

## EXAMPLE OF A PROPOSAL OUTLINE AND PROPOSAL

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#### **OUTLINE:**

**One or Two Sentence Description:** We will study the effects of university exam scheduling policies on the number of student exam conflicts.

**Models:** Here are some **models** we plan to use:

- (1) A simple model: We are given  $n$  classes and a list of pairs of classes,  $P$ , that have common students. We want to know the smallest  $m$  such that one can schedule all classes in  $m$  exam slots so that any two classes with common students are in different slots. This is an IP, and can be viewed as a graph colouring problem.
- (2) A more advanced model: same as the previous model, except for each pair of classes with common students we are given the number of common students. For any  $m$  we want to schedule the exams to minimize the sum, over all pairs of classes assigned to the same slot, of the number of students taking both classes. This is an IP with a parameter,  $m$ .
- (3) Another more advanced model: we are given a list of  $n$  classes,  $s$  students, and for each student we will list the classes this student is taking. For each  $m$  we want to schedule class exams in a way that minimizes the total number of students with exam conflicts. This is an IP with a parameter,  $m$ .

**Software:** We plan to use Gurobi software to solve the above IP's.

**Some questions we may study:** When the exam period shrinks, how gradually do the number of exam conflicts increase? How easily can we model *exam hardships* (i.e., having three exams in any 24 hour period)? If large classes are broken up into smaller sections for the purposes of exams, does this significantly improve the number of exam conflicts? Can the university impose some additional constraints to make the number of exam conflicts significantly reduce (this may include deciding the exam schedule before any students can register)?

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Research supported in part by an NSERC grant.

**OUR BASIC LP/IP/QP/ETC.: Given Information:** We have (1)  $n$  classes, (2)  $m$  possible exam periods, and (3) a set,

$$P = \{(i, j) \mid \text{classes } i \text{ and } j \text{ have students in common}\} .$$

**OUR BASIC LP/IP/QP/ETC.: Optimization Problem:** We wish to find if there is a schedule of exams where no two classes with common students are assigned the same exam period. This is **equivalent to a graph colouring problem**. [If your basic optimization problem is not equivalent to a standard type of problem, describe a standard problem that is similar to your problem.]

**OUR BASIC LP/IP/QP/ETC.: The Decision Variables and Their Intuitive Meaning:**

For  $i = 1, \dots, n$  and  $k = 1, \dots, m$  we introduce a decision variable  $x_{ik}$  whose intuitive meaning is

$$x_{ik} = \begin{cases} 1 & \text{if class } i \text{ is assigned exam slot number } k, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

**OUR BASIC LP/IP/QP/ETC.: Formal Description:** The **objective** is unimportant, since this problem is one of feasibility. The **constraints** are:  $x_{ij} \in \{0, 1\}$  and

$$\sum_{k=1}^m x_{ik} = 1, \quad \text{for all } i = 1, \dots, n$$

[meaning that each class is assigned one exam slot],

$$x_{ik} + x_{jk} \leq 1 \quad \text{for all } (i, j) \in P \text{ and } k = 1, \dots, m$$

[meaning that if  $(i, j) \in P$ , then at most one of classes  $i$  and  $j$  are assigned one slot]. In view of the above constraints, it is enough to require that  $x_{ij} \in \mathbb{Z}$  and  $x_{ij} \geq 0$  for all  $i, j$ .

Our **research group** includes: Jimi Hendrix (JH), Paul McCartney (PM), and Ann Wilson (AW).

The **tasks** we anticipate in this project include:

- (1) Modeling optimization problems based on synthetic data. [JH is responsible]
- (2) Harvesting online information regarding UBC courses, making educated guesses regarding course overlap based on degree requirements. [PM is responsible]
- (3) Generating synthetic data sets based on harvested information and various policies. [AW is responsible]
- (4) Running optimization algorithms. [JH is responsible]
- (5) Statistical analysis of the results over various data sets. [PM is responsible]
- (6) Taking writeups of the above and organizing them into a coherent report. [AW is responsible]

**PROPOSAL:**

**We will study** the effects of university final exam scheduling policies on the number of student exam conflicts. Our **motivation** is that students and teaching staff tend to prefer a short exam period, but exam conflicts creates additional work for the teaching staff; we seek to study these tradeoffs. **The specific policies** we study include the length of the exam period and requiring multi-sectioned classes to have separate exams for each section.

Our simplest model of exam scheduling is an IP (integer program) that is a special case of graph colouring. We are given  $n$  classes and the set,  $P$ , of pairs  $(i, j)$  with  $1 \leq i, j \leq n$  such that classes  $i$  and  $j$  have common students. We want to know the smallest value of  $m$  such that it is possible to schedule exams in  $m$  exam slots so that any pair of classes with common students are scheduled for different slots.

Given  $n$ ,  $P$ , and  $m$ , we formulate the following IP: we introduce decision variables  $x_{ik}$  for  $i = 1, \dots, n$  and  $k = 1, \dots, m$ , whose values are in  $\{0, 1\}$ , and are meant to be

$$x_{ik} = \begin{cases} 1 & \text{if class } i \text{ is assigned colour } k, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

To enforce this, we impose the constraints

$$(1) \quad \sum_{k=1}^m x_{ik} = 1, \quad \text{for all } i = 1, \dots, n$$

so that each class is assigned one exam slot; the constraints

$$(2) \quad x_{ik} + x_{jk} \leq 1 \quad \text{for all } (i, j) \in P \text{ and } k = 1, \dots, m$$

requires at most one of each pair  $(i, j)$  of classes with common students to be assigned to each exam slot  $k$ . So a conflict free schedule is equivalent to the feasibility of  $x_{ik} \in \{0, 1\}$  subject to (1) and (2).

A more advanced model is to replace (2) by introducing variables  $y_{ijk} = 0, 1$  subject to

$$x_{ik} + x_{jk} \leq 1 + y_{ijk}.$$

Then the minimum value of  $y_{ijk}$  is 0 if  $i, j$  aren't both assigned the colour  $k$ , but otherwise  $y_{ijk}$  must be 1; one can then seek to minimize

$$\sum_k \sum_{(i,j) \in P} y_{ijk} w_{ij}$$

where  $w_{ij} > 0$  is some measure of how bad it is to assign classes  $i$  and  $j$  to the same exam period (such as the number of common students).

An even more advanced model would be to consider which individual students are assigned to which classes and to try to prevent *exam hardships*, i.e., three exams in a 24 hour period.

Our data will be based on UBC requirements for the math B.A., from which we will generate some typical course schedules for students. We will then look at the number and schedule of the sections of each class. Time permitting, we will do this for other UBC degrees. It is likely that we will synthetically create larger data sets.

The simplest result we seek is the trade-off between the number of exam slots and the number of student conflicts, and if this trade-off has a threshold phenomenon. We will look at a number of data sets to see if our conclusions are robust and what principles can be drawn from our results.

Should time permit, we will impose some university policies that would change the results. For example, one could schedule individual exams for each section of a class. Another example is to require all students of one major (of one faculty, etc.) to take a specific section of a course.

Our research group consists of (1) JH, in charge of creating models and running optimization algorithms, (2) PM, in charge of harvesting online information, (3) AW, in charge of generating synthetic data sets and of coordinating and editing the final report.

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