

HOMEWORK #1, MATH 441, FALL 2018

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Please note:

- (1) You may work together on homework, but you must write your own code/software; you must write up your own solutions individually.
- (2) You must acknowledge with whom you worked. You must also acknowledge any sources you have used beyond the textbook and class material.
- (3) In all these problems you must **justify your answer** unless the problem states otherwise; you will not be given any credit for stating the correct answer without a **written justification** that your answer is correct.
- (4) Submit the entire homework as a single PDF file to `canvas.ubc.ca`.

Homework Problems.

- (1) Say that you allow yourself at most 12 hours of TV per day, which consists of
 - (a) x_1 hours of “The Expanse” reruns,
 - (b) x_2 hours of “The X-Files” reruns,
 - (c) x_3 hours of “The Walking Dead” reruns,
 - (d) x_4 hours of other documentary programs,which gives you a utility of

$$U(\mathbf{x}) = 10x_1 + 9x_2 + 5x_3 + 2x_4 .$$

In addition, you are allowed to watch at most 2.3 hours of The Expanse, and at most 3.9 hours of The X-Files.

- (a) Write an LP that finds the TV viewing that maximizes your utility given the the above constraints; no justification is needed for this part.
- (b) What is the optimal solution of the above optimization problem? **Justify your answer.**
- (c) Use Gurobi (or some other program) to solve this LP. Provide a print-out of your code/software (encoding this LP), and of the solution Gurobi provides. If you don’t use Gurobi, you have to indicate what your code/software is doing.
- (d) What is the optimal solution and objective (do not use scientific notation)?

- (e) Use Gurobi or your software to solve the same problem with the additional constraint that x_1, \dots, x_4 have to be integers.
- (2) Consider the following variant of Exercise 1: x_1, \dots, x_4 are the same and satisfy the same constraints, but your utility function involves a parameter $B \in \mathbb{R}$ and is given as

$$U(\mathbf{x}) = 10x_1 + 9x_2 + 5x_3 + 2x_4 + B(12 - x_1 - x_2 - x_3 - x_4),$$

[which reflects an additional utility of B per each hour of TV that you don't watch]. Describe the optimal solutions for all values of $B \in \mathbb{R}$, and **justify your answer** (explain this in words; your justification can appeal to the simplex method, but this is not recommended or required). [Hint: The optimal solution changes as B passes through certain values, such as $B = 10$: the optimal solution for $B \geq 10$ is different than that for $10 \geq B \geq 9$.]

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