HOMEWORK #3, MATH 441, FALL 2018

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Please note:

(1) You may work together on homework, but you must write your own code/software; you must write up your own solutions individually.
(2) You must acknowledge with whom you worked. You must also acknowledge any sources you have used beyond the textbook and class material.
(3) In all these problems you must **justify your answer** unless the problem states otherwise; you will not be given any credit for stating the correct answer without a **written justification** that your answer is correct.
(4) Submit the entire homework as a single PDF file to `canvas.ubc.ca`.

Homework Problems

Class presentations will take place during the last three weeks of November, during class; each day will have at most three time slots, and each group must present once. (So the first possible day is Nov 12, the second is Nov 14, etc., and the ninth day is Nov 30.) Each group will submit a preference, and we will form a utility function as described in class: for example, if group number 12 expresses the preferences 456123987, then

(1) day 4 (Nov 19) is their first choice, day 5 (Nov 21) their second choice, and day 7 (Nov 26) is their last choice, and
(2) we add the utility term of group 12

$$9x_{12,4} + 8x_{12,5} + 7x_{12,6} + 6x_{12,1} + 5x_{12,2} + 4x_{12,3} + 3x_{12,9} + 2x_{12,8} + x_{12,7}$$

to the utility function.

(In general $x_{i,j}$ is the utility of day j to group i, based on the declared preferences of group i.) Assume that there are 18 groups and answer the following questions.

(1) Say that each group has the same preferences, say 123456789. Can each group get its first choice in an optimal schedule? Could some group get its last choice in an optimal schedule? Is there a unique optimal schedule in this case?

Research supported in part by an NSERC grant.

(2) Under any set of preferences, can a group be assigned its last choice in an optimal schedule?
(3) Under any set of preferences and in any optimal schedule, is it possible that groups 1 and 2 would rather switch days? (In other words, group 1 prefers the day assigned to 2 than the day assigned to them, and group 2 prefers the day assigned to group 1 than the day assigned to them.)
(4) Say that we multiply the utility term of group 1 by a real number α with $\alpha > 1000$ (and don't change the other groups' utility terms). Describe how this changes the optimal solution.
(5) Same question for $0 < \alpha < 1/1000$.
(6) Same question for $\alpha = -1$.

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