, otherwise I am not making any claim"*

PDC Semantics: Knowledge Base (KB)

- A **knowledge base KB** is true in I if and only if every clause in KB is true in I.

	p	q	r	s
I_1	*true*	*true*	*false*	*false*

Which of the three KB below are True in I_1 ?

A

p
r
$s \leftarrow q \wedge p$

B

p
q
$s \leftarrow q$

C

p
$q \leftarrow r \wedge s$

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

Definition (model)
A model of a set of clauses (a KB) is an interpretation in which all the clauses are ***true***.

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*false*	*false*	*false*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*true*	*false*
I_5	*true*	*true*	*false*	*true*

Which interpretations are models?

iclicker.

A. 1 only
B. 1, 3 and 5
C. 3 and 4
D. 1, 3 and 4
E. none

Logical Consequence

Definition (logical consequence)
If *KB* is a set of clauses and *G* is a conjunction of atoms, *G* is a logical consequence of *KB*, written $KB \models G$, if *G* is *true* in every model of *KB*.

- we also say that *G* logically follows from *KB*, or that *KB* entails *G*.
- In other words, $KB \models G$ if there is no interpretation in which *KB* is *true* and *G* is *false*.

Example: Logical Consequences

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*true*	*true*	*true*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*false*	*true*
I_5	*false*	*true*	*true*	*true*
I_6	*false*	*true*	*true*	*false*
I_7	*false*	*true*	*false*	*false*
I_8	*false*	*true*	*false*	*true*
…	….	…	…	…

Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

Of the 2^4 interpretations, only 3 are models

Which of the following is/are true?

- $KB \models p$, $KB \models q$, $KB \models r$, $KB \models s$

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One simple way to prove that G logically follows from a KB

- Collect all the models of the KB
- Verify that G is true in all those models

Any problem with this approach?

- The goal of proof theory is to find proof procedures that allow us to prove that a logical formula follows from (i.e. is logically entailed by) a KB while avoiding the above

Soundness and Completeness

- Suppose I tell you I have a **proof procedure for PDCL**; what do I need to show you in order for you to trust my procedure?

  - KB ⊢ G means *G* can be derived by my proof procedure from *KB*.
  - Recall *KB* ⊨ *G* means *G* is true in all models of *KB*.

Definition (soundness)

A proof procedure is sound if *KB* ⊢ *G* implies *KB* ⊨ *G*.

Definition (completeness)

A proof procedure is complete if *KB* ⊨ *G* implies *KB* ⊢ *G*.

Bottom-up Ground Proof Procedure

One rule of derivation, a generalized form of ***modus ponens***:

If "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" is a clause in the knowledge base, and each b_i has been derived, then h can be derived.

You are forward chaining on this clause.

(This rule also covers the case when ***m=0*** - i.e., when the entire clause consists only of an atom)

Bottom-up proof procedure

KB ⊦ *G* if $G \subseteq C$ at the end of this procedure:

C :={};
repeat

- **select** clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in *KB* such that $b_i \in C$ for all i, and $h \notin C$;
- $C := C \cup \{ h \};$

until no more clauses can be selected.

KB: $e \leftarrow a \wedge b$

$e \leftarrow d$

a

$b \leftarrow a$

$d \leftarrow g$

← in-class Activity

Bottom-up proof procedure: Example

KB

$z \leftarrow f \wedge e$

$q \leftarrow f \wedge g \wedge z$

$e \leftarrow a \wedge b$

a

b

r

f

C :={};
repeat
  select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in KB such that $b_i \in C$ for all i, and $h \notin C$;
  $C := C \cup \{ h \}$
until no more clauses can be selected.

Which is correct?

A. KB ⊢ {a,q,z}

B. KB ⊢ {b,r,z}

C. KB ⊢ {a,q}

Next class

(still section 5.3)

- Soundness and Completeness of Bottom-up Proof Procedure
- Using PDC Logic to model the electrical domain
- Reasoning in the electrical domain