Stochastic Local Search

CPSC 322 Lecture 14

Successful application of SLS

Scheduling of Hubble Space Telescope

“multistart stochastic repair”

Johnston, Mark D., and Glenn Miller. "Spike: Intelligent scheduling of hubble space telescope observations.“ *Intelligent Scheduling* (1994): 391-422.

Lecture Overview

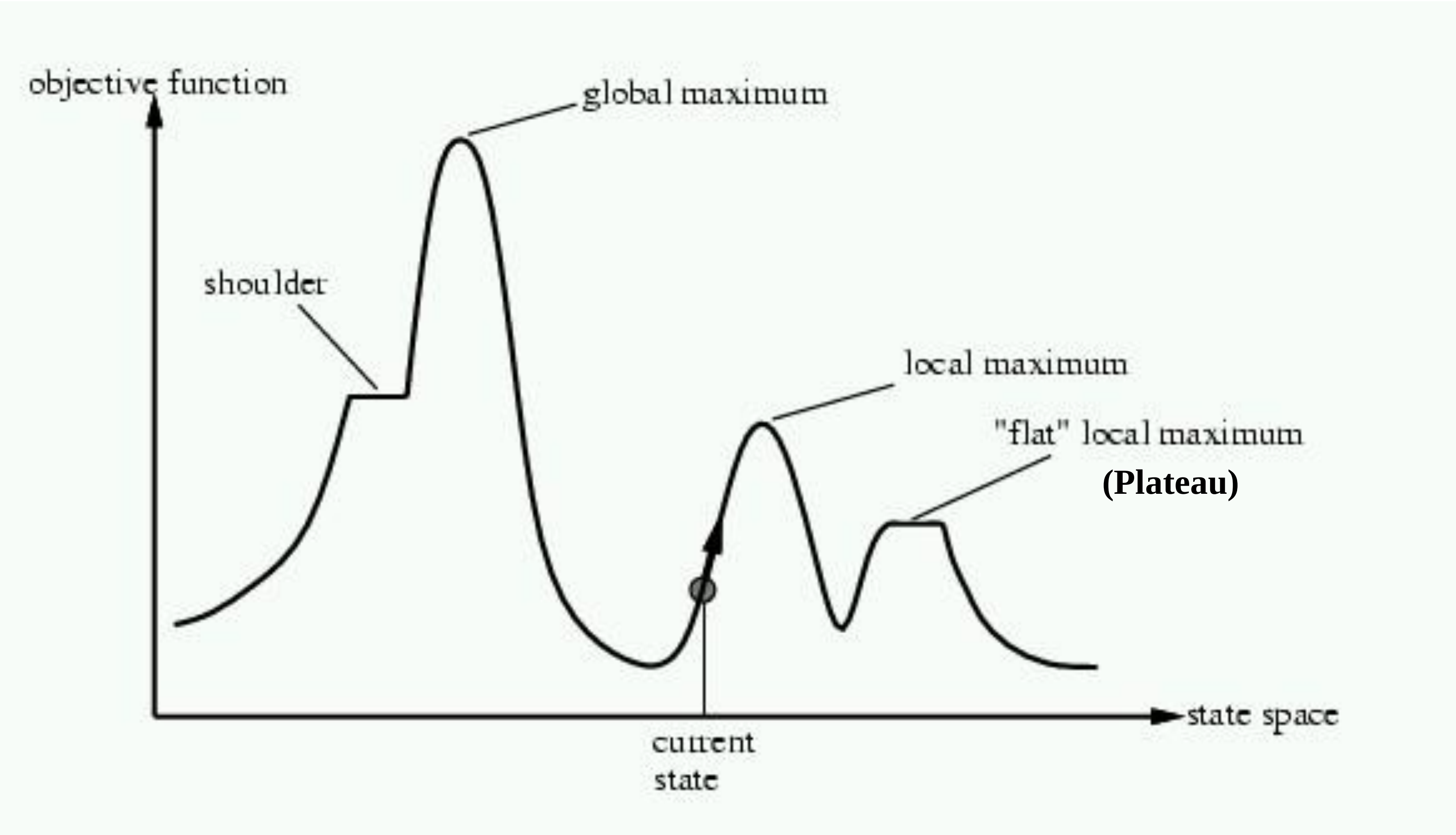
- **Recap Local Search in CSPs**
- Stochastic Local Search (SLS)
- Comparing SLS algorithms

Local Search: Summary

- A useful method for large CSPs
  - Start from a possible world (often randomly chosen)
  - Generate some neighbors (“similar” possible worlds)
  - Move from current node to a neighbor, selected to minimize/maximize a scoring function which combines:
    - Information about how many constraints are violated
    - Information about the cost/quality of the solution (you want the best solution, not just a solution)

Problems with Hill Climbing

- Local Maxima
- Plateaus & Shoulders



Learning Goals for today's class

You can:

- Implement SLS with
  - random steps (1-step, 2-step versions)
  - random restart
- Compare SLS algorithms with runtime distributions

Lecture Overview

- **Recap Local Search in CSPs**
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Stochastic Local Search

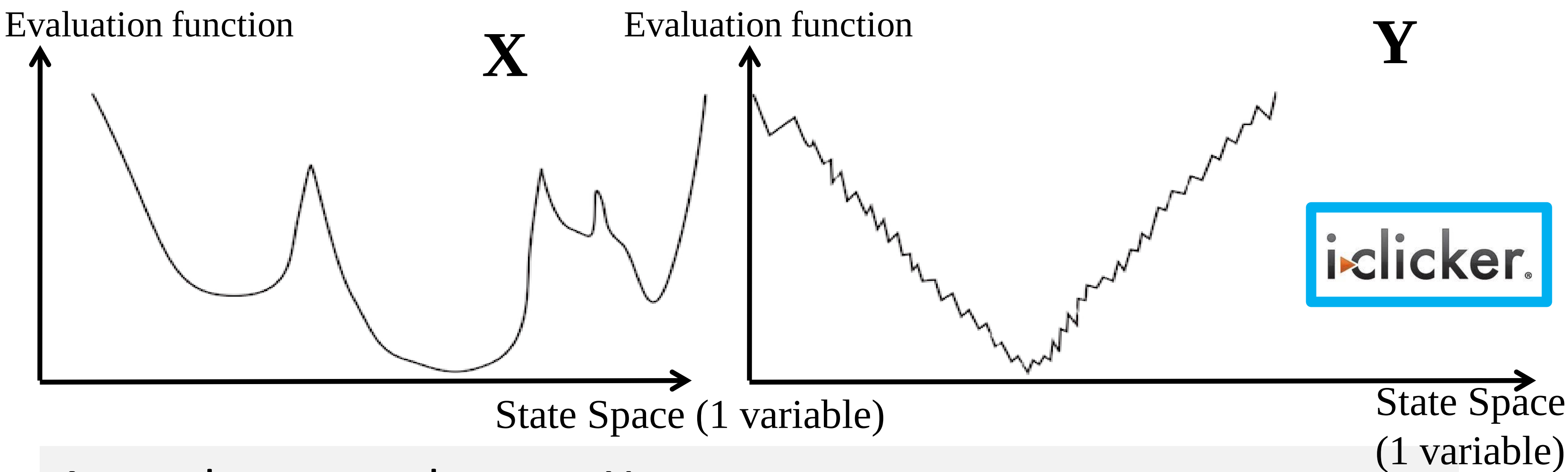
GOAL: We want our local search

- to be guided by the scoring function
- Not to get stuck in local maxima/minima, plateaus etc.

SOLUTION: We can alternate

a) Hill-climbing steps
b) Random steps: move to a random neighbor.
c) Random restart: reassign random values to all variables.

Which randomized variant of Greedy Descent would work best in each of these two search spaces?



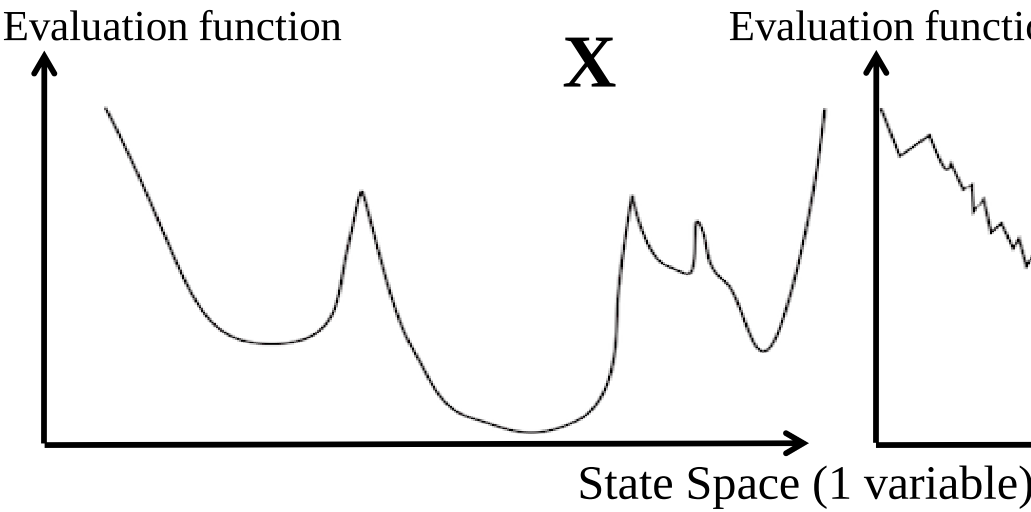
A. random steps best on X
   random restart best on Y

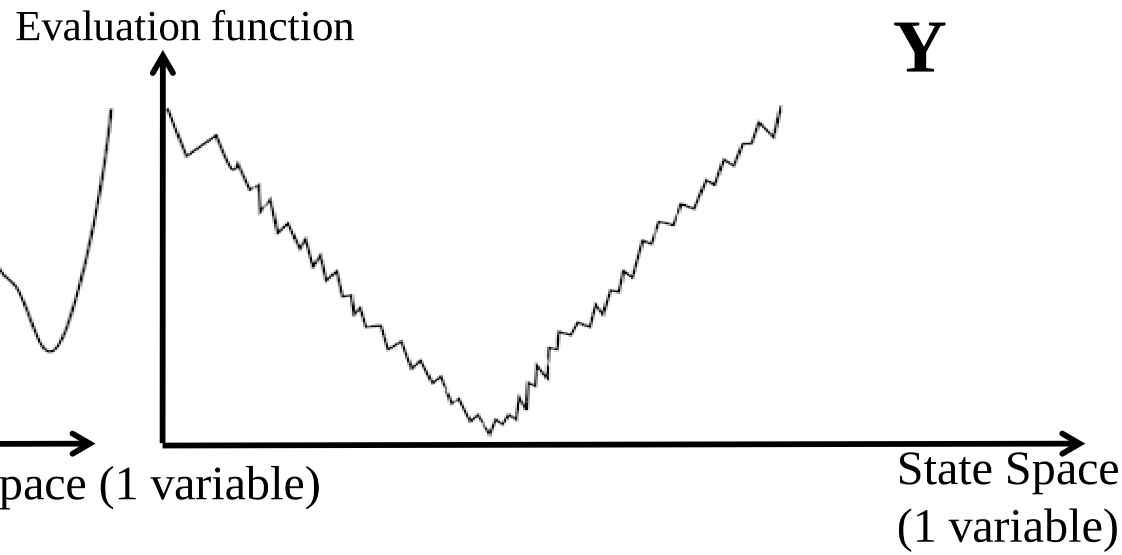
B. random steps best on Y
   random restart best on X

C. The two methods are equivalent on X and Y

D. Use the Force, Luke

Which randomized method would work best in each of the these two search spaces?





- But these examples are simplified extreme cases for illustration
  - in practice, you don't know what your search space looks like
- Usually integrating both kinds of randomization works best

Random Steps (Walk): one-step

Let's assume that neighbors are generated as

- assignments that differ in one variable's value

How many neighbors there are given n variables with domains with d values? ***n(d-1)***

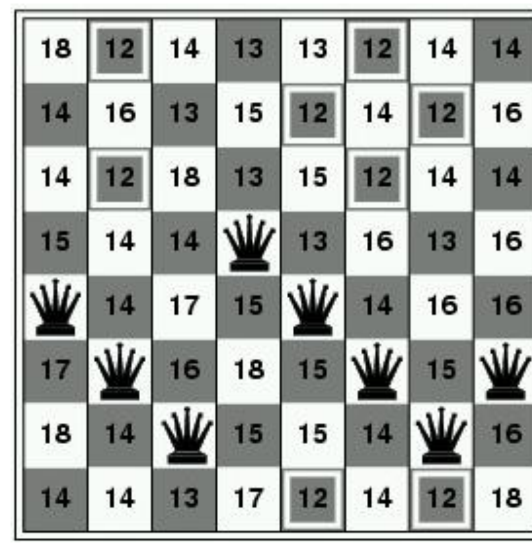
One strategy to add randomness to the selection of the variable-value pair. Sometimes choose the pair:

1. According to the scoring function
2. A random one

Eg. in 8-queen

- How many neighbors?

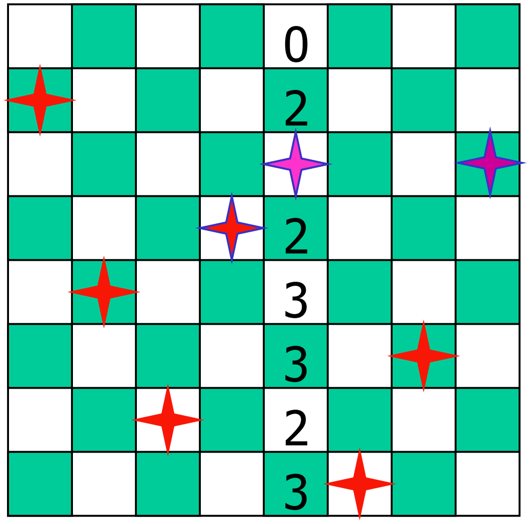
1. Choose one of the best neighbors
2. Choose a random neighbor



Random Steps (Walk): two-step

Another strategy: select a variable first, then a value:

- Sometimes select variable:
  1. that participates in the largest number of conflicts.
  2. at random, any variable that participates in some conflict
  3. at random
- Sometimes choose value
  a) That minimizes # of conflicts
  b) at random



Example: SLS for RNA secondary structure design

RNA strand made up of four bases: cytosine (C), guanine (G), adenine (A), and uracil (U)

2D/3D structure RNA strand folds into is important for its function

Predicting structure for a strand is “easy”: $O(n^3)$

But what if we want a strand that folds into a certain structure?

- Local search over strands
  - ✓ Search for one that folds into the right structure
- Evaluation function for a strand
  - ✓ Run $O(n^3)$ prediction algorithm
  - ✓ Evaluate how different the result is from our target structure
  - ✓ Only defined implicitly, but can be evaluated by running the prediction algorithm

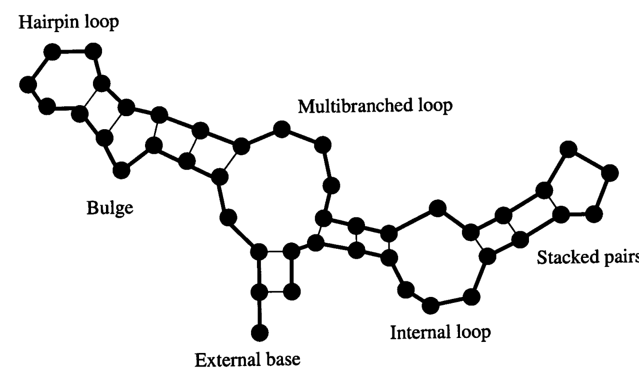
RNA strand

GUCCCAUAGGAUGUCCCAUAGGA

Easy

Hard

Secondary structure



Example: Local search algorithm RNA-SSD developed at UBC

[Andronescu, Fejes, Hutter, Condon, and Hoos, Journal of Molecular Biology, 2004]

CSP/logic: formal verification

Hardware verification
(e.g., IBM)

Software verification
(small to medium programs)

CPSC 422

Most progress in the last 10 years based on:
Encodings into propositional satisfiability (SAT)

Stochastic Local search (SLS) advantage: Online setting

- **When the problem can change** (particularly important in scheduling)
- **E.g., schedule for airline:** thousands of flights and thousands of personnel assignment
  - Storm can render the schedule infeasible
- **Goal:** Repair with minimum number of changes
- This can be easily done with a local search starting form the current schedule
- Other techniques usually:
  - require more time
  - might find solution requiring many more changes

SLS Advantage: anytime algorithms

- When should the algorithm be stopped ?
  - When a solution is found
    (e.g. no constraint violations)
  - Or when we are out of time: you have to act NOW
  - Anytime algorithm:
    - ✓ maintain the node with best h found so far (the “incumbent”)
    - ✓ given more time, can improve its incumbent

SLS limitations

- **Typically no guarantee to find a solution even if one exists**
  - SLS algorithms can sometimes stagnate
    - ✓ Get caught in one region of the search space and never terminate
  - Very hard to analyze theoretically

- **Not able to show that no solution exists**
  - SLS simply won’t terminate
  - You don’t know whether the problem is infeasible or the algorithm has stagnated

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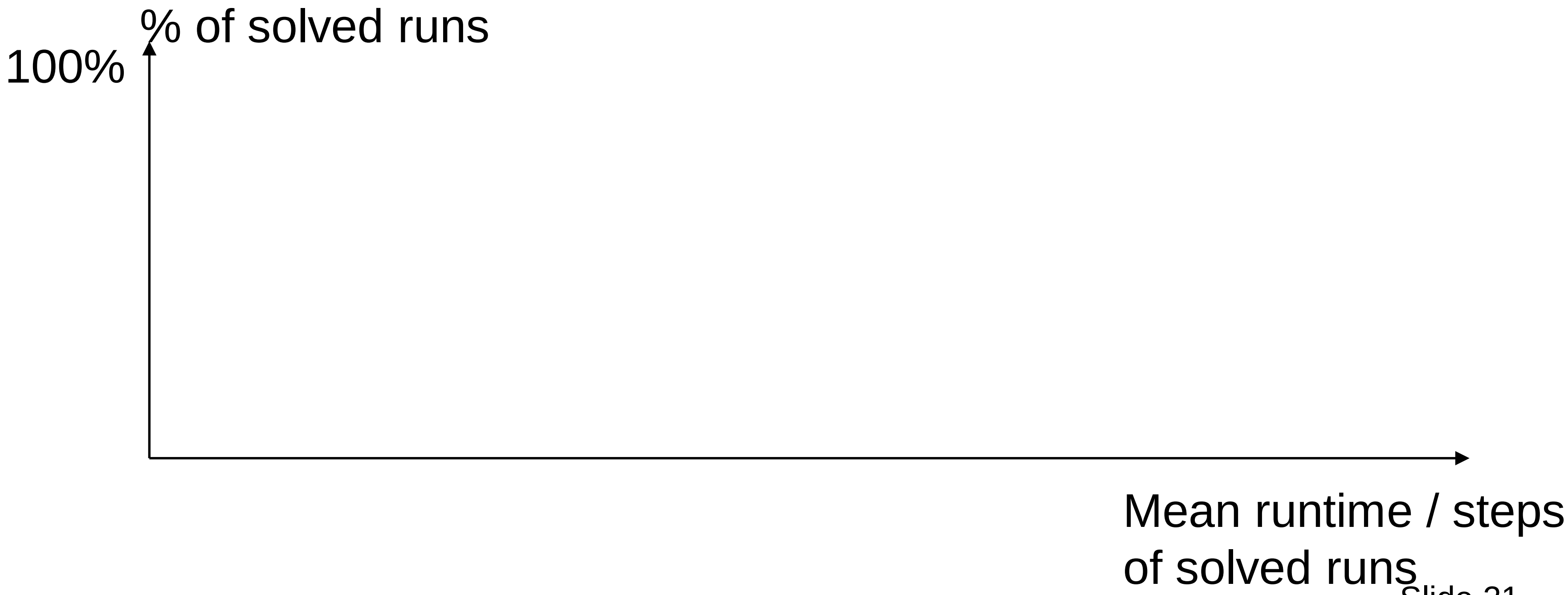
Evaluating SLS algorithms

- SLS algorithms are randomized
  - The time taken until they solve a problem is a random variable
  - It is entirely normal to have runtime variations of 2 orders of magnitude in repeated runs!
    - ✓ E.g. 0.1 seconds in one run, 10 seconds in the next one
    - ✓ On the same problem instance (only difference: random seed)
    - ✓ Sometimes SLS algorithm doesn't even terminate at all: stagnation
- If an SLS algorithm sometimes stagnates, what is its mean runtime (across many runs)?
  - **Infinity!**
  - In practice, one often counts timeouts as some fixed large value X
  - Still, summary statistics, such as **mean** run time or **median** run time, don't tell the whole story
    - ✓ E.g. would penalize an algorithm that often finds a solution quickly but sometime stagnates

First attempt at SLS algorithm evaluation

- How can you compare three algorithms (A, B, C) when
  A. solves the problem 30% of the time very quickly but doesn't halt for the other 70% of the cases
  B. solves 60% of the cases reasonably quickly but doesn't solve the rest
  C. solves the problem in 100% of the cases, but slowly?



Runtime Distributions are even more effective

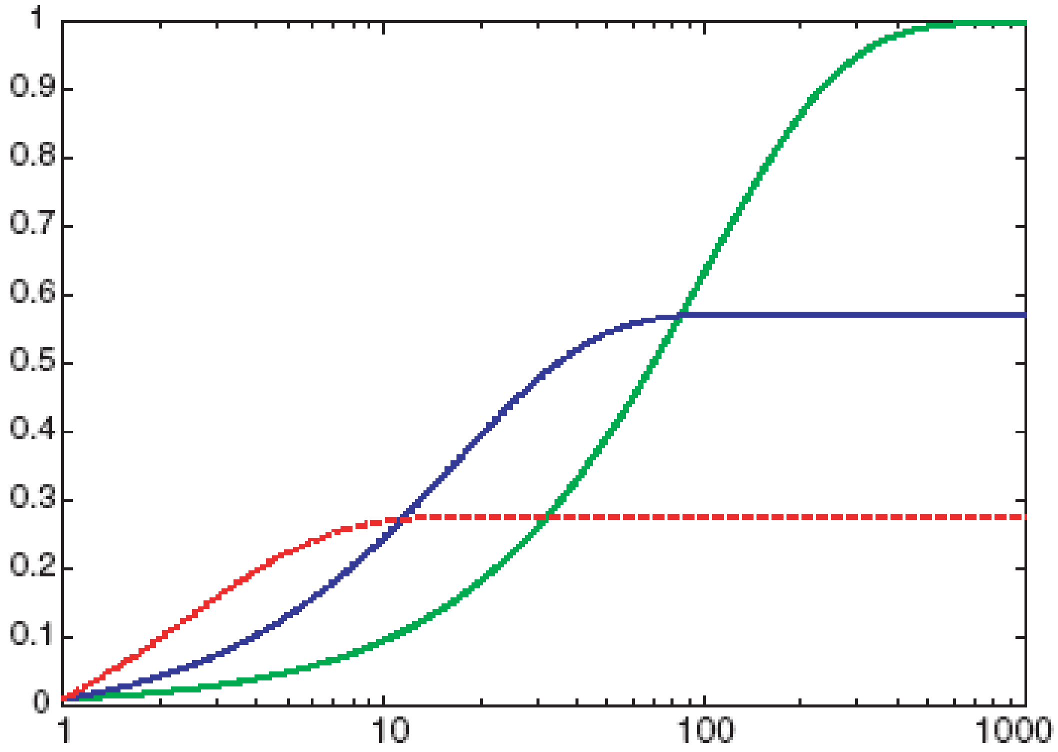
Plots runtime (or number of steps) and the proportion (or number) of the runs that are solved within that runtime.

- log scale on the *x* axis is commonly used

Fraction of solved runs,

i.e.

P(solved in n steps)



n (# of steps)

Comparing runtime distributions

x axis: runtime (or number of steps)

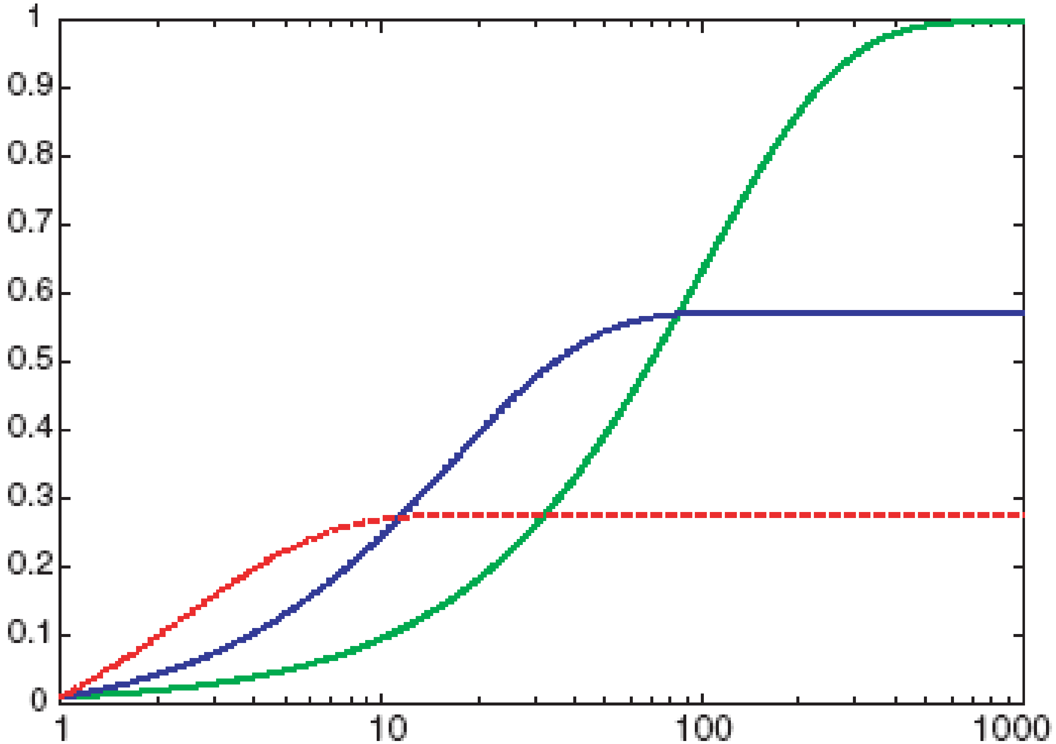
y axis: proportion (or number) of runs solved in that runtime

- Typically use a log scale on the x axis

Fraction of solved runs,

i.e.

P(solved in n steps)



n (# of steps)

Which algorithm is most likely to solve the problem within 7 steps?

A. blue

B. red

C. green

Comparing runtime distributions

- Which algorithm has the best median performance?
  - i.e., which algorithm takes the fewest number of steps to be successful in 50% of the cases?

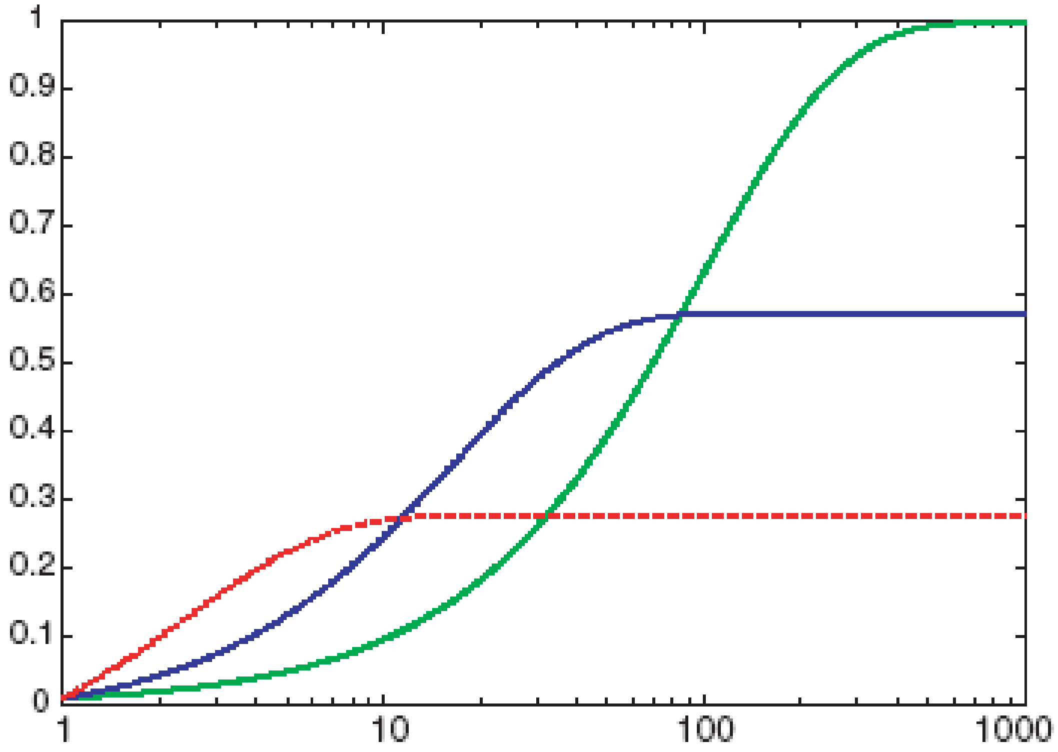
A. blue

B. red

C. green

Fraction of solved runs, i.e.

P(solved by this # of steps/time)

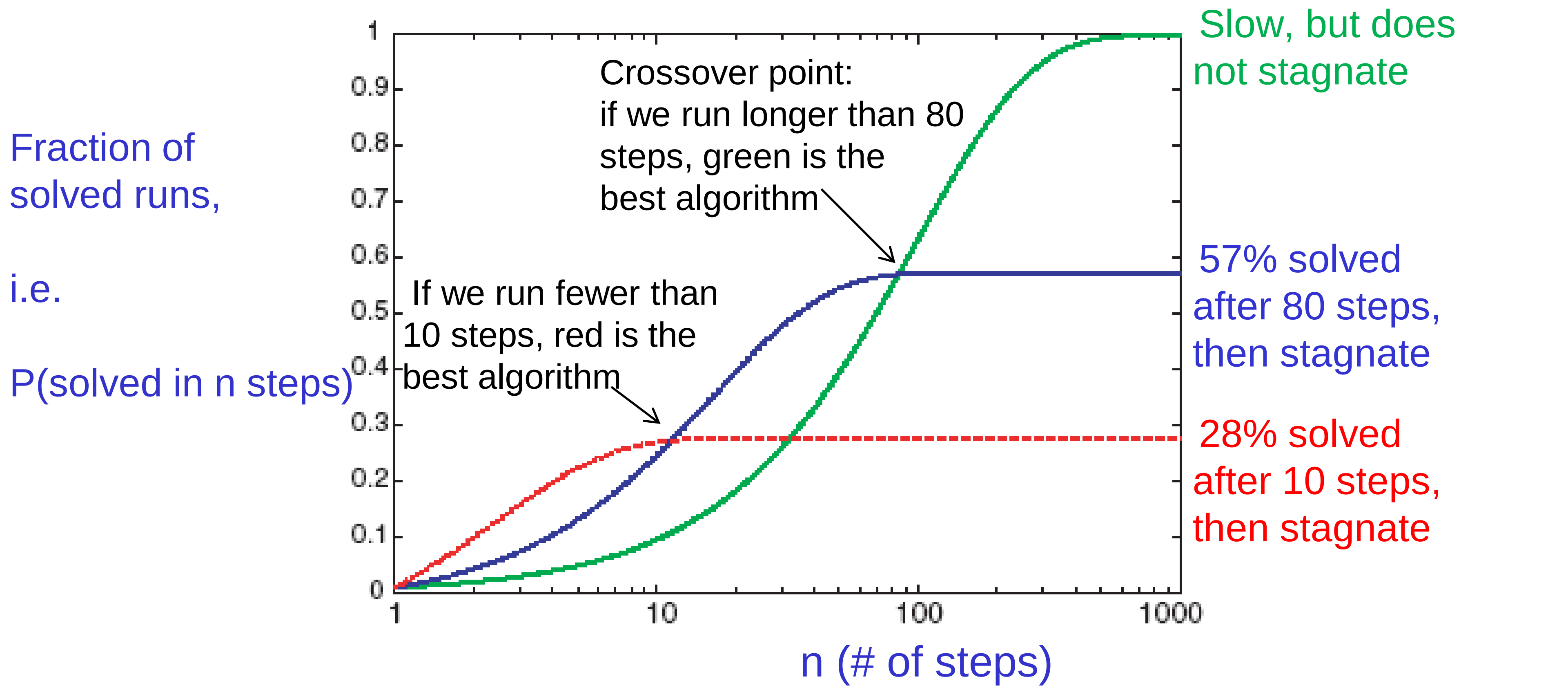


# of steps

Comparing runtime distributions

x axis: runtime (or number of steps)
y axis: proportion (or number) of runs solved in that runtime

- Typically use a log scale on the x axis



Runtime distributions in AIspace

- Let's look at some algorithms and their runtime distributions:
  1. Greedy Descent
  2. Random Sampling
  3. Random Walk
  4. Greedy Descent with random walk

- Scheduling problem 2 in AIspace:

What are we going to look at in AIspace

During two-stage selection:

- Sometimes select variable:
  1) that participates in the largest number of conflicts.
  2) at random, any variable that participates in some conflict.
  3) at random
- Sometimes choose value:
  a) That minimizes # of conflicts
  b) at random

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AIspace terminology

Random sampling	restarts
Random walk	3b
Greedy Descent	1a
Greedy Descent Min Conflict	2a
Greedy Descent with random walk	1-2 a-b

Stochastic Local Search

- **Key Idea:** combine greedily improving moves with randomization
  - As well as improving steps we can allow a "small probability" of:
    - Random steps: move to a random neighbor.
    - Random restart: reassign random values to all variables.
  - Always keep best solution found so far
  - Stop when
    - Solution is found (in vanilla CSP, all constraints satisfied)
    - Run out of time (return best solution so far)

Next:

- Finish CSPs: More SLS variants Ch. 4.7.3, 4.8