Math 441: Outline, Starting September 6, 2017

Joel Friedman

University of British Columbia
www.math.ubc.ca/∼jf

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Course Overview

Course summary:

- Prerequisite: Math 340.
- New material (20% of the grade): new algorithms and modelling.
- Projects (80% of grade): preliminary reports (20%), presentation (10%), final project (50%).

Schedule:

- September: Info needed to design a project.
- October and November: Further remarks on algorithms and modelling; presentations.

Optimization

Course prerequisite is Math 340, one term of linear programming.

- Linear Programing (LP): maximize a linear function of some real variables subject to linear constraints.
- Standard form (sometimes called something else): maximize $\vec{c}^{\mathrm{T}}\vec{x}$ (sometimes $\vec{c}\cdot\vec{x}$) subject to $A\vec{x} \le \vec{b}$ and $\vec{x} \ge \vec{0}$.
- New: Integer Linear Programming (ILP): same as linear programming, but we insist that $\vec{x} \in \mathbb{Z}^n$, i.e., $\vec{x}$ has integer components.
- New: Mixed Integer Programming (MIP): some variables must have integer components, others variables are real valued.
- New: Comibinatorial Optimization: Umbrella term for a number of related optimization problems, usually involving integeral variables (encompassing ILP, often using LP's).

Vague Project Ideas

- What happens to exam scheduling when we break up large classes into individual sections?
- What happens when we schedule employees for longer shifts?
- In an election, some voters prefer Ronald, some Hillary. What happens when Ronald get to draw the voting district maps?
- What happens when one grocery store increases its wage for a certain job?
- What happens to dairy prices and production in Canada if NAFTA is renegotiated?

"What happens" hopefully turns into a number of questions you could ask, some easy ones, some more difficult ones.
Some models can completely describe a situation; some models try to conclude principles from a complicated situation.

Rough Description of a Project

- Model(s). Precise formulation. Optimization should involve LP's on some level.
- "What happens" should be made precise; hopefully there are many questions that involve solving a number of LP's, ILP's, etc. [Even if your model is a very rough approximation, you may be able to infer some interesting principles.]
- Data: precise data or realistic synthetic data. Organize data for input to optimization software.
- Run one or more results.
- Interpret results by hand or curve fitting.

LP Example (See Oct 30, 2015 notes)

This is our first example. We will cover a number of aspects of this LP. Idea:

$$\max\ 4x_1 + 5x_2, \quad \text{s.t. } x_1 + 2x_2 \le 8, \quad x_1 + x_2 \le 5,$$

$$2x_1 + x_2 \le 8, \quad x_1, x_2 \ge 0.$$

Interpretation: making x_1 lattés, and x_2 espresso macchiatos. Latté: 1 unit stomach acidity tolerance, 1 unit coffee, 2 units milk. Macchiato: 2 unit SAT, 1 coffee, 1 milk. Constraints: SAT ≤ 8, Coffee ≤ 5, Milk ≤ 8.

Setup: Either max $z = \vec{c} \cdot \vec{x}$, s.t. $A\vec{x} \le \vec{b}$, $\vec{x} \ge \vec{0}$, or write out.

Introduce slack variables: $\vec{x}_{\text{slack}} = \vec{b} - A\vec{x}_{\text{dec}}$. Get BFS.

Properties: All A entries non-negative, $\vec{b}, \vec{c} \ge 0$, typical in some pure "resource" constraint problems. Implies that $\vec{x} = 0$ is always feasible, and other special and helpful properties.

Before running simplex: Claim $z = 23$, $\vec{x} = (2, 3)$ is optimal. Proof: Use dual. Then setup simplex method.