Notes Starting Nov 7

- Branch and band

Nov 7, 2018

- Branch & Bound (e.g. drive UBC to SFU; bin packing)
- Presentations: Day 1: Nov 14
  Day 2: Nov 16
  3: 19
  4: 21
  5: 23
  6: 26
  7: 28
  8: 30

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Rest of November:

- Presentations
- Gurobi (a bit)

- Branch and bound
- " " cut
- Other LP/IP's in projects and applications

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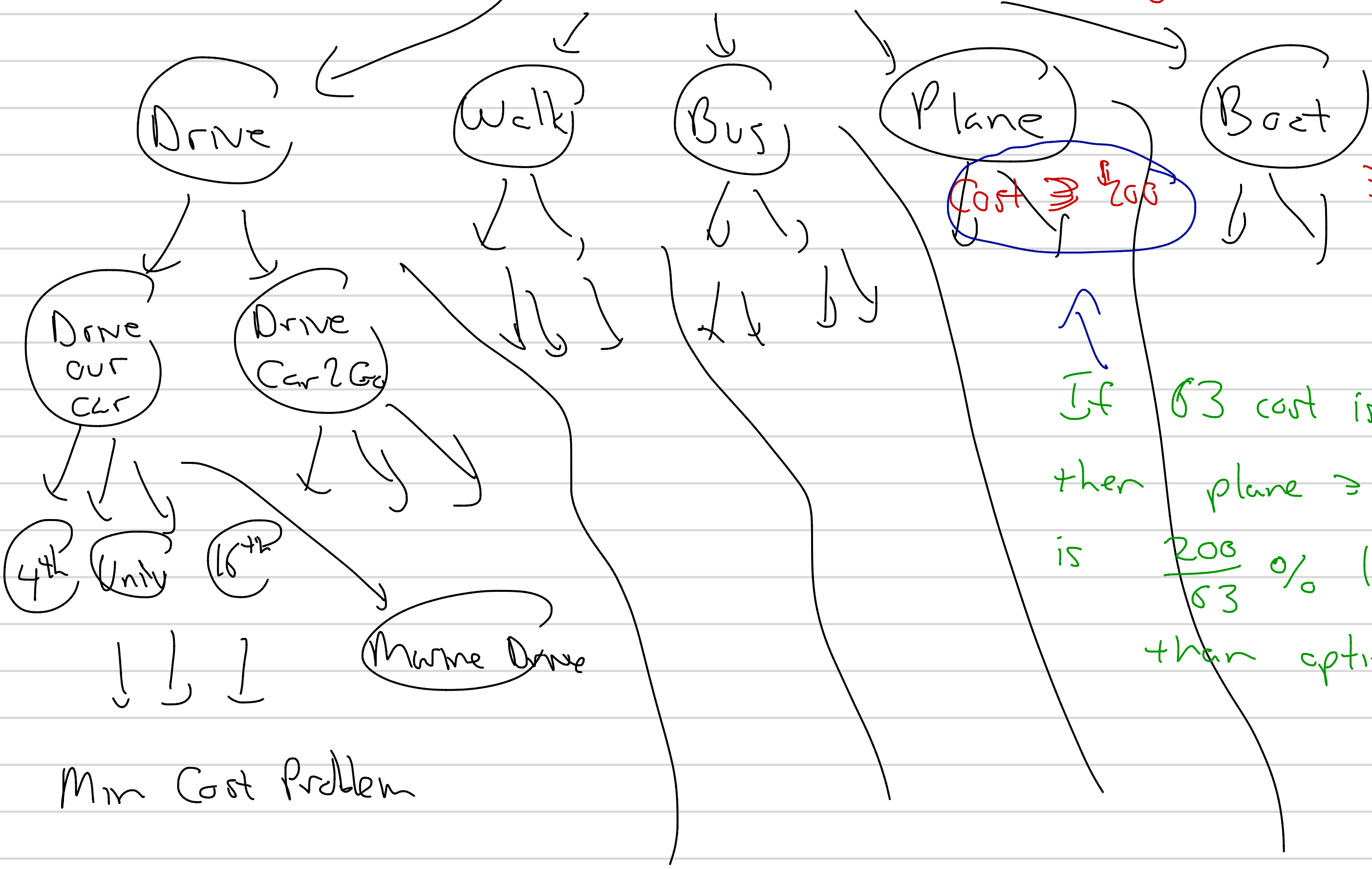
Branch & bound:

We want to go from UBC to ... $\boxed{\text{SFU}}$ Consider

or U Toronto

or ...

Our decisions?



If 63 cost is feasible, then plane ≥ 200 is $\frac{200}{63}$ % larger than optimal

Min Cost Problem

Objective: min $\underset{\uparrow \text{ min}}{(\text{time})} + \alpha \underset{\uparrow \text{ \$}}{(\text{cost})}$ fix $\alpha = 1$ $/min

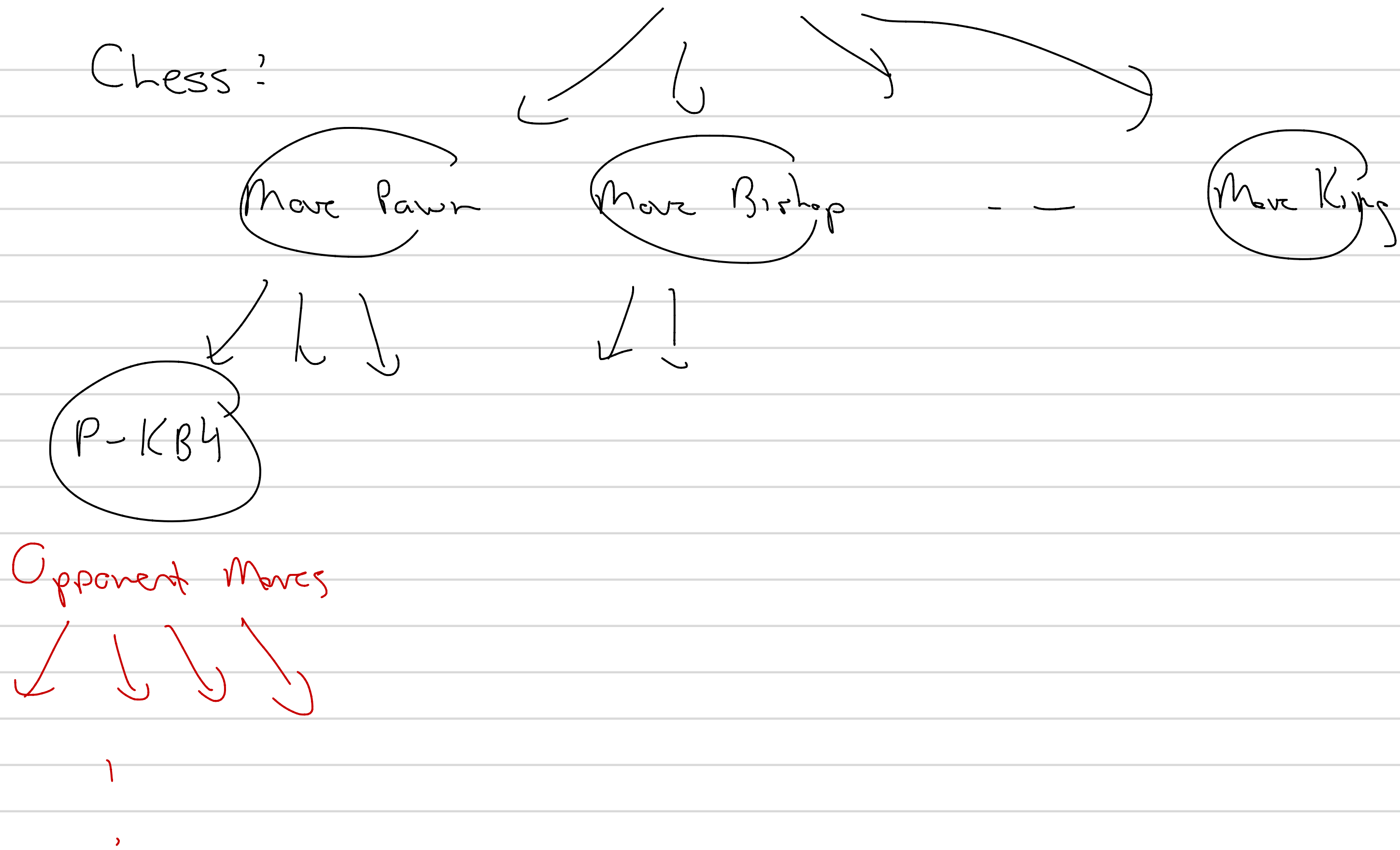
~~$3 + 60~~

① Find some feasible solution ⟷ You know the cost

② At as many nodes as possible, have some lower bound

63

Chess:

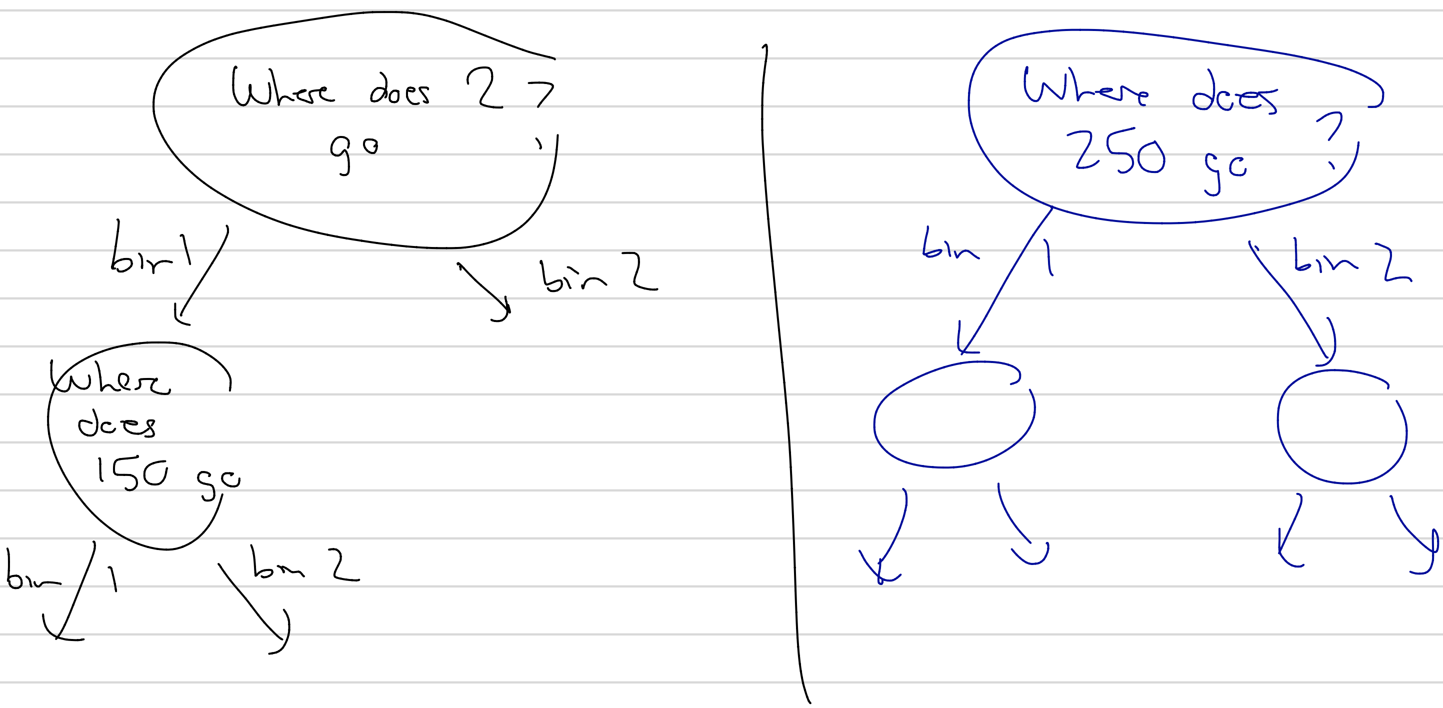


Bin Packing:

2 bins,

Items size 250, 150, 100, 100, 100, 100, 4, 3, 2 ...

2nd ... 1st



① Want to choose a reasonable decision tree ...

Now Q!

- Branch and bound — very general idea.
- Integer Programming: special case of branch & bound

"branch and cut"

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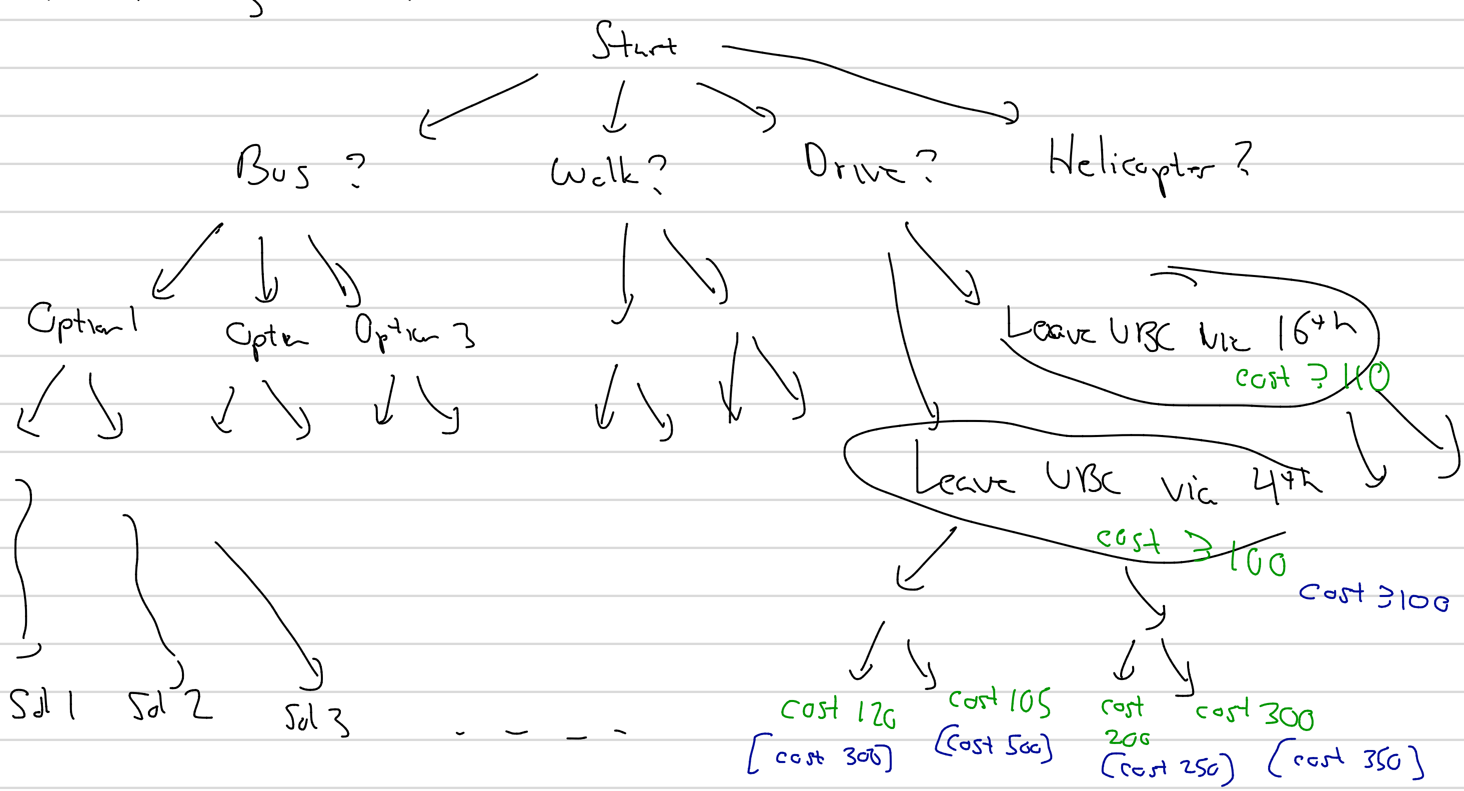
Branch & bound: how do I get from UBC to SFU minimizing cost:



Say there are too many solutions to check all of them out

Branch & Bound: (1) You decide how good a solution you want; maybe with 10% of optimal, 2%, etc.

(2) You should have some feasible solution

(3) You have some way of bounding enough nodes of the tree

Example:

Say you have IP:

Bin packing into 2 bins of equal size.

Problem: Given n items and their sizes

e.g. $n = 10$, size 150, 150, 100, 100, 100, 3, 2, 1, 1, 1

I want

$$x_i = \begin{cases} 1 & \text{if item } i \text{ goes into bin } \#1 \\ 0 & \text{" " " " " bin } \#2 \end{cases}$$

Sizes $s_1, \ldots, s_n$ given, want

First bin holds: $s_1 x_1 + s_2 x_2 + \ldots + s_n x_n \leq w$

2nd " " $s(1-x_1) + s_2(1-x_2) + \ldots + s_n(1-x_n) \leq w$

Minimize w.

Decision vars: $x_1, \ldots, x_n, w$ Given $n, s_1, \ldots, s_n$.

Another strategy: sort $s_1, \ldots, s_n$

So 150, 150, 100, 100, 100

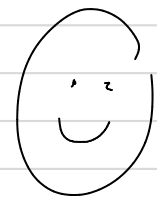
Bin 1: 150, 100, 100 ← size 350

Bin 2: 150, 100 ← size 250

Better:

Bin 1: 150, 150 ← size 300

Bin 2: 100, 100, 100 ←



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For $IP \overset{\text{relax}}{\rightsquigarrow} LP$

$\vec{x} \in \mathbb{Z}$ $\qquad$ $\vec{x} \in \mathbb{R}$

or

$x_i = 0, 1$ $\qquad$ $0 \leq x_i \leq 1$

Solving the LP version (relaxation) is much faster to do

This gives a bound on the IP

Example: min w s.t.

$$150x_1 + 150x_2 + 100x_3 + 100x_4 + 98x_5 \le w$$

$$150(1-x_1) + 150(1-x_2) + 100(1-x_3) + 100(1-x_4) + 100(1-x_5) \le w$$

s.t.

x_i are 0, 1 $\leftarrow$ $0 \le x_i \le 1$ and x_i integers

relax $\longrightarrow$ $\underbrace{0 \le x_i \le 1 \text{ but } x_i\text{'s can be real}}_{\text{could set } x_i = \frac{1}{2}}$

Nov 14: Branch & Bound:

In LP/IP this is called $\boxed{\text{Branch \& Cut}}$

$$\left.\begin{aligned} \max\ & 4x_1 + 5x_2 \\ \text{s.t.}\ & \\ & x_1 + 2x_2 \le 9.6 \\ & x_1 + x_2 \le 5.5 \\ & 2x_1 + x_2 \le 8 \\ & x_1, x_2 \ge 0 \end{aligned}\right\}$$

How does branch and cut work?

$x_1, x_2 \in \mathbb{Z}$ ← IP

Relax $x_1, x_2 \in \mathbb{R}$, $\boxed{x^* = (1.4, 4.1)}$

What to do?

Maybe $x_2 = 4$?

IP

$x_2 \le 4$ → Run LP add $x_2 \le 4$

$x_2 \ge 5$ → Run LP add $x_2 \ge 5$ ← Infeasible

Opt: $x_2 = 4$, $x_1 = 1.5$ [obj = 26]

$x_1 \le 1$ → $\boxed{\text{Obj } 24 \quad x_1 = 1,\ x_2 = 4}$ feasible obj 24

$x_1 \ge 2$ → Obj 25.5, $x_1 = 2$, $x_2 = 3.5$

Now

$x_2 \le 3$

$x_2 \ge 4$, i.e. $x_2 = 4$

Nov 19:

What do branch and cut nodes mean?

(1) In general, hard to tell... There can be good news.

(2) There are 2 specific cases that are easy to understand

- bin packing
- graph colouring

Bin Packing

Graph Colouring

⋮

Int Prog: Decision vars $x_{ij} \in 0,1$

Node in Branch and Cut

$x_{11}=1,\ x_{12}=0$

$x_{31}=0,\ x_{32}=1$

$x_{12,1}=1,\ x_{12,2}=0$

Fixed

$x_{11}+x_{12}=1$

For IP, $x_{ij} = 0,1$ what's happening?

???

??

LP relaxation: $0 \le x_{ij} \le 1$

continuous, (real, $\mathbb{R}$)

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Bin Packing: 100 total | total 40 | 12, 6, 1, 1 ← sums to 20

Bin 1 | Bin 2 | Undecided

Fixed | Solution

100 | 40 + 20

Say

100 | 90 | 12, 6, 1, 1 total 20

Bin 1 | Bin 2 | undecided

Relation: 105 | 105 | 5 into bin 1

Graph colouring

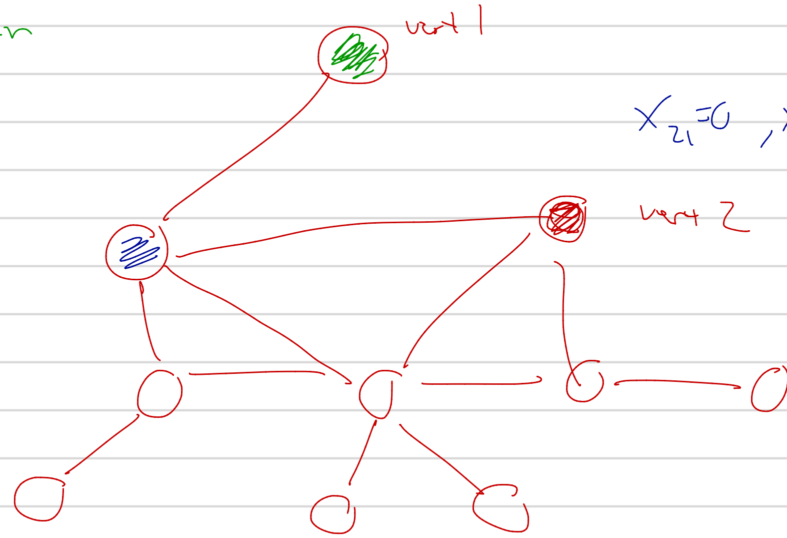
Node: 3 colours $\quad X_{i1} + X_{i2} + X_{i3} = 1$

for all i

$X_{11} = 1$, $X_{12} = 0$, $X_{13} = 0$

1st color green

We know:



$X_{21} = 0$, $X_{22} = 1$, $X_{23} = 0$