CSPs: Search and Arc Consistency

CPSC 322 Lecture 11

Lecture Overview

- **Recap CSPs**
- Generate-and-Test
- Search
- Consistency
- Arc Consistency

Constraint Satisfaction Problems: definitions

Definition (Constraint Satisfaction Problem)

A constraint satisfaction problem consists of

- a set of variables
- a domain for each variable
- a set of constraints

Definition (model / solution)

A model of a CSP is an assignment of values to variables (i.e. **possible worlds**) that satisfies all of the constraints.

Learning Goals for today's class

You can:

- Implement the Generate-and-Test Algorithm. Explain its disadvantages.
- Solve a CSP by search (specify neighbors, states, start state, goal state). Compare strategies for CSP search. Implement pruning for DFS search in a CSP.
- Build a constraint network for a set of constraints.
- Verify whether a network is arc-consistent.
- Make an arc arc-consistent.

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Generate-and-Test Algorithm

- **Algorithm:**
  - Generate possible worlds one at a time
  - Test them to see if they violate any constraints

```
for a in domA
  for b in domB
     for c in domC
         if (abc) satisfies all constraints
             return (abc)
return NULL
```

- This procedure is able to solve any CSP
- However, the running time is proportional to the number of possible worlds
  - always exponential in the number of variables
  - far too long for many CSPs ☹

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CSPs as search problems

- **states:** assignments of values to a **subset** of the variables
- **start state:** the empty assignment (no variables assigned values)
- **neighbours** of a state: nodes in which values are assigned to one additional variable
- **goal state:** a state which **assigns a value to each variable**, and **satisfies all of the constraints**

Note: the path to a goal node is not important

CSPs as Search Problems

What search strategy will work well for a CSP?

If there are n variables, every solution is at depth n

Is there a role for a heuristic function?

A. Yes B. No

iclicker.

The search space is always…

A. Finite with cycles B. Infinite without cycles

C. Finite without cycles D. Infinite with cycles

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So which search strategy is better?

A. BFS
B. IDS
C. A*
D. DFS

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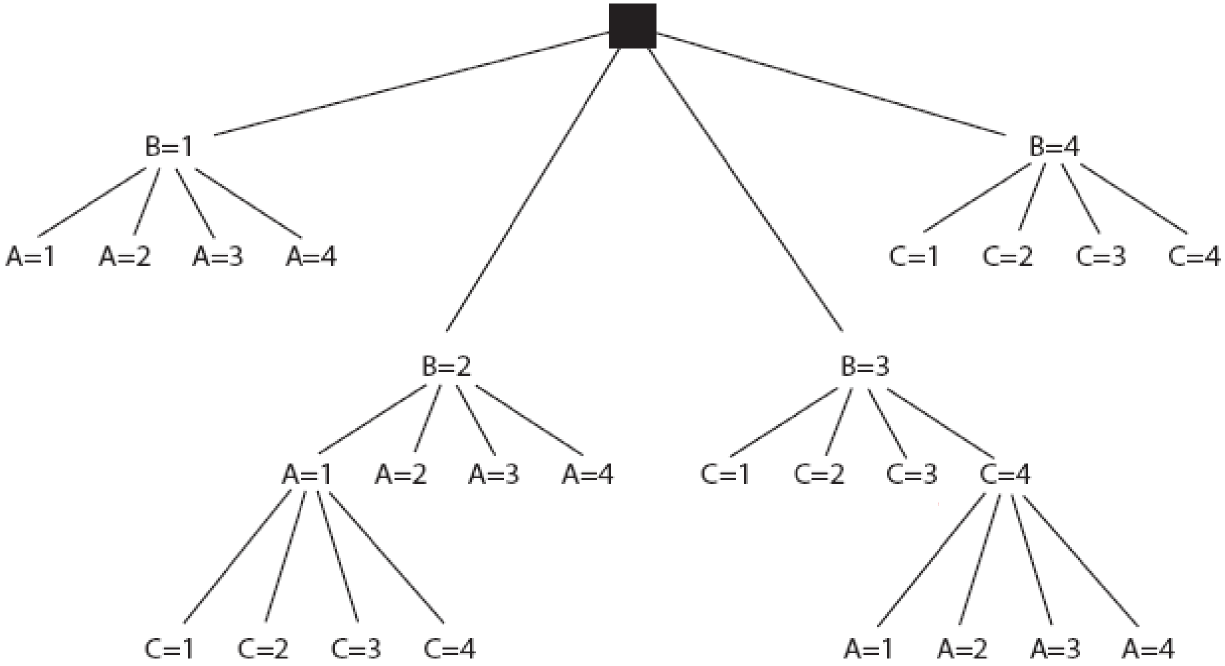
CSPs as search problems

Simplified notation



CSPs as Search Problems

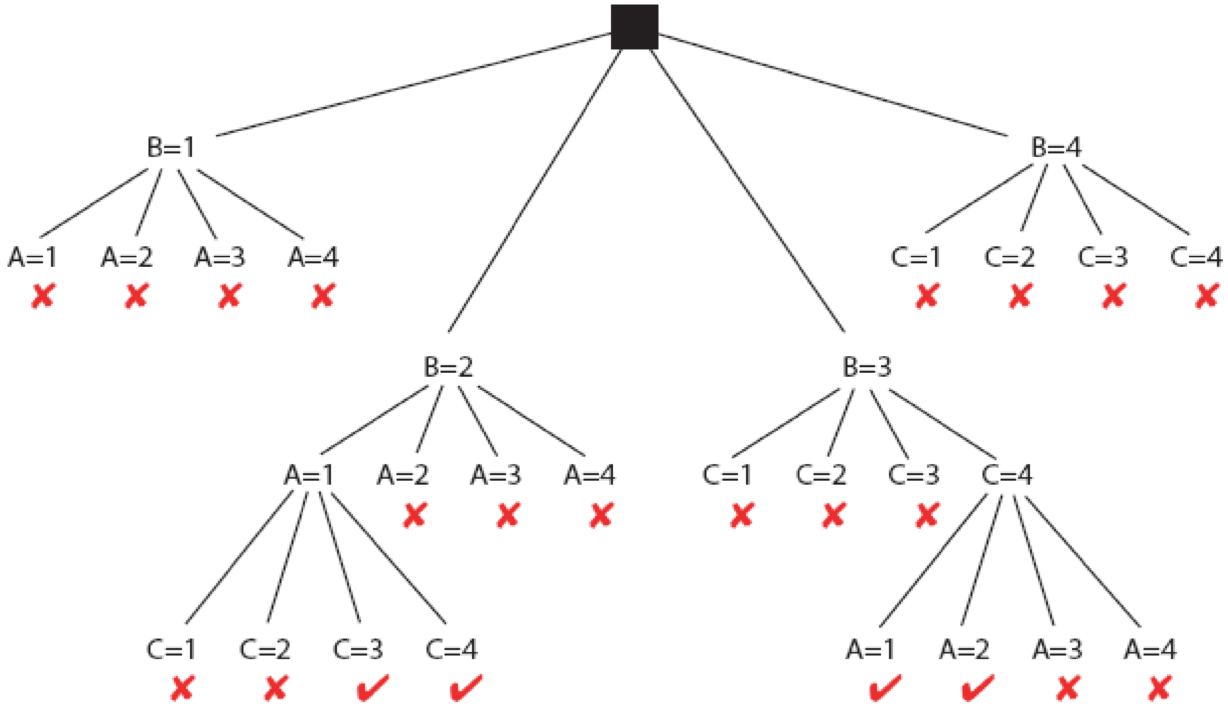
How can we avoid exploring some sub-trees i.e., prune the DFS Search tree?

- once we consider a path whose end node violates one or more constraints, we know that **a solution cannot exist below that point**
- thus we should **remove that path** rather than continuing to search

Solving CSPs by DFS: Example

Problem:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C



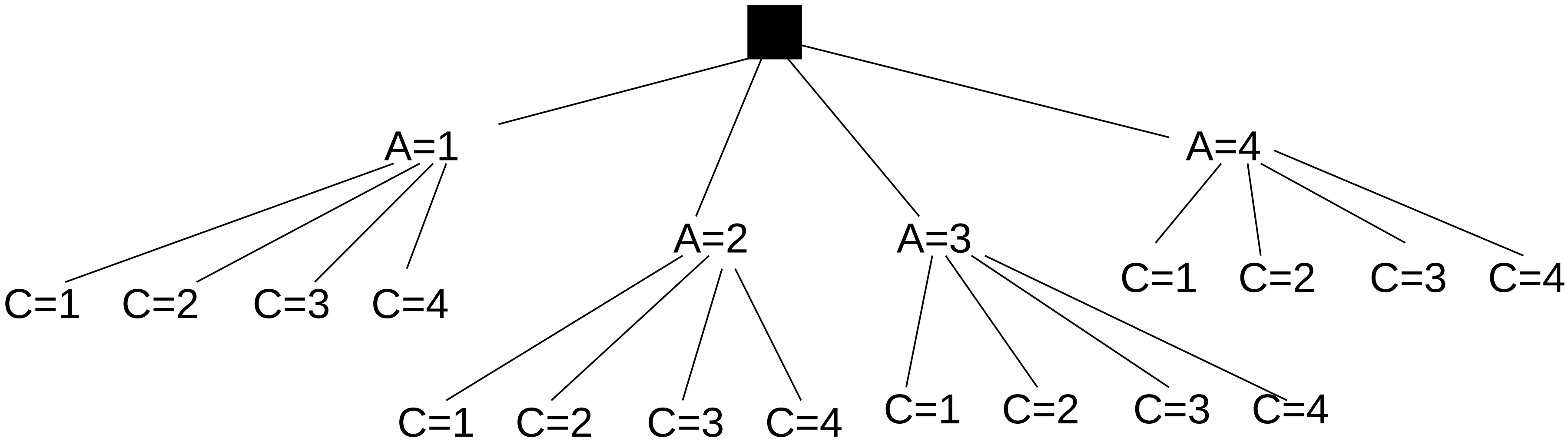
Solving CSPs by DFS: Example Efficiency

Problem:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C

Note: the algorithm's efficiency depends on the order in which variables are expanded

Degree "Heuristics"



Standard Search vs. Specific R&R systems

Constraint Satisfaction (Problems):

- State: assignments of values to a subset of the variables
- Successor function: assign values to a "free" variable
- Goal test: set of constraints
- Solution: possible world that satisfies the constraints
- Heuristic function: *none (all solutions at the same distance from start)*

Planning :

- State
- Successor function
- Goal test
- Solution
- Heuristic function

Inference

- State
- Successor function
- Goal test
- Solution
- Heuristic function

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Can we do better than Search?

Key ideas:

- **prune the domains** as much as possible **before "searching"** for a solution.

Simple when using constraints involving single variables (technically enforcing **domain consistency**)

Definition: A variable is domain consistent if no value of its domain is ruled impossible by any unary constraints.

- Example: if we have the constraint $B \neq 3$, then **D_B** = {1, 2, 3, 4} _____________ domain consistent.

How do we deal with constraints involving multiple variables?

Definition (**constraint network**)

A constraint network is defined by a graph, with

- one **node** for every **variable**
- one **node** for every **constraint**

and undirected edges running between variable nodes and constraint nodes whenever a given variable is involved in a given constraint.

Note: strictly speaking, when all of the constraints are binary, constraint nodes are not necessary: we can drop constraint nodes and use edges to indicate that a constraint holds between a pair of variables. However, **in this course we will always show constraint nodes.**

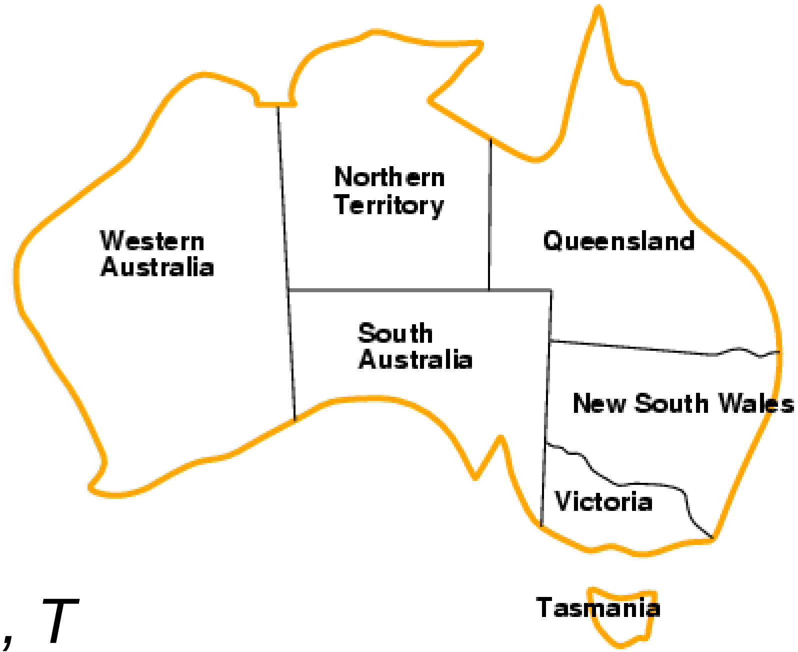
Example Constraint Network

Recall Example:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C, B=1

What would a constraint network for this example look like?

Example: Constraint Network for Map Coloring



Variables *WA, NT, Q, NSW, V, SA, T*

Domains D_i = {red,green,blue}

Constraints: adjacent regions must have different colors

Group activity: draw a constraint network for this example

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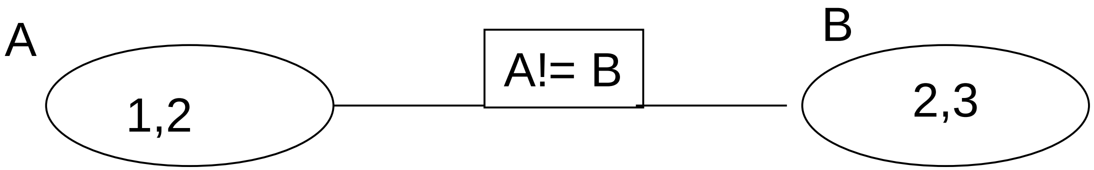
Arc Consistency

Definition (arc consistency)

An arc $\langle X, r(X,Y)\rangle$ is arc consistent if for each value x in $dom(X)$ there is some value y in $dom(Y)$ such that $r(x,y)$ is satisfied.



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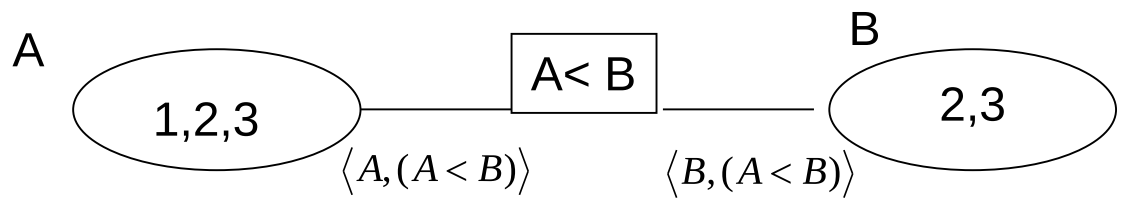
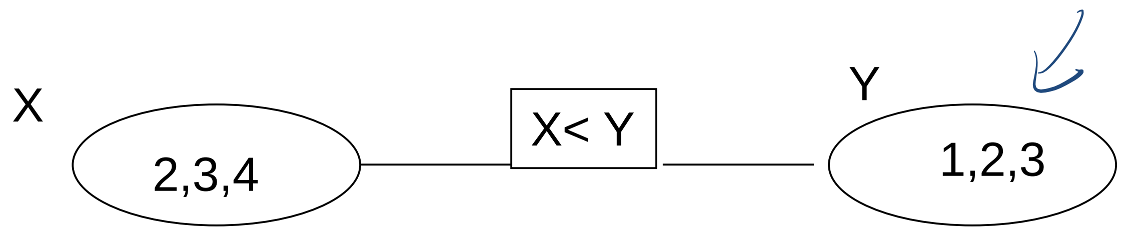


A. Both arcs are consistent
B. Left consistent, right inconsistent
C. Right consistent, left inconsistent
D. Both arcs are inconsistent
E. I consistently fall asleep in this class

How can we enforce Arc Consistency?

- If an arc $\langle X, r(X,Y) \rangle$ is not arc consistent, all values x in $dom(X)$ for which there is no corresponding value in $dom(Y)$ may be deleted from $dom(X)$ to make the arc $\langle X, r(X,Y) \rangle$ consistent.
  - This removal can never rule out any models/solutions



- **A network is arc consistent** if all its arcs are arc consistent.